Example Candidate Responses

Cambridge O Level

Cambridge O Level Additional Mathematics

4037

Paper 1

Cambridge Secondary 2

CAMBRIDGE International Examinations

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Version 1.1

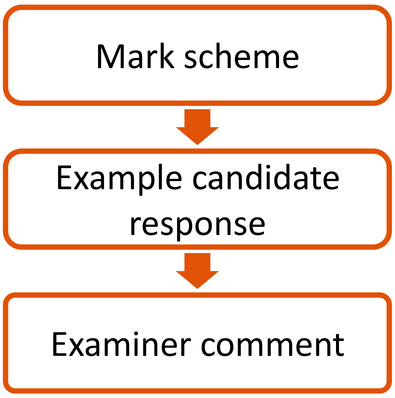
Contents

Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge O Level Additional Mathematics (4037), and to show how different levels of candidates' performance relate to the subject's curriculum and assessment objectives.

In this booklet candidate responses have been chosen to exemplify a range of answers. Each response is accompanied by a brief commentary explaining the strengths and weaknesses of the answers.

For ease of reference the following format for each component has been adopted:



For each question an extract from the mark scheme, as used by examiners, is followed by examples of marked candidate responses, each with an examiner comment on performance. Comments are given to indicate where and why marks were awarded, and how additional marks could have been obtained. In this way, it is possible to understand what candidates have done to gain their marks and what they still have to do to improve them.

This document illustrates the standard of candidate work for those parts of the assessment which help teachers assess what is required to achieve marks beyond what should be clear from the mark scheme.

Past papers, Examiner Reports and other teacher support materials are available on Teacher Support at **http://teachers.cie.org.uk**

Assessment at a glance

All candidates will take two written papers.

The syllabus content will be assessed by Paper 1 and Paper 2.

Paper 1	Duration	Marks
10–12 questions of various lengths There will be no choice of question.	2 hours	80

Paper 2	Duration	Marks
10–12 questions of various lengths There will be no choice of question.	2 hours	80

Teachers are reminded that the latest syllabus is available on our public website at **www.cie.org.uk** and Teacher Support at **http://teachers.cie.org.uk**

Paper 1

Question 1

Mark scheme

1	$k^2 - 4(2k+5) \quad (<0)$ $k^2 - 8k - 20 \quad (<0)$ $(k-10)(k+2) \quad (<0)$ critical values of 10 and –2 $-2 < k < 10$	M1 M1 A1 A1	use of $b^2 - 4ac$, (not as part of quadratic formula unless isolated at a later stage) with correct values for *a*, *b* and *c* Do not need to see < at this point attempt to obtain critical values correct critical values correct range
	Alternative 1: $\frac{\mathrm{d}y}{\mathrm{d}x} = 2(2k+5)x + k$ When $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, $x = \frac{-k}{2(2k+5)}$, $y = \frac{8k+20-k^2}{4(2k+5)}$ When $y = 0$, obtain critical values of 10 and –2 $-2 < k < 10$	M1 M1 A1 A1	attempt to differentiate, equate to zero and substitute *x* value back in to obtain a *y* value consider $y = 0$ in order to obtain critical values correct critical values correct range
	Alternative 2: $y = (2k+5)\left(\left(x + \frac{k}{2(2k+5)}\right)^2 - \frac{k^2}{4(2k+5)}\right) + 1$ Looking at $1 - \frac{k^2}{4(2k+5)} = 0$ leads to critical values of 10 and –2 $-2 < k < 10$	M1 M1 A1 A1	attempt to complete the square and consider '$1 - \frac{k^2}{4(2k+5)}$' attempt to solve above = to 0, to obtain critical values correct critical values correct range

Example candidate response – high

1 Given that the graph of $y = (2k+5)x^2 + kx + 1$ does not meet the x-axis, find the possible values of k. [4]

$y = (2k+5)x^2 + kx + 1$

$b^2 - 4ac < 0$

$a = 2k+5$

$b = k$

$c = 1$

$\cancel{(2k+5)^2 - 4(2k+5)(1) <}$

$(k)^2 - 4(2k+5)(1) < 0$

$k^2 - 8k - 20 < 0$

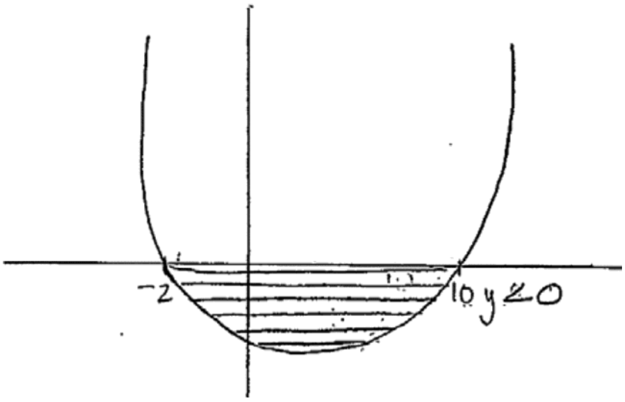
$k^2 - 10k + 2k - 20 < 0$

$k(k-10) + 2(k-10) < 0$

$(k+2)(k-10) < 0$

$k > 2$ and $k < 10$

$\boxed{2 < k < 10}$ Ans.



Total marks awarded = 3 out of 4

Examiner comment – high

The candidate produced a well thought-out solution, showing clearly the use of the discriminant of the quadratic equation which had been obtained by substituting in the correct values in the correct place. A clear attempt to solve the quadratic equation was made, with the correct critical values being shown on the sketch. The first three marks (M1, M1, A1) were obtained. A missing sign in the final answer meant that the candidate was unable to gain the last accuracy mark.

Example candidate response – middle

1 Given that the graph of $y = (2k+5)x^2 + kx + 1$ does not meet the x-axis, find the possible values of k. [4]

$y = ax^2 + bx + c$

$y = (2k+5)x^2 + kx + 1$

$\Rightarrow a = 2k+5,\ b = k,\ c = 1$

Since graph does not meet the x-axis

$b^2 - 4ac < 0$

$(k)^2 - 4(2k+5)(1) < 0$

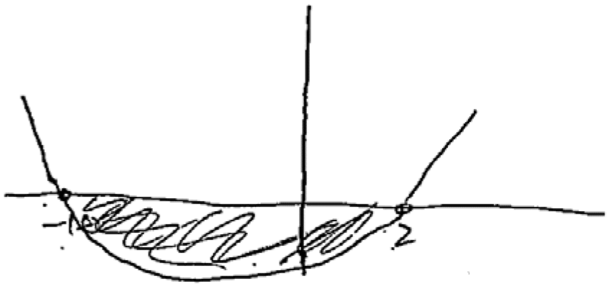
$k^2 - 8k - 20 < 0$

$k^2 + 10k - 2k - 20 < 0$

$k(k+10) - 2(k+10) < 0$

$(k+10)(k-2) < 0$

$k = 2,\ k = -10$



$k > -10 \quad k < 2$

$-10 < k < 2$

Total marks awarded = 2 out of 4

Examiner comment – middle

The candidate correctly made use of the discriminant of the quadratic equation together with correct substitution of correct values (M1). An attempt to solve the resulting quadratic equation resulted in the candidate gaining the second method mark (M1), but incorrect critical values were obtained from the solution meaning that the candidate was unable to gain the last 2 accuracy marks (A0, A0).

Example candidate response – low

1 Given that the graph of $y = (2k+5)x^2 + kx + 1$ does not meet the x-axis, find the possible values of k. [4]

$$y = (2k+5)x^2 + (k)x + 1$$

$$b^2 - 4ac < 0$$

$$k^2 - 4(2k+1)(1) < 0$$

$$k^2 - 8k - 4 < 0$$

$$k < 8.472 \quad \text{or} \quad k < -0.472$$

$$k < 8.47 \quad \text{or} \quad k < -0.472$$

Total marks awarded = 0 out of 4

Examiner comment – low

An attempt to use the discriminant from the quadratic equation was made, however an incorrect substitution was made so the candidate was unable to obtain the first method mark (M0). An attempt to solve the resulting incorrect quadratic was made, but because the candidate did not show where the values obtained came from (it was assumed that the candidate had made use of the polynomial equation solver on a calculator) e.g. use of factorisation if possible or use of the quadratic formula, no method mark could be awarded as no method was evident (M0). This meant that no accuracy marks could subsequently be awarded (A0, A0).

Question 2

Mark scheme

2	$\dfrac{\tan\theta+\cot\theta}{\operatorname{cosec}\theta}=\dfrac{\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}}$	M1	for $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$, $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$ and $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$; allow when used
	$=\dfrac{\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}{\dfrac{1}{\sin\theta}}$	M1	dealing correctly with fractions in the numerator; allow when seen
	$=\dfrac{1}{\cos\theta}$	M1	use of the appropriate identity; allow when seen
	$=\sec\theta$	A1	must be convinced it is from completely correct work (beware missing brackets)
	Alternative:		
	$\dfrac{\tan\theta+\cot\theta}{\operatorname{cosec}\theta}=\dfrac{\dfrac{\tan^2\theta+1}{\tan\theta}}{\operatorname{cosec}\theta}$	M1	for either $\tan\theta=\dfrac{1}{\cot\theta}$ or $\cot\theta=\dfrac{1}{\tan\theta}$ and $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$; allow when used
	$=\dfrac{\sec^2\theta}{\tan\theta\dfrac{1}{\sin\theta}}$	M1	dealing correctly with fractions in numerator; allow when seen
	$=\dfrac{\sec^2\theta}{\sec\theta}$	M1	use of the appropriate identity; allow when seen
	$=\sec\theta$	A1	must be convinced it is from completely correct work

Example candidate response – high

2 Show that $\dfrac{\tan\theta + \cot\theta}{\operatorname{cosec}\theta} = \sec\theta$. [4]

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \div \frac{1}{\sin\theta}$$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \times \sin\theta$$

$$\frac{\sin^2\theta + \cos^2\theta}{(\cos\theta)(\sin\theta)} \times \sin\theta$$

$$\frac{1\,(\cancel{\sin\theta})}{(\cos\theta)(\cancel{\sin\theta})}$$

$$\frac{1}{\cos\theta} = \sec\theta \quad \text{- shown.}$$

Total marks awarded = 3 out of 4

Examiner comment – high

The candidate worked through the proof in a correct fashion, making use of the correct trigonometric relationships (M1), dealing with fractions correctly (M1) and making use of the appropriate trigonometric identity (M1). The final accuracy mark was withheld as the candidate had failed to make a correct use of brackets (none were used in this case) in the first two lines of the solution (A0).

Example candidate response – middle

2 Show that $\dfrac{\tan\theta + \cot\theta}{\operatorname{cosec}\theta} = \sec\theta$. [4]

$$\text{L.H.S} \Rightarrow \frac{\tan\theta + \cot\theta}{\operatorname{cosec}\theta} = 0$$

$$\Rightarrow \frac{\tan\theta + \frac{1}{\tan\theta}}{1/\sin\theta} = 0$$

$$\Rightarrow \frac{\tan^2\theta + 1}{1/\sin\theta} = 0$$

$$\Rightarrow \frac{\sec^2\theta}{\sec\theta} = 0$$

$$\Rightarrow \sec\theta \qquad [\text{shown}]$$

Total marks awarded = 2 out of 4

Examiner comment – middle

The candidate erroneously equated the left hand side of the given expression to zero, but this was condoned. The candidate made use of the correct trigonometric relationships mentioned in the alternative mark scheme (M1), but did not deal with fractions correctly, the denominator being omitted from the resulting fraction (M0). However, a correct use of a correct identity meant that the candidate was able to gain another method mark (M1) but was then unable to obtain the final accuracy mark (A0).

Example candidate response – low

2 Show that $\dfrac{\tan\theta + \cot\theta}{\operatorname{cosec}\theta} = \sec\theta$. [4]

$$\frac{\tan\theta + \cot\theta}{1 + \cot^2\theta}$$

$$\tan\theta \div 1 + \cot^2\theta + \frac{\cancel{\cot}\,\theta}{1 + \cancel{\cot}\,\theta}$$

$$\tan\theta \times \frac{1 + \tan\theta}{1}$$

$$1 + \tan^2\theta$$

$$= \sec\theta$$

Total marks awarded = 0 out of 4

Examiner comment – low

An attempt to split the left hand side of the expression was made. The correct identity appeared to have been used initially but the index number 2 was omitted from the term involving cosec *x* seemed to have been crossed out. Some incorrect cancelling of terms resulted in a completely incorrect result. It was felt that there was no work of any merit (M0, M0, M0, A0).

Question 3

Mark scheme

3	$\mathbf{A}^{-1} = \frac{1}{2}\begin{pmatrix} 3 & -2 \\ -5 & 4 \end{pmatrix}$	B1 B1	$\frac{1}{2}$ multiplied by a matrix for matrix
	$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & -2 \\ -5 & 4 \end{pmatrix}\begin{pmatrix} 8 \\ 9 \end{pmatrix}$	M1	attempt to use the inverse matrix, must be pre-multiplication
	$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 6 \\ -4 \end{pmatrix}$		
	$x = 3, y = -2$	A1, A1	

Example candidate response – high

3 Find the inverse of the matrix $\begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}$ and hence solve the simultaneous equations

$$4x + 2y - 8 = 0,$$
$$5x + 3y - 9 = 0.$$

[5]

$$\frac{1}{(4\times3)-(2\times5)}\begin{pmatrix} 3 & -2 \\ -5 & 4 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 3 & -2 \\ -5 & 4 \end{pmatrix} = \begin{pmatrix} 3/2 & -1 \\ -5/2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3/2 & -1 \\ -5/2 & 2 \end{pmatrix}\begin{pmatrix} 8 \\ 9 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$\therefore x = 3$
$y = -2$ (Ans)

Total marks awarded = 5 out of 5

Examiner comment – high

The candidate successfully produced the inverse matrix as requested (B1, B1). This was then used in a correct manner i.e. using post multiplication by the inverse matrix to produce a fully correct solution (M1, A1, A1).

Example candidate response – middle

3 Find the inverse of the matrix $\begin{pmatrix}4 & 2\\5 & 3\end{pmatrix}$ and hence solve the simultaneous equations

$$4x + 2y - 8 = 0,$$
$$5x + 3y - 9 = 0.$$

[5]

$$\begin{pmatrix}4 & 2\\5 & 3\end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix}d & -b\\-c & a\end{pmatrix}$$

$$= \frac{1}{4\cdot 3 - 5\cdot 2}\begin{pmatrix}3 & -2\\-5 & 4\end{pmatrix}$$

$$= \frac{1}{12-10}\begin{pmatrix}3 & -2\\-5 & 4\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}3 & -2\\-5 & 4\end{pmatrix}$$

$$\begin{pmatrix}4 & 2\\5 & 3\end{pmatrix}^{-1}\begin{pmatrix}4 & 2\\5 & 3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}8\\9\end{pmatrix}\begin{pmatrix}4 & 2\\5 & 3\end{pmatrix}^{-1}$$

$$\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{2}\cancel{\begin{pmatrix}8\\9\end{pmatrix}}\begin{pmatrix}3 & -2\\-5 & 4\end{pmatrix}\begin{pmatrix}8\\9\end{pmatrix}$$

$$= \begin{pmatrix}3(8)-2(9)\\-5(8)+4(9)\end{pmatrix}$$

$$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}6\\-4\end{pmatrix}$$

Total marks awarded = 3 out of 5

Examiner comment – middle

The candidate successfully produced the inverse matrix as requested (B1, B1). This was then used in a correct manner i.e. using post multiplication by the inverse matrix, however the candidate did not take account of the $\frac{1}{2}$ from the inverse matrix and thus failed to obtain the correct solutions (M1, A0, A0).

Example candidate response – low

3 Find the inverse of the matrix $\begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}$ and hence solve the simultaneous equations

$$4x + 2y - 8 = 0,$$
$$5x + 3y - 9 = 0.$$

[5]

Inverse :- $= \frac{1}{(4\times3)-(5\times2)} \begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix}$

$= \frac{1}{2} \begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1.50 & -1 \\ -2.50 & 2 \end{bmatrix}$

$4x + 2y = 8$ — (i)
$5x + 3y = 9$ — (ii)

From equation (i), $x = \frac{8-2y}{4}$

Put in eq. (ii)

$5\left[\frac{8-2y}{4}\right] + 3y = 9$

$\therefore \boxed{y = -2}$

Put $y = -2$ in $x = \frac{8-2y}{4}$

$x = \frac{8-2(-2)}{4}$
$= \frac{12}{4} = 3$
$\boxed{x = 3}$

Total marks awarded = 2 out of 5

Examiner comment – low

The candidate successfully produced the inverse matrix as requested (B1, B1). The important word from the question is the word ‘hence’. This implies that the inverse matrix found needs to be used to help solve the simultaneous equations. In this case the candidate chose to use an alternative method and thus was unable to gain any credit (M0, A0, A0).

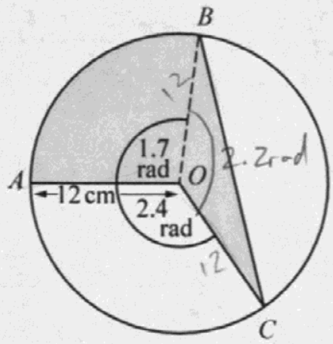
Question 4

Mark scheme

4 **(i)**	Area = $\left(\frac{1}{2}\times 12^2\times 1.7\right)+\left(\frac{1}{2}\times 12^2\sin(2\pi-1.7-2.4)\right)$	B1,B1 M1	B1 for sector area, allow unsimplified B1 for correct angle *BOC*, allow unsimplified correct attempt at area of triangle, allow unsimplified using *their* angle *BOC* (Their angle *BOC* must not be 1.7 or 2.4)
	= awrt 181	A1	
(ii)	$BC^2 = 12^2+12^2-(2\times 12\times 12\cos 2.1832)$ or $BC = 2\times 12\times\sin\left(\frac{2\pi-4.1}{2}\right)$	M1	correct attempt at *BC*, may be seen in **(i)**, allow if used in **(ii)**. Allow use of *their* angle *BOC*.
	$BC = 21.296$	A1	
	Perimeter = $(12\times 1.7)+12+12+21.296$	B1 M1	for arc length, allow unsimplified for a correct 'plan' (an arc + 2 radii and *BC*)
	= 65.7	A1	

Example candidate response – high

4



The diagram shows a circle, centre O, radius 12 cm. The points A, B and C lie on the circumference of this circle such that angle AOB is 1.7 radians and angle AOC is 2.4 radians.

(i) Find the area of the shaded region. [4]

$A_{\text{sector}} = \frac{1}{2} r^2 \theta$

$A_{\triangle OBC} = \frac{1}{2}(12\times 12)(\sin 2.2)$

$A_{\text{shaded}} :- \left(\frac{1}{2}(12)^2(1.7)\right) + \left(\frac{1}{2}(144 \times \sin 2.18)\right)$

$A_{\text{shaded}} = 122.4 + 59.047$

$\text{Area} = 181.4 = \underline{181\ \text{cm}^2}\ (3s.f.)$

(ii) Find the perimeter of the shaded region. [5]

Arc length $= r\theta$

Arc length $= 12 \times 1.7 = 20.4$

$20.4 + AO + OC + BC$

$\underline{20.4 + 12 + 12 + BC}$

$BC^2 = 12^2 + 12^2 - 2(12 \times 12 \times \cos 2.2\text{rad})$

$BC^2 = 288 + 169.49$

$BC = \sqrt{288 + 169.49} = \boxed{21.4}$

$20.4 + 12 + 12 + 21.4 = \text{Perimeter}_{\text{shaded}}$

$\text{Perimeter}_{\text{shaded}} = \underline{65.8\ \text{cm}}$

Examiner comment – high

(i) The candidate correctly found the angle *BOC*, and was given credit for 2.2 radians even though this was not correct to 3 significant figures (B1). The area of the triangle *BOC* and the sector *AOB* were found correctly and an answer rounding to 181 was obtained (M1, B1, A1).

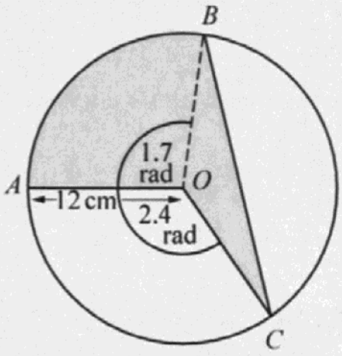
(ii) The arc length *AB* was found correctly (B1). The candidate made use of the cosine rule to obtain a correct value for the length of *BC*, and then used a correct strategy to obtain the required perimeter. At this point, the lack of accuracy in the use of 2.2 radians was penalised by withholding the final accuracy mark (M1, A1, M1, A0).

Marks awarded = **(i) 4/4, (ii) 4/5**

Total marks awarded = 8 out of 9

Example candidate response – middle

4



The diagram shows a circle, centre O, radius 12 cm. The points A, B and C lie on the circumference of this circle such that angle AOB is 1.7 radians and angle AOC is 2.4 radians.

(i) Find the area of the shaded region. [4]

~~Area~~ $\frac{1}{2}(12)^2 \times 1.7$ ~~rad~~ $+ \frac{1}{2} \times 12 \times 12 \times \sin(2\pi - 1.7 - 2.4)$

$= 122.4 + \cancel{2.74}\ 58.9$

~~$= 125\ \text{cm}^2$~~

$= 181.3\ \text{cm}^2$

(ii) Find the perimeter of the shaded region. [5]

$s = r\theta$

$= 12 \times 1.7$

$= 20.4$

$BC^2 = 12^2 + 12^2 - 2(12)(12)\sin(2\pi - 1.7 - 2.4)$

$= 52.3$

$\therefore BC = 7.23\ \text{cm}$

$\therefore \text{Perimeter} = 20.4\ \text{cm} + 7.23\ \text{cm} + 12 + 12$

~~$= 27.6\ \text{cm}$~~

$= 51.63\ \text{cm}$

Examiner comment – middle

(i) The candidate correctly found the angle *BOC*, never showing the exact value used (B1). The area of the triangle *BOC* and the sector *AOB* were found correctly and an answer rounding to 181 was obtained (M1, B1, A1).

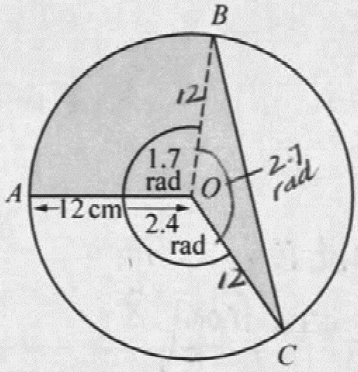
(ii) The arc length *AB* was found correctly (B1). The candidate appeared to attempt to use the cosine rule to obtain a value for the length of *BC*, but used sin *BOC* rather than cos *BOC* (M0, A0). A correct plan gained the candidate further credit (M1), but the last accuracy mark was not available (A0).

Marks awarded = **(i) 4/4, (ii) 2/5**

Total marks awarded = 6 out of 9

Example candidate response – low

4



The diagram shows a circle, centre O, radius 12 cm. The points A, B and C lie on the circumference of this circle such that angle AOB is 1.7 radians and angle AOC is 2.4 radians.

(i) Find the area of the shaded region. [4]

Shaded region area = $\left[\frac{1}{2} \times 12^2 \times 1.7\right] + \left[\frac{1}{2} \times 12 \times 12 \times \sin 2.7\,\text{rad}\right]$

$= 122.40 + 30.77$

$= 15\cancel{2}3.17$

$\boxed{= 153\,\text{cm}^2}$

(ii) Find the perimeter of the shaded region. [5]

$BC = \sqrt{12^2 + 12^2 - 2(12)(12)(\cos 2.7)}$

$BC = 23.4\,\text{cm}$

$OC = 12$

$AO = 12$

$AB = r\theta = 12 \times 1.7 = 20.4$

Total Perimeter of Shaded region $= 23.4 + 20.4 + 12 + 12$

$\boxed{= 67.80\,\text{cm}}$

Examiner comment – low

(i) A correct sector area was found (B1), however the candidate did not calculate the magnitude of angle *BOC* correctly, but made use of their incorrect angle correctly to attempt to find the area of the triangle *BOC* (B0, M1). As a result the final answer was incorrect (A0).

(ii) A correct use of the cosine rule using the candidate's incorrect angle *BOC* gained a method mark for the attempt to find the length *BC* (M1, A0). A correct arc length for *AC* was found (B1) and a correct plan was made use of (M1). The candidate was unable to gain the final accuracy mark (A0).

Marks awarded = **(i) 2/4, (ii) 3/5**

Total marks awarded = 5 out of 9

Question 5

Mark scheme

5 (a) (i)	20160	B1	
(ii)	$3 \times {}^6P_4 \times 2$ $= 2160$	B1,B1	B1 for ${}^6P_4$ (must be seen in a product) B1 for all correct, with no further working
(iii)	$5 \times 2 \times {}^6P_4$ $= 3600$	B1,B1 B1	B1 for ${}^6P_4$ (must be seen in a product) B1 for 5 (must be in a product) B1 for all correct, with no further working
	Alternative 1: ${}^6C_4 \times 5! \times 2$ $= 3600$	B2 B1	for ${}^6C_4 \times 5!$ for ${}^6C_4 \times 5! \times 2$
	Alternative 2: $\left({}^7P_5 - {}^6P_5\right) \times 2$ $= 3600$	B2 B1	for $\left({}^7P_5 - {}^6P_5\right)$ for $\left({}^7P_5 - {}^6P_5\right) \times 2$
	Alternative 3: $2!\left({}^6P_4 + \left({}^6P_1 \times {}^5P_3\right) + \left({}^6P_2 \times {}^4P_2\right) + \left({}^6P_3 \times {}^3P_1\right) + {}^6P_4\right)$ $= 3600$	B2 B1	4 terms correct or omission of 2! in each term all correct

(b) (i)	${}^{14}C_4 \times {}^{10}C_4$ or ${}^{14}C_8 \times {}^8C_4$ (or numerical or factorial equivalent) $= 210210$	B1,B1	B1 for either ${}^{14}C_4$ or ${}^{14}C_8$ as part of a product B1 for correct answer, with no further working
(ii)	${}^8C_4 \times {}^6C_4$ $= 1050$	B1,B1	B1 for either ${}^8C_4$ or ${}^6C_4$ as part of a product B1 for correct answer with no further working

Example candidate response – high

5 **(a)** A security code is to be chosen using 6 of the following:

- the letters A, B and C
- the numbers 2, 3 and 5
- the symbols * and $.

None of the above may be used more than once. Find the number of different security codes that may be chosen if

(i) there are no restrictions, [1]

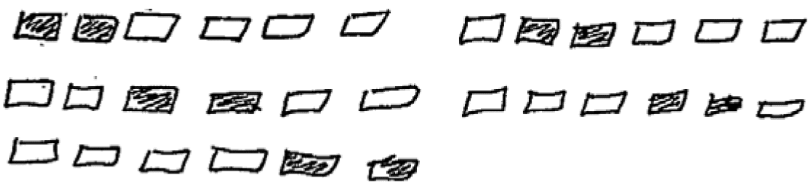
$^8C_6 = \boxed{28}$ Ans

(ii) the security code starts with a letter and finishes with a symbol, [2]

$\cancel{^3C_1 \times {}^6C_4 \times {}^2C_1 = 90}$

$^3P_1 \times {}^6P_4 \times {}^2P_1 = \boxed{2160}$ Am.

(iii) the two symbols are next to each other in the security code. [3]



$({}^2P_2 \times {}^6P_4) + ({}^6P_1 \times {}^2P_2 \times {}^5P_3) +$

$({}^6P_2 \times {}^2P_2 \times {}^4P_2) + ({}^6P_3 \times {}^2P_2 \times {}^3P_1) +$

$({}^6P_4 \times {}^2P_2) = \boxed{3600}$ Am

Example candidate response – high, continued

(b) Two teams, each of 4 students, are to be selected from a class of 8 boys and 6 girls. Find the number of different ways the two teams may be selected if

(i) there are no restrictions, [2]

$$\left({}^{14}C_4 \times {}^{10}C_4\right) = \boxed{210210} \text{ Ans}$$

(ii) one team is to contain boys only and the other team is to contain girls only. [2]

$$\left({}^{8}C_4 \times {}^{6}C_4\right) = \boxed{1050} \text{ Ans}$$

Examiner comment – high

(a) (i) The candidate considered combinations rather than permutations (B0).

(ii) A fully correct response using the expected permutations was given (B1, B1).

(iii) A fully correct response using the expected permutations was given using the method for Alternative 3 (B2, B1).

Marks awarded = **(a) (i) 0/1, (a) (ii) 2/2, (a) (iii) 3 /3**

(b) (i) A fully correct response using the combinations expected was given (B1, B1).

(ii) A fully correct response using the combinations expected was given (B1, B1).

Marks awarded = **(b) (i) 2/2, (b) (ii) 2/2**

Total marks awarded = 9 out of 10

Example candidate response – middle

5 (a) A security code is to be chosen using 6 of the following:

- the letters A, B and C
- the numbers 2, 3 and 5
- the symbols * and $

None of the above may be used more than once. Find the number of different security codes that may be chosen if

(i) there are no restrictions, [1]

$^8C_6$ $\quad$ $^8P_6 = 20160$

(ii) the security code starts with a letter and finishes with a symbol, [2]

A B C $\qquad$ * \$

$3\left({}^1P_1 \times {}^1P_1 \times {}^6P_4\right)$

$3 \times 1 \times 1 \times 360 = 1080$

(iii) the two symbols are next to each other in the security code. [3]

${}^1P_1 \times {}^6P_4 =$

$1 \times 360 = 360$

Example candidate response – middle, continued

(b) Two teams, each of 4 students, are to be selected from a class of 8 boys and 6 girls. Find the number of different ways the two teams may be selected if

(i) there are no restrictions, [2]

$${}^{14}C_4 = 1001$$

(ii) one team is to contain boys only and the other team is to contain girls only. [2]

BOYS GIRLS

$${}^{8}C_4 + {}^{6}C_4$$

$$= 70 + 15$$

$$= 85$$

Examiner comment – middle

(a) (i) The candidate considered permutations to obtain the correct answer (B1).

(ii) The candidate realised that the solution involved the permutation of 4 items from 6 items but did not consider the correct multiples of this permutation (B1, B0).

(iii) The candidate again realised that the solution involved the permutation of 4 items from 6 items but again did not consider the correct multiples of this permutation (B1, B0, B0).

Marks awarded = **(a) (i) 1/1, (a) (ii) 1/2, (a) (iii) 1/3**

(b) (i) Only the first team was considered so the candidate was unable to gain any credit (B0, B0).

(ii) The correct combinations were found, but the candidate chose to add them together rather than multiply them together (B0, B0).

Marks awarded = **(b) (i) 0/2, (b) (ii) 0/2**

Total marks awarded = 3 out of 10

Example candidate response – low

5 **(a)** A security code is to be chosen using 6 of the following:

- the letters A, B and C – 3
- the numbers 2, 3 and 5 – 3
- the symbols * and $. – 2

None of the above may be used more than once. Find the number of different security codes that may be chosen if

(i) there are no restrictions, [1]

→ No restrictions.
→ 6 to be used from 8

$\therefore$ $\boxed{{}^{8}P_{6} = 20{,}160 \text{ different security codes.}}$

(ii) the security code starts with a letter and finishes with a symbol, [2]

Ans = $3! \times 4! \times 2!$

$\boxed{= ~~[illegible]~~ \; 288 \text{ different security codes.}}$

(iii) the two symbols are next to each other in the security code. [3]

Two Symbols can be next to eachother in 2! ways. The symbols can be placed together in 5! ways.

$\therefore$ Ans = $2! \times 5!$

$\boxed{\text{Ans} = 240}$

Example candidate response – low, continued

(b) Two teams, each of 4 students, are to be selected from a class of 8 boys and 6 girls. Find the number of different ways the two teams may be selected if

(i) there are no restrictions, [2]

$4 \times 2 = 8$ students to be selected from 14 students.

~~Hence, $2 \times {}^{14}P_{4} = 2 \times 24024$~~

~~$= 48{,}048$.~~

Hence, ${}^{14}C_{8} = \boxed{3003 \text{ combinations}}$

(ii) one team is to contain boys only and the other team is to contain girls only. [2]

Team 1 = 4 boys to be selected from 8 boys

Team 2 = 4 girls to be selected from 6 girls.

$$\therefore \text{Ans} = {}^{8}C_{4} + {}^{6}C_{4}$$

$$= 70 + 15$$

$$\boxed{= 85 \text{ ways}}$$

Examiner comment – low

(a) (i) The candidate considered permutations to obtain the correct answer (B1).

(iii) No evidence of the correct use of permutations was evident (B0, B0).

(iv) No evidence of the correct use of permutations or a correct approach was seen (B0, B0).

Marks awarded = **(a) (i) 1/1, (a) (ii) 0/2, (a) (iii) 0 /3**

(b) (i) The candidate considered the number of ways 8 people could be chosen from 14 which was an incorrect approach (B0, B0).

(ii) The correct combinations were found, but the candidate chose to add them together rather than multiply them together (B0, B0).

Marks awarded = **(b) (i) 0/2, (b) (ii) 0/2**

Total marks awarded = 1 out of 10

Question 6

Mark scheme

6 (i)	10ln4 or 13.9 or better	B1	
(ii)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}=\right)\frac{20t}{t^2+4}-4$	M1 B1	attempt to differentiate and equate to zero $\frac{20t}{t^2+4}$ or equivalent seen
	When $\frac{\mathrm{d}x}{\mathrm{d}t}=0,\ \frac{20t}{t^2+4}=4$	DM1	attempt to solve *their* $\frac{\mathrm{d}x}{\mathrm{d}t}=0$, must be a 2 or 3 term quadratic equation with real roots
	leading to $t^2-5t+4=0$ $t=1, t=4$	A1	for both

(iii)	If $(v=)\ \frac{20t}{t^2+4}-4$		
	$(a=)\ \frac{20(t^2+4)-20t(2t)}{(t^2+4)^2}$	M1 A1 A1	attempt to differentiate *their* $\frac{\mathrm{d}x}{\mathrm{d}t}$ $20(t^2+4)$ $20t\,(2t)$
	$20(4-t^2)$ or $80-20t^2$ or $4-t^2$ or equivalent expression involving $-t^2$	A1	$20(4-t^2)$ or $80-20t^2$ or $4-t^2$
	When acceleration is 0, $t=2$ only	B1	$t=2$, dependent on obtaining first and second A marks
	Alternative 1 for first 3 marks: If $(v=)\ \frac{20t-4t^2-16}{t^2+4}$	M1	attempt to differentiate *their* $\frac{\mathrm{d}x}{\mathrm{d}t}$
	$(a=)\frac{(t^2+4)\ (20-8t)-(20t-4t^2-16)\ (2t)}{(t^2+4)^2}$	A1 A1	for $(t^2+4)(20-8t)$ for $(20t-4t^2-16)(2\mathrm{t})$
	Alternative 2 for M1 mark: If $(v=)\ 20t(t^2+4)^{-1}-4$ $(a=)20t\left(-2t(t^2+4)^{-2}\right)+20(t^2+4)^{-1}$	M1	attempt to differentiate *their* $\frac{\mathrm{d}x}{\mathrm{d}t}$
	Alternative 3 for the first 3 marks If $(v=)\ (20t-4t^2-16)(t^2+4)^{-1}$ $(a=)(20t-4t^2-16)\left(-2t(t^2+4)^{-2}\right)+(20-8t)(t^2+4)^{-1}$ Numerator $=-2t(20t-4t^2-16)+(20-8t)\ (t^2+4)$	M1 A1 A1	attempt to differentiate *their* $\frac{\mathrm{d}x}{\mathrm{d}t}$ for $2t(20t-4t^2-15)$ for $(20-8t)(t^2+4)$

Example candidate response – high

6 A particle moves in a straight line such that its displacement, x m, from a fixed point O after t s, is given by $x = 10\ln(t^2 + 4) - 4t$.

(i) Find the initial displacement of the particle from O. [1]

Initial displacement is at time $t = 0$:-

$$x = 10 \ln(0^2 + 4) - 4(0)$$
$$= 10 \ln(4)$$
$$= 13.862 \approx \boxed{13.9 \text{ m}}$$

(ii) Find the values of t when the particle is instantaneously at rest. [4]

$$v = \frac{dx}{dt} = \frac{10 \cdot 2t}{t^2+4} - 4$$

$$v = \frac{2t \cdot 10 - 4(t^2+4)}{t^2+4}$$

particle at rest $\rightarrow$ $v = 0$:-

$$\frac{2t \cdot 10 - 4t^2 - 16}{t^2+4} = 0$$

$$\cancel{-4t^2 - 6 = 0}$$
$$\cancel{4t^2 + 6 = 0}$$
$$\cancel{4t^2 = -6}$$
$$\cancel{t^2 = \frac{-6}{4} = \frac{-3}{2}}$$

$$20t - 4t^2 - 16 = 0$$
$$4t^2 - 20t + 16 = 0$$
$$t^2 - 5t + 4 = 0$$
$$(t-4)(t-1) = 0$$

$$\boxed{t = 4 \text{ or } t = 1}$$

$\downarrow$

Ans.

Example candidate response – high, continued

(iii) Find the value of t when the acceleration of the particle is zero. [5]

$$v = \frac{20t - 4t^2 - 16}{t^2 + 4}$$

$$a = \frac{dv}{dt} = \frac{-(-4t^2 + 20t - 16)(2t) + (t^2+4)(20-8t)}{(t^2+4)^2}$$

$$= \frac{+8t^3 - 40t^2 + 32t + (20t^2 - 8t^2 + 80 - 32t)}{(t^2+4)^2}$$

$$a = \frac{+8t^3 - 40t^2 + 32t + 12t^2 + 80 + 32t}{(t^2+4)^2}$$

$a = 0$:-

$$-8t^3 + 28t^2 - 64t - 80 = 0$$
$$8t^3 - 28t^2 + 64t + 80 = 0$$
$$2t^3 - 7t^2 + 16t + 20 = 0$$

$$\boxed{t = -0.854 \text{ or } t = 4.82 \text{ or } t = -0.465}$$

Examiner comment – high

(i) A clear and correct response was given (B1).

(ii) The candidate realised that differentiation was needed to obtain the velocity. This was done correctly for both terms, and the result equated to zero (B1, M1). The resulting equation was then solved correctly (M1, A1).

(iii) The candidate correctly attempted to differentiate the velocity expression to obtain an expression for the acceleration. The initial expression obtained was correct and marked using Alternative 1 (M1, A1, A1). However an error in the simplification of this expression meant that an incorrect cubic equation was subsequently obtained and hence incorrect solutions (A0, B0).

Marks awarded = **(i) 1/1, (ii) 4/4, (iii) 3 /5**

Total marks awarded = 8 out of 10

Example candidate response – middle

6 A particle moves in a straight line such that its displacement, x m, from a fixed point O after t s, is given by $x = 10\ln(t^2 + 4) - 4t$.

(i) Find the initial displacement of the particle from O. [1]

$t = 0$

$x = 10 \ln(0 + 4) - 4(0)$

$x = 10 \ln(4)$

$x = 13.86$ m

(ii) Find the values of t when the particle is instantaneously at rest. [4]

$\frac{dx}{dt} = v$

$x = 10 \ln(t^2 + 4) - 4t$

$\frac{dx}{dt} = 10 \times \frac{1}{t^2+4} \times 2t - 4$

$v = \frac{20t}{t^2+4} - 4$

instantaneous rest $\rightarrow v = 0$

$\frac{20t}{t^2+4} - 4 = 0$

$20t = 4(t^2 + 4)$

$20t = 4t^2 + 16$

$4t^2 - 20t + 16 = 0$

$\boxed{\div 4}$

$t^2 - 5t + 4 = 0$

$t^2 - 4t - t + 4 = 0$

$t(t-4) - 1(t-4) = 0$

$(t-4)(t-1) = 0$

$t = 4$ s, $t = 1$ s

Example candidate response – middle, continued

(iii) Find the value of t when the acceleration of the particle is zero. **[5]**

acceleration $= \dfrac{dv}{dt} = a$

$v = \underbrace{20t}_{u}\underbrace{(t^2+4)^{-1}}_{v} - 4$ Product rule

$\dfrac{dv}{dt} = \underbrace{20t}_{u}\underbrace{(-1(t^2+4)^{-2}(2t))}_{v'} + \underbrace{(t^2+4)^{-1}}_{v}\underbrace{(20)}_{u'} - 4$

$a = 20t(-2t(t^2+4)^{-2}) + \dfrac{20}{t^2+4} - 4$

$a = \dfrac{-40t^2}{(t^2+4)^2} + \dfrac{20}{t^2+4} - 4$

$\dfrac{-40t^2 + 20(t^2+4)}{(t^2+4)^2} = 4$

$-40t^2 + 20t^2 + 80 = 4(t^2+4)^2$

$80 - 20t^2 = 4(t^4 + 16 + 8t^2)$

$80 - 20t^2 = 4t^4 + 64 + 32t^2$

$4t^4 + 32t^2 + 20t^2 + 64 - 80 = 0$

$4t^4 + 52t^2 - 16 = 0$

$\div 4$

$t^4 + 13t^2 - 4 = 0$

$\boxed{\text{Let } t^2 = x}$

$x^2 + 13x - 4 = 0$

$a = 1, b = 13, c = -4$ $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-13 \pm \sqrt{13^2 - 4(1)(-4)}}{2(1)}$

$x = \dfrac{-13 + \sqrt{185}}{2}, \quad x = \dfrac{-13 - \sqrt{185}}{2}$

$t^2 = 0.3, \quad t^2 = -13$ ✗ rejected because negative (square-root)

$t = \pm 0.548$

$\boxed{t = 0.548\text{ s}}$

Examiner comment – middle

(i) A clear and correct response was given (B1).

(ii) The candidate realised that differentiation was needed to obtain the velocity. This was done correctly for both terms, and the result equated to zero (B1, M1). The resulting equation was then solved correctly (M1, A1).

(iii) An attempt to differentiate the expression obtained for the velocity was made. The candidate did not use a correct method of differentiation. An attempt at a quotient or equivalent product was necessary (M0, A0, A0, A0, B0). There were no marks available in this question for the solution of a quadratic equation.

Marks awarded = **(i) 1/1, (ii) 4/4, (iii) 0 /5**

Total marks awarded = 5 out of 10

Example candidate response – low

6 A particle moves in a straight line such that its displacement, x m, from a fixed point O after t s, is given by $x = 10\ln(t^2+4) - 4t$.

(i) Find the initial displacement of the particle from O. [1]

$x = 10\ln(0^2+4) - 4(0)$

$x = 10\ln(4) - 0$

$x = 13.86 = \boxed{13.9\text{ m}} \rightarrow$ initial displacement.

(ii) Find the values of t when the particle is instantaneously at rest. [4]

$\cancel{\int 10\ln(t^2+4) - 4t}$

$\cancel{\int \frac{10}{t^2+4} - \frac{4t^{1+1}}{2} \rightarrow \text{velocity}}$

$\frac{dx}{dt} = 10\ln(t^2+4) - 4t$

$\frac{10}{(t^2+4)} - 4 = 0$

$\frac{10}{(t^2+4)} = 4$

$10 = 4t^2 + 16$

$-4t^2 = 10 + 16$

$-4t^2 = +6$

$-t^2 = \frac{+6}{4}$

$t^2 = \frac{+3}{2}$

$\sqrt{t^2} = \sqrt{\frac{+3}{2}}$

$\boxed{t = \pm 1.225}$

AM

$\cancel{\frac{10}{t^2+4} - 2t^2 = 0}$

$\cancel{\frac{10 - 2t^2(t^2+4)}{t^2+4} = 0}$

$\cancel{\frac{10 - 2t^4 + 8t^2}{(t^2+4)} = 0}$

$\cancel{-2t^4 + 8t^2 + 10 = 0}$

$\cancel{+2t^4 + 8t^2 = -10}$

$\cancel{\frac{t^2}{t}(-2t^2 + 8) = \frac{-10}{t^2}}$

$\cancel{-2t^2 + 8 = \frac{-10}{t^2}}$

$\cancel{-2t^2 + 8 = -10t^{-2}}$

$\cancel{-2t^2 + 10t^{-2} = -8}$

$\cancel{\frac{2}{2}(-t^2 + 5t^{-2}) = \frac{-8}{2}}$

$\cancel{-1(-t^2 + 5t^{-2}) = (-4) - 1}$

$\cancel{t^2 - 5t^{-2} = 4}$

$\cancel{-4t^{-2} = 4}$

$\cancel{t^{-2} = \frac{-4}{4}}$

$\cancel{\sqrt{t^{-2}} = \sqrt{-1}}$

$\cancel{\boxed{t = \mp 1}}$ AM

Example candidate response – low, continued

(iii) Find the value of t when the acceleration of the particle is zero. **[5]**

$$\frac{d^2x}{d^2t} = \frac{10}{(t^2+4)} - 4 = 0$$

$$\frac{d^2x}{d^2t} = -10(t^2+4)^{-1-1} \Rightarrow -10(t^2+4)^{-2}$$

$$\frac{-10}{(t^2+4)^2} = 0$$

$$-10 = 0$$

$$\boxed{t = 0} \quad \text{Am}$$

Examiner comment – low

(i) A clear and correct response was given (B1).

(ii) The candidate realised that differentiation was needed to obtain the velocity. However the logarithm term was differentiated incorrectly, there was a missing term in the numerator of the fraction given by the candidate (The denominator was correct) (M1, B0). The resulting quadratic equation obtained when the candidate had equated their expression to zero did not have any real roots, so no credit was available (M0, A0).

(iii) Again, an attempt at differentiation of the velocity expression was made. The candidate did not differentiate their fraction correctly using the chain rule, so no marks were available in this part (M0, A0, A0, A0, B0).

Marks awarded = **(i) 1/1, (ii) 1/4, (iii) 0/5**

Total marks awarded = 2 out of 10

Question 7

Mark scheme

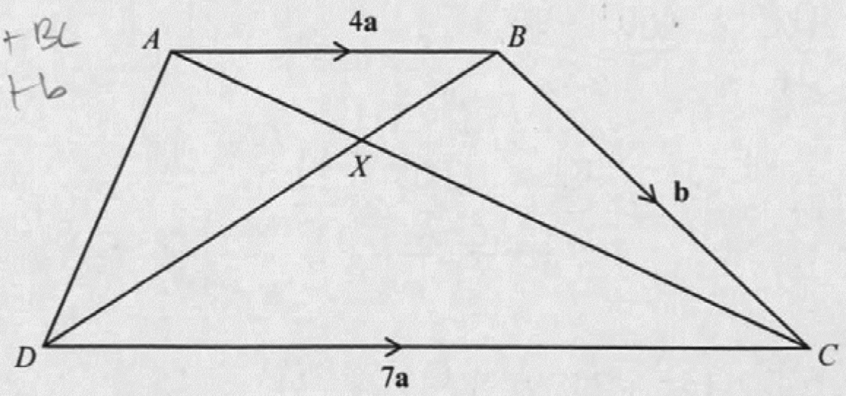
7 (i)	$\overrightarrow{DA} = 3\mathbf{a} - \mathbf{b}$	B1	mark final answer, allow unsimplified
(ii)	$\overrightarrow{DB} = 7\mathbf{a} - \mathbf{b}$	B1	mark final answer, allow unsimplified
(iii)	$\overrightarrow{AX} = \lambda(4\mathbf{a} + \mathbf{b})$	B1	mark final answer, allow unsimplified
(iv)	$\overrightarrow{DX} = 3\mathbf{a} - \mathbf{b} + \lambda(4\mathbf{a} + \mathbf{b})$	M1 A1	*their* **(i)** + *their* **(iii)** or equivalent valid method or $3\mathbf{a} - \mathbf{b}$ + *their* **(iii)** Allow unsimplified

(v)	$3\mathbf{a} - \mathbf{b} + \lambda(4\mathbf{a} + \mathbf{b}) = \mu(7\mathbf{a} - \mathbf{b})$ Equate like vectors: $3 + 4\lambda = 7\mu$ $-1 + \lambda = -\mu$ leads to $\lambda = \frac{4}{11}$, $\mu = \frac{7}{11}$	M1 DM1 A1,A1	equating *their* **(iv)** and $\mu \times$ *their* **(ii)** for an attempt to equate like vectors and attempt to solve 2 linear equations for λ and μ A1 for each

Example candidate response – high

7

$\vec{AC} = AB + BC$
$= 4a + b$



In the diagram $\overrightarrow{AB} = 4\mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{DC} = 7\mathbf{a}$. The lines AC and DB intersect at the point X. Find, in terms of **a** and **b**,

(i) $\overrightarrow{DA}$, [1]

$= \vec{DC} - \vec{AC}$

$= 7a - (AB + BC)$

$= 7a - 4a - b = 3a - b$

(ii) $\overrightarrow{DB}$. [1]

$= \vec{DC} + \vec{CB}$

$= 7a - b$

Given that $\overrightarrow{AX} = \lambda\overrightarrow{AC}$, find, in terms of **a**, **b** and λ,

(iii) $\overrightarrow{AX}$, [1]

$= \lambda(4a + b)$

$= 4a\lambda + \lambda b$

(iv) $\overrightarrow{DX}$. [2]

$= \vec{DA} + \vec{AX}$

$= 3a - b + 4a\lambda + \lambda b$

$= (3 + 4\lambda)a + b(\lambda - 1)$

Example candidate response – high, continued

Given that $\overrightarrow{DX} = \mu\overrightarrow{DB}$,

(v) find the value of λ and of μ. **[4]**

$\overrightarrow{DX} = \mu(7a - b)$

$a(3+4\lambda) + b(\lambda - 1) = (7\mu)a - b(\mu)$

$3 + 4\lambda = 7\mu \rightarrow ①$

$\lambda - 1 = \mu \rightarrow ②$

$① \rightarrow \quad 3 + 4\lambda = 7(\lambda - 1)$

$3 + 4\lambda = 7\lambda - 7$

$10 = 3\lambda$

$\boxed{\lambda = \frac{10}{3}}$

$② \rightarrow \quad \mu = \lambda - 1$

$= \frac{10}{3} - 1$

$\boxed{\mu = \frac{7}{3}}$

Examiner comment – high

(i) A correct vector was obtained using correct reasoning (B1).

(ii) A correct vector was obtained using correct reasoning (B1).

(iii) A correct vector was obtained using the given information correctly (B1).

(iv) The candidate made use of the previous parts of the question as well as the given information, to obtain a correct vector (M1, A1).

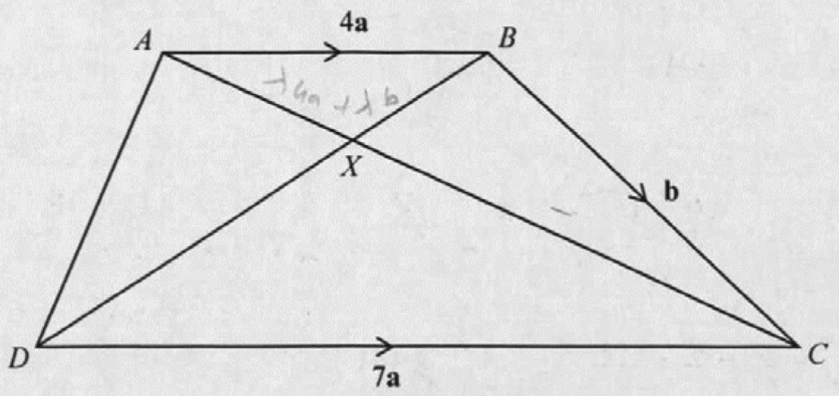
(v) The candidate realised that use of parts (ii) and (iv) and equating the results appropriately yielded a vector equation (M1). An attempt to equate like vectors was made, but the omission of a negative sign meant that the candidate was unable to gain accuracy marks from the solution of their simultaneous equations (M1, A0, A0).

Marks awarded = **(i) 1/1, (ii) 1/1, (iii) 1 /1, (iv) 2/2, (v) 2/4**

Total marks awarded = 7 out of 9

Example candidate response – middle

7



In the diagram $\overrightarrow{AB} = 4\mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{DC} = 7\mathbf{a}$. The lines AC and DB intersect at the point X. Find, in terms of **a** and **b**,

(i) $\overrightarrow{DA}$, [1]

$DA = 7a - 4a - b$

$= 3a - b$

(ii) $\overrightarrow{DB}$. [1]

$7a - b$

Given that $\overrightarrow{AX} = \lambda\overrightarrow{AC}$, find, in terms of **a**, **b** and λ,

(iii) $\overrightarrow{AX}$, [1]

$\overrightarrow{AC} = \cancel{+b + 4a}$

$= 4a + b$

$= \lambda(4a + b)$

$= \lambda 4a + \lambda b$

(iv) $\overrightarrow{DX}$. [2]

$= 7a - (4a + b - \lambda a + \lambda b)$

$= 7a - 4a - b + \lambda 4a - \lambda b$

$= 3a - b + \lambda 4a - \lambda b$

Example candidate response – middle, continued

Given that $\overrightarrow{DX} = \mu\overrightarrow{DB}$,

(v) find the value of λ and of μ. **[4]**

$$3a - b + \lambda 4a - \lambda b = \mu 7a - \mu b$$

$$3a - b + \lambda(4a - b) = \mu(7a - b)$$

$$\lambda(4a - b) = \mu(7a - b) - 3a + b$$

$$\lambda = \frac{\mu(7a - b) - 3a + b}{(4a - b)} \quad \text{— (i)}$$

$$3a - b + \left(\frac{\mu 7a - b) - 3a + b}{4a - b}\right) = \mu(7a - b)$$

$$\frac{(4a - b)(3a - b) + (\mu 7a - b) - 3a + b}{4a - b}$$

$$\frac{12a^2 - 4ab - 3ab + b^2 - 3a + b + (\mu 7a - b)}{4a - b} = \mu(7a - b)$$

$$12a^2 - 7ab - 3a + b + b^2 + \mu(7a - b) = \mu(7a - b)(4a - b)$$

$$\frac{\cancel{12a^2} - \cancel{7ab} - 3a + b \cancel{+ b^2}}{\cancel{28a^2} - \cancel{7ab} - 4ab \cancel{+ b^2}} = \mu - \mu(7a - b)$$

$$\frac{3 - 3a + b}{7 - 4ab} = \mu - 7\mu a + \mu b$$

$$\frac{3 - 3a + b}{7 - 4ab} = \mu(-7a + b)$$

$$\frac{\frac{3 - 3a + b}{7 - 4ab}}{-7a + b} = \mu$$

Examiner comment – middle

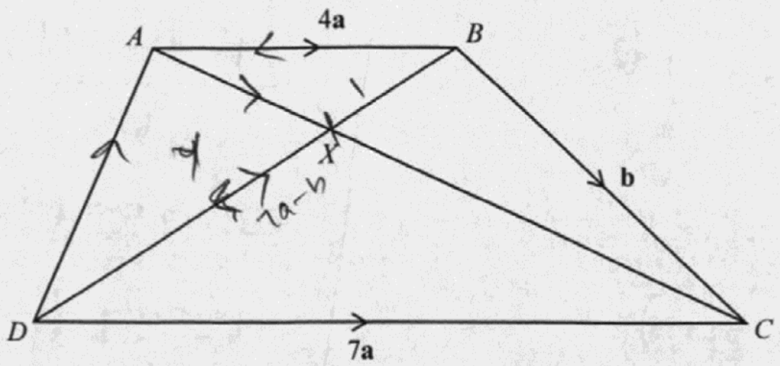
- **(i)** A correct vector was obtained using correct reasoning (B1).
- **(ii)** A correct vector was obtained using correct reasoning (B1).
- **(iii)** A correct vector was obtained using the given information correctly (B1).
- **(iv)** The candidate did not make full use of parts (i) and (iii) as intended. An omission of a number and sign errors meant that an incorrect method was assumed (M0, A0).
- **(v)** The candidate realised that use of parts (ii) and (iv), even though an incorrect (iv) had been obtained, and equating the results appropriately yielded a vector equation (M1). No attempt to equate like vectors appeared in the subsequent working (M0, A0, A0).

Marks awarded = **(i) 1/1, (ii) 1/1, (iii) 1/1, (iv) 0/2, (v) 1/4**

Total marks awarded = 4 out of 9

Example candidate response – low

7



In the diagram $\overrightarrow{AB} = 4\mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{DC} = 7\mathbf{a}$. The lines AC and DB intersect at the point X. Find, in terms of **a** and **b**,

(i) $\overrightarrow{DA}$, [1]

$-7a + b - 4a$

$\boxed{-11a + b}$

$\cancel{B}\ BD = -7a + b$

$\overrightarrow{DA} = (-7a + b) - 4a$

(ii) $\overrightarrow{DB}$. [1]

$\cancel{7a - b}$

$-(-7a + b)$

$\boxed{7a - b}$

Given that $\overrightarrow{AX} = \lambda\overrightarrow{AC}$, find, in terms of **a**, **b** and λ,

$\frac{AX}{AC} = \frac{\lambda}{1}$

(iii) $\overrightarrow{AX}$, [1]

$(-11 + b) + \lambda(7a - b)$

$-11 + b + 7a\lambda - \lambda b$

$-11 + (1 - \lambda)b + 7a\lambda$

(iv) $\overrightarrow{DX}$. [2]

$\cancel{7a}\ (-11a + b) + (-11 + b) + 7a\lambda - \lambda b)$

$-11a + b - 11 + b - 7a\lambda - \lambda b$

$-11a + 2b - 11 - 7a\lambda - \lambda b$

$-11a + 2b - 11 - \lambda(a + b)$

Example candidate response – low, continued

Given that $\overrightarrow{DX} = \mu\overrightarrow{DB}$,

(v) find the value of λ and of μ. [4]

$$\frac{-11a + 2b - 11 - x(a+b)}{7a - b} = 0$$

Examiner comment – low

(i) Incorrect directions and reasoning meant that an incorrect vector was obtained (B0).

(ii) A correct vector was written down, as there was only one mark available, no extra working needed to be seen in this case (B1).

(iii) No correct work or reasoning was seen, the expression also contained a scalar quantity, so the result given was not a vector (B0).

(iv) An attempt to use the results obtained in parts (i) and (iii) was made and the method was given credit even though the answer to part (iii) was not a vector quantity (M1, A0).

(v) It was felt that this response did not merit a mark as a correct vector equation was never written down and no attempt to equate like vectors was made (M0, M0, A0, A0).

Marks awarded = **(i) 0/1, (ii) 1/1, (iii) 0/1, (iv) 1/2, (v) 0/4**

Total marks awarded = 2 out of 9

Question 8

Mark scheme

8	**(i)**	$5e^{2x} - \frac{1}{2}e^{-2k} \quad (+c)$	B1, B1	B1 for each term, allow unsimplified
	(ii)	$\left(5e^{2k} - \frac{1}{2}e^{-2k}\right) - \left(5e^{-2k} - \frac{1}{2}e^{2k}\right)$	M1 A1	use of limits provided integration has taken place. Signs must be correct if brackets are not included. allow any correct form
	(iii)	$\left(5e^{2k} - \frac{1}{2}e^{-2k}\right)\left(5e^{-2k} - \frac{1}{2}e^{2k}\right) = -60$ or $\frac{11}{2}e^{2k} - \frac{11}{2}e^{-2k} = -60$ or equivalent leading to $11e^{2k} - 11e^{-2k} + 120 = 0$	B1 DB1	correct expression from **(ii)** either simplified or unsimplified equated to -60, must be first line seen. must be convinced as **AG**
	(iv)	$11y^2 + 120y - 11 = 0$ $(11y - 1)(y + 11) = 0$ leading to $k = \frac{1}{2}\ln\frac{1}{11}, \ \ln\frac{1}{\sqrt{11}}, \quad -\ln\sqrt{11}, -\frac{1}{2}\ln 11$	M1 DM1 A1	attempt to obtain a quadratic equation in y or e^{2k} and solve to get y or e^{2k} (only need positive solution) attempt to deal with e to get $k =$. any of given answers only.

Example candidate response – high

8 **(i)** Find $\int (10e^{2x} + e^{-2x})dx$. [2]

$(10e^{2x} + e^{-2x})$

$\left((10e^{2x})(\tfrac{1}{2}) + (e^{-2x})(-\tfrac{1}{2})\right)$

~~$(20e^{2x} - 2e^{-2x}) \rightarrow$ Ans~~

$\boxed{5e^{2x} - \frac{e^{-2x}}{2}}$

Ans

(ii) Hence find $\int_{-k}^{k} (10e^{2x} + e^{-2x})dx$ in terms of the constant k. [2]

$\left(5e^{2(k)} - \frac{e^{-2(k)}}{2}\right) - \left(5e^{2(-k)} - \frac{e^{-2(-k)}}{2}\right)$

$\left(5e^{2k} - \frac{e^{-2k}}{2}\right) - \left(5e^{-2k} - \frac{e^{2k}}{2}\right)$

$5e^{2k} - \frac{e^{-2k}}{2} - 5e^{-2k} + \frac{e^{2k}}{2}$

$\boxed{5.5e^{2k} - 5.5e^{-2k}}$

Ans

(iii) Given that $\int_{-k}^{k} (10e^{2x} + e^{-2x})dx = -60$, show that $11e^{2k} - 11e^{-2k} + 120 = 0$. [2]

~~$22e^{2k} - 22e^{-2k} = -60$~~

~~$22e^{2k} - 22e^{-2k} + 60 = 0$~~

~~$\frac{2}{2}(11e^{2k} - 11e^{-2k} + 30) = 0$~~

$5.5e^{2k} - 5.5e^{-2k} = -60$

$2(5.5e^{2k} - 5.5e^{-2k} + 60) = 0 \times 2$

$\boxed{11e^{2k} - 11e^{-2k} + 120 = 0} \rightarrow$ proven

Example candidate response – high, continued

(iv) Using a substitution of $y = e^{2k}$ or otherwise, find the value of k in the form $a \ln b$, where a and b are constants. [3]

$11e^{2k} - \frac{11}{e^{2k}} + 120 = 0$

$11y - \frac{11}{y} + 120 = 0$

$\frac{11y^2 - 11 + 120y}{y} = 0$

$11y^2 + 120y - 11 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{-120 \pm \sqrt{120^2 - 4(11)(-11)}}{2(11)}$

$\frac{-120 \pm \sqrt{14400 + 484}}{22}$

$\frac{-120 \pm \sqrt{14884}}{22}$

$\frac{-120 + 122}{22} = 0.09$

$\frac{-120 - 122}{22} = -11$

$e^{2k} = y$

$\ln e^{-11} = 2k$

$2k = -11$

$\boxed{k = -5.5}$ Ans

$\ln e^{0.09} = 2k$

$\boxed{k = 0.05}$ Ans

Examiner comment – high

(i) Correct integration of both terms was done, omission of an arbitrary constant was condoned (B1, B1).

(ii) A correct substitution of the given limits was made into the response from part **(i)**. The candidate did go on to simplify this correctly but this was not essential for this part (M1, A1).

(iii) With the simplification done in part **(ii)**, the candidate did not have much work to do in order to gain full marks for showing the expected response (B1, DB1).

(iv) A correct quadratic equation in terms of *y* was obtained together with a valid attempt to solve this quadratic equation (M1). The candidate was unable to deal correctly with the exponential term obtained when attempting to solve for *x* (DM0, A0).

Marks awarded = **(i) 2/2, (ii) 2/2, (iii) 2/2, (iv) 1/3**

Total marks awarded = 7 out of 9

Example candidate response – middle

8 **(i)** Find $\int (10e^{2x} + e^{-2x})dx$. [2]

$$= \frac{5\cancel{10}e^{2x}}{\cancel{2}} + \frac{e^{-2x}}{-2}$$

$$= 5e^{2x} - \frac{e^{-2x}}{2}$$

(ii) Hence find $\int_{-k}^{k} (10e^{2x} + e^{-2x})dx$ in terms of the constant k. [2]

$$= \left[5e^{2x} - \frac{e^{-2x}}{2}\right]_{-k}^{k}$$

$$= \left\{5e^{2k} - \frac{e^{-2k}}{2}\right\} - \left(5e^{-2k} + \frac{e^{2k}}{2}\right)$$

$$= \frac{10e^{2k} - e^{-2k}}{2} - \left\{\frac{10e^{-2k} + e^{2k}}{2}\right.$$

(iii) Given that $\int_{-k}^{k} (10e^{2x} + e^{-2x})dx = -60$, show that $11e^{2k} - 11e^{-2k} + 120 = 0$. [2]

$$= \frac{10e^{2k} - e^{-2k} - 10e^{-2k} + e^{2k}}{2} = -60$$

$$\cancel{10e\left(\frac{2k}{-2k}\right) + e\left(\frac{2k}{-2k}\right) = -120}$$

$$11e^{2k} - 11^{-2k} = 2(-60)$$

$$11e^{2k} - 11^{-2k} + 120 = 0$$

$\therefore$ Shown

Example candidate response – middle, continued

(iv) Using a substitution of $y = e^{2k}$ or otherwise, find the value of k in the form $a \ln b$, where a and b are constants. [3]

if $y = e^{2k}$,

$11y - \frac{11}{y} + 120 = 0$

$11y + 120 = \frac{11}{y}$

$11y^2 + 120y - 11 = 0$

$11y^2 + 121y - y - 11 = 0$

$11y(y+11) - 1(y+11) = 0$

$(y+11)(11y-1)$

$y = -11$ or $y = \frac{1}{11}$

$e^{2k} = -11$

∴ not possible

$e^{2k} = \frac{1}{11}$

$\ln e^{2k} = \ln\left(\frac{1}{11}\right)$

$2k = \ln\left(\frac{1}{11}\right)$

$k = \frac{1}{2} \ln\left(\frac{1}{11}\right)$

$a = \frac{1}{2}$, $b = \frac{1}{11}$

Examiner comment – middle

(i) Correct integration of both terms was done, omission of an arbitrary constant was condoned (B1, B1).

(ii) An incorrect sign obtained when substituting in the given limits implied an incorrect method (M0, A0).

(iii) As there was incorrect result from part **(ii)**, the candidate was unable to obtain the given result using correct working. Incorrect manipulation of the signs of terms lead to a result which appeared to be correct but had not been obtained from completely correct work (B0, DB0).

(iv) The candidate used the given result from part **(iii)**, obtaining a correct quadratic equation and its solution. The candidate was able to deal with the exponential term and provide a completely correct solution (M1, DM1, A1).

Marks awarded = **(i) 2/2, (ii) 0/2, (iii) 0/2, (iv) 3/3**

Total marks awarded = 5 out of 9

Example candidate response – low

8 **(i)** Find $\int(10e^{2x}+e^{-2x})\,dx$. [2]

$\cancel{10e^{2x}}$ $= 10 \cdot \frac{e^{2x+1}}{2x+1} + \frac{e^{1-2x}}{1-2x}$

$\boxed{= \frac{10-20x(e^{2x+1})+(2x+1)(e^{1-2x})}{(2x+1)(1-2x)}} = \frac{10(1-2x)(e^{2x+1})+(e^{1-2x})(2x+1)}{(2x+1)(1-2x)}$

Ans.

(ii) Hence find $\int_{-k}^{k}(10e^{2x}+e^{-2x})\,dx$ in terms of the constant k. [2]

$= \frac{10-20k(e^{2k+1})+e^{1-2k}(2k+1)}{(2k+1)(1-2k)}$ $= \cancel{\frac{10-20k e^{2k+1}/2k e^{1-2k}+e^{1-2k}}{1-4k^2}}$

$= 11e^{2k} - 11e^{-2k} + 60$

(iii) Given that $\int_{-k}^{k}(10e^{2x}+e^{-2x})\,dx = -60$, show that $11e^{2k}-11e^{-2k}+120=0$. [2]

$11e^{2k} - 11e^{-2k} + 60 = -60$

$\boxed{11e^{2k} - 11e^{-2k} + 120 = 0}$

Ans.

Example candidate response – low, continued

(iv) Using a substitution of $y = e^{2k}$ or otherwise, find the value of k in the form $a \ln b$, where a and b are constants. [3]

$$11e^{2k} - 11e^{-2k} + 120 = 0$$

$$y = e^{2k}$$

$$11y - 11y^{-1} + 120 = 0$$

$$11y - \frac{11}{y} + 120 = 0$$

$$11y^2 + 120y - 11 = 0$$

$$y = \frac{-120 \pm \sqrt{(120)^2 - 4(11)(-11)}}{2(11)}$$

$$y + 11 = 0 \quad \text{or} \quad 11y - 1 = 0$$

$$y = -11 \quad \text{or} \quad y = \frac{1}{11}$$

$$\frac{1}{11} = e^{2k}$$

$$2k = \ln\left(\frac{1}{11}\right)$$

$$k = \frac{\ln\left(\frac{1}{11}\right)}{2}$$

$$\boxed{k = -1.20}$$

OR

$$-11 = e^{2k}$$

$$2k = \ln(-11)$$

$$k = \frac{\ln(-11)}{2}$$

$$\boxed{k = 1.20}$$

Examiner comment – low

(i) The candidate was unable to integrate terms involving the exponential function (B0, B0).

(ii) An integration of the correct form had not been obtained in part **(i)** so the candidate was unable to obtain marks for substitution into an expression that was not in the form $ae^{2x} + be^{-2x}$ (M0, A0).

(iii) As the working from the previous parts was incorrect, achieving the given result was not possible (B0, DB0).

(iv) The candidate used the given result from part **(iii)**, obtaining a correct quadratic equation and its solution. The candidate was able to deal with the exponential term and provide a completely correct solution for the positive root of the quadratic equation, but failed to discount the solution from the negative root of the quadratic equation which gave an undefined result. This was penalised by withholding the accuracy mark (M1, DM1, A0).

Marks awarded = **(i) 0/2, (ii) 0/2, (iii) 0/2, (iv) 2/3**

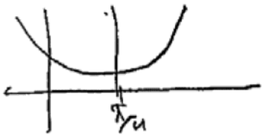
Total marks awarded = 2 out of 3

Question 9

Mark scheme

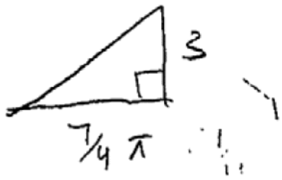
9	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4 - 6\sin 2x$	M1,A1	M1 for attempt to differentiate A1 for all correct
	When $x = \frac{\pi}{4}$, $y = \pi$	B1	for y
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -2$ so gradient of normal $= \frac{1}{2}$	DM1	for substitution of $x = \frac{\pi}{4}$ into *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ and use of '$m_1 m_2 = -1$', dependent on first M1
	Normal equation $y - \pi = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$	DM1	correct attempt to obtain the equation of the normal, dependent on previous DM mark
	When $x = 0$, $y = \frac{7\pi}{8}$	A1	must be terms of π
	When $y = 0, x = -\frac{7\pi}{4}$	A1	must be terms of π
	Area $= \frac{1}{2} \times \frac{7\pi}{4} \times \frac{7\pi}{8} \quad = \frac{49\pi^2}{64}$	B1ft	Follow through on *their* x and y intercepts; must be exact values

Example candidate response – high



9 A curve has equation $y = 4x + 3\cos 2x$. The normal to the curve at the point where $x = \frac{\pi}{4}$ meets the x- and y-axes at the points A and B respectively. Find the exact area of the triangle AOB, where O is the origin. [8]

$$\frac{dy}{dx} = 4 - 3\sin 2x(2)$$
$$= 4 - 6\sin 2x$$
$$= 4 - 6\sin 2\left(\frac{\pi}{4}\right)$$



m of tangent = ~~4~~ -2

m of normal = $\frac{1}{2}$

at $x = \frac{\pi}{4}$, $y = 4\left(\frac{\pi}{4}\right) + 3\cos 2\left(\frac{\pi}{4}\right)$

$y = \pi$

equation of normal:

$y - y_1 = m(x - x_1)$

$y - \pi = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

$y = \frac{x}{2} - \frac{\pi}{8} + \pi$

$y = \frac{x}{2} + \frac{7}{8}\pi$

at A, $y = 0$

$0 = \frac{x}{2} + \frac{7}{8}\pi$

$-\frac{7}{8}\pi = \frac{x}{2}$

$x = -\frac{7}{4}\pi$

$A = \left(-\frac{7}{4}\pi, 0\right)$

at B, $x = 0$

$y = 4(0) + 3\cos 2(0)$

$y = 3$

$B = (0, 3)$

Area of AOB $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times \frac{7}{4}\pi \times 3$

$= \frac{21}{8}\pi$

$= 8.25$ sq units

(to 3 s.f.)

Total marks awarded = 6 out of 8

Examiner comment – high

A correct strategy was used by the candidate. The given function was differentiated correctly (M1, A1). The gradient of the normal at the given point was calculated correctly, together with the value of y at the given point (DM1, B1). A correct equation of the normal to the curve at the given point was subsequently obtained (DM1). As the question asked for the exact value of the area of the triangle AOB, it was expected that candidates find the exact coordinates of the point A and the point B. The candidate was able to do this for the point A (A1). However the candidate did not substitute the value of $x = 0$ into the normal equation but erroneously used the curve equation, thus not being able to obtain the correct coordinates for the point B (A0). As the candidate had not found the correct intercept (the normal with the y-axis, not the curve with the y-axis) the follow through mark for finding the area of the triangle AOB using the candidate's values was not available (A0).

Example candidate response – middle

9 A curve has equation $y = 4x + 3\cos 2x$. The normal to the curve at the point where $x = \frac{\pi}{4}$ meets the x- and y-axes at the points A and B respectively. Find the exact area of the triangle AOB, where O is the origin. [8]

Normal's equation: $y - y_1 = -\frac{1}{dy/dx} \times (x - x_1)$

$y = 4x + 3\cos 2x$

$\frac{dy}{dx} = 4 - 3\sin 2x \cdot 2$

$\frac{dy}{dx} = 4 - 6\sin 2x$

$\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ $= 4 - 6\sin(2 \times \frac{\pi}{4})$

$\frac{dy}{dx} = -2$

$y = 4 \times \frac{\pi}{4} + 3\cos(2 \times \frac{\pi}{4})$

$y = \pi + 3\cos\frac{\pi}{2} = \pi$

$\therefore (\frac{\pi}{4}, \pi)$

Normal: $(y - \pi) = \frac{+1}{+2}(x - \frac{\pi}{4})$

$2(y - \pi) = x - \frac{\pi}{4}$

$2y = x - \frac{\pi}{4} + 2\pi$

$y = \frac{1}{2}x + \frac{7}{4}\pi$ → Normal

cuts x-axis

$\frac{1}{2}x + \frac{7}{4}\pi = 0$

$2x + 7\pi = 0$

$2x = -7\pi$

$x = -\frac{7}{2}\pi$

$\boxed{A(-\frac{7}{2}\pi, 0)}$

cuts y-axis

$y = \frac{1}{2}(0) + \frac{7}{4}\pi$

$y = \frac{7}{4}\pi$

$\boxed{B(0, \frac{7}{4}\pi)}$

$O(0,0)$

$A_{AOB} = \frac{1}{2}\begin{vmatrix} -\frac{7}{2}\pi & 0 & 0 & -\frac{7}{2}\pi \\ 0 & 0 & \frac{7}{4}\pi & 0 \end{vmatrix}$

$A = \frac{1}{2}\left[(0+0+0) - (-\frac{7}{2}\pi \times \frac{7}{4}\pi + 0 + 0)\right]$

$A = \frac{1}{2}(-(-60.45))$

$A = \frac{1}{2} \times 60.45$

Area $= 30.23 = \boxed{30.2 \text{ unit}^2}$

Total marks awarded = 5 out of 8

Examiner comment – middle

A correct strategy was used throughout by the candidate. The given function was differentiated correctly (M1, A1). The gradient of the normal at the given point was calculated correctly, together with the value of y at the given point (DM1, B1). A correct equation of the normal to the curve at the given point was subsequently obtained (DM1). When the candidate simplified the equation of the normal an error was made which resulted in the incorrect (but exact) coordinates for the points A and B (A0, A0). A correct method of calculating the area of the triangle was used, but the candidate did not give an exact answer and so was unable to gain the last follow through mark (B0).

Example candidate response – low

9 A curve has equation $y = 4x + 3\cos 2x$. The normal to the curve at the point where $x = \frac{\pi}{4}$ meets the x- and y-axes at the points A and B respectively. Find the exact area of the triangle AOB, where O is the origin. [8]

Normal to the curve points: $x =$ $\frac{\pi}{4}$ and $y = \pi$

$$y = 4\left(\frac{\pi}{4}\right) + 3\cos 2\left(\frac{\pi}{4}\right)$$

$$y = \pi$$

Equation of normal: $y = \frac{-1}{4}x + c$

$$\pi = \frac{-\pi}{16} + c$$

$$c = \pi + \frac{\pi}{16} = 3.34$$

$$y = \frac{-1}{4}x + 3.34.$$

Point $A \rightarrow (13.4, 0)$

Point $B \rightarrow (0, 3.34)$

$OA = 13.4$

$OB = 3.34$

$A\hat{O}B = 90°$

$\therefore$ Area of Δ $AOB = \frac{1}{2} \times 3.34 \times 13.4$

$= 22.4$ units^2

Total marks awarded = 1 out of 8

Examiner comment – low

No attempt to use differentiation was evident anywhere. This meant that the candidate was unable to obtain any method marks as they were all dependent on the candidate using differentiation (M0, A0, DM0, DM0). The candidate did find the value of the *y*-coordinate at the given point (B1). As no exact values were used the final follow through mark was unavailable even though the candidate had attempted to use the coordinates of the intercepts on the axes of their incorrect normal equation (A0, A0, B0).

Question 10

Mark scheme

10 (a)	$\cos^2 3x = \frac{1}{2}$, $\cos 3x = (\pm)\frac{1}{\sqrt{2}}$ $3x = 45°, 135°, 225°, 315°$ $x = 15°, 45°, 75°, 105°$	M1 A1,A1	complete correct method, dealing with sec and 3, correctly A1 for each correct pair
(b)	$3(\cot^2 y + 1) + 5\cot y - 5 = 0$ Leading to $3\cot^2 y + 5\cot y - 2 = 0$ or $2\tan^2 y - 5\tan y - 3 = 0$ $(3\cot y - 1)(\cot y + 2) = 0$ or $(\tan y - 3)(2\tan y + 1) = 0$	M1 M1	use of a correct identity to get an equation in terms of one trig ratio only for $\cot y = \frac{1}{\tan y}$ to obtain either a quadratic equation in $\tan y$ or solutions in terms of $\tan y$; allow where appropriate
	$\tan y = 3$, $\tan y = -\frac{1}{2}$ $y = 71.6°, 251.6°$ $153.4°, 333.4°$	M1 A1,A1	for solution of a quadratic equation in terms of either $\tan y$ or $\cot y$ A1 for each correct 'pair'
(c)	$\sin\left(z + \frac{\pi}{3}\right) = \frac{1}{2}$ $z + \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$ $z = \frac{\pi}{2}, \frac{11\pi}{6}$ (allow 1.57, 5.76)	M1 A1 M1 A1	completely correct method of solution one correct solution in range correct attempt to obtain a second solution within the range second correct solution (and no other)

Example candidate response – high

10 **(a)** Solve $2\cos 3x = \sec 3x$ for $0° \leqslant x \leqslant 120°$. [3]

$2\cos 3x = \sec 3x$

$\dfrac{\cos 3x}{\sec 3x} = \dfrac{1}{2}$

$\cos 3x \div \sec 3x = \dfrac{1}{2}$

$\cos 3x \div \dfrac{1}{\cos 3x} = \dfrac{1}{2}$

$\cos 3x \times \cos 3x = \dfrac{1}{2}$

$\cos^2 3x = \dfrac{1}{2}$

$\cos 3x = \sqrt{\dfrac{1}{2}}$

$\cos 3x = \dfrac{\sqrt{2}}{2}$

$\cos \alpha = \dfrac{\sqrt{2}}{2}$

$\alpha = \cos^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$ $\boxed{0° \leq 3x \leq 360°}$

$\boxed{\alpha = 45°}$

$3x = 45°, 360° - 45°$

$3x = 45°, 315°$

$\boxed{x = 15°, 105°}$ Ans.

(b) Solve $3\operatorname{cosec}^2 y + 5\cot y - 5 = 0$ for $0° \leqslant y \leqslant 360°$. [5]

$3\operatorname{cosec}^2 y + 5\cot y - 5 = 0$

$3(1 + \cot^2 y) + 5\cot y - 5 = 0$

$3 + 3\cot^2 y + 5\cot y - 5 = 0$

$3\cot^2 y + 5\cot y - 2 = 0$

$3\cot^2 y + 6\cot y - \cot y - 2 = 0$

$3\cot y(\cot y + 2) - 1(\cot y + 2) = 0$

$(3\cot y - 1)(\cot y + 2) = 0$

$\boxed{\cot y = \frac{1}{3}, -2}$

$\dfrac{1}{\tan y} = \dfrac{1}{3}, -2$

reciprocating both...

$\boxed{\tan y = 3, \ \tan y = -\frac{1}{2}}$

$\tan y = 3$

$\tan y = 3$

$\tan \alpha = 3$

$\alpha = \tan^{-1}(3)$

$\alpha = 71.565°$

$\boxed{\approx \alpha = 71.6°}$

$\boxed{0° \leq y \leq 360°}$

$y = 71.565°, 180° + 71.565°$

$y = 71.565°, 251.565°$

$\boxed{\approx y = 71.6°, 251.6°}$

$\tan y = -\dfrac{1}{2}$

$\tan \alpha = \dfrac{1}{2}$

$\alpha = \tan^{-1}\left(\dfrac{1}{2}\right)$

$\alpha = 26.565°$

$\boxed{\approx \alpha = 26.6°}$

$y = 180° - 26.565°, 360° - 26.565°$

$y = 153.435°, 333.435°$

$\boxed{\approx y = 153.4°, 333.4°}$

$\boxed{y = 71.6°, 153.4°, 251.6°, 333.4°}$

Ans.

Question 10(c) is printed on the next page.

Example candidate response – high, continued

(c) Solve $2\sin\left(z+\frac{\pi}{3}\right)=1$ for $0 \leqslant z \leqslant 2\pi$ radians. [4]

$\Rightarrow 2\sin z = 1$

$\sin z = \frac{1}{2}$

S | A
T | C

$\sin\alpha = \frac{1}{2}$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right)$

$\boxed{\alpha = \frac{\pi}{6} \text{ rad.}}$

$0 \leq z \leq 2\pi$

$\frac{\pi}{3} \leq z+\frac{\pi}{3} \leq 2\pi + \frac{\pi}{3}$

$\boxed{\frac{\pi}{3} \leq z+\frac{\pi}{3} \leq 2\pi+\frac{\pi}{3}}$

$z + \frac{\pi}{3} = \frac{\pi}{6}, \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}$

$z = \frac{\pi}{6} - \frac{\pi}{3}, \frac{5}{6}\pi - \frac{\pi}{3}, \frac{13}{6}\pi - \frac{\pi}{3}$

$z = -\frac{\pi}{6}, \frac{1}{2}\pi, \frac{11}{6}\pi$

$\boxed{z = \frac{1}{2}\pi, \frac{11}{6}\pi}$ ~~rad~~ rad.

Ans.

Examiner comment – high

(a) A completely correct method dealing with sec $3x$ correctly was used by the candidate. However the candidate did not give all the possible solutions, omitting to consider the solutions from the negative root of the equation obtained (M1, A1, A0).

Marks awarded = **(a) 2/3**

(b) Use of the correct trigonometric identity together with a correct solution of the resulting quadratic equation in cot y and the correct method of dealing with cot y gained the candidate all the available method marks (M1, M1, M1). All the possible solutions to the correct level of accuracy were then given (A1, A1).

Marks awarded = **(b) 5/5**

(c) A clearly well thought out approach, showing all the relevant steps and correct work concerning the necessary range of values for z in radians, gained the candidate full marks (M1, A1, M1, A1).

Marks awarded = **(c) 4/4**

Total marks awarded = 11 out of 12

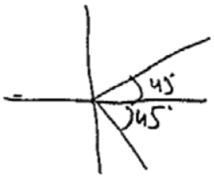
Example candidate response – middle

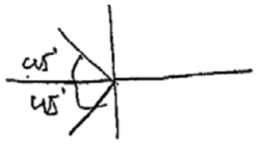
10 **(a)** Solve $2\cos 3x = \sec 3x$ for $0° \leqslant x \leqslant 120°$. [3]

$2\cos 3x = \dfrac{1}{\cos 3x}$

$\sqrt{\cos^2 3x} = \sqrt{\dfrac{1}{2}}$

$\cos 3x = \pm\sqrt{\dfrac{1}{2}}$





$\cos 3x = \sqrt{\dfrac{1}{2}}$ or $\cos 3x = -\sqrt{\dfrac{1}{2}}$

$\alpha = 45°$ $\alpha = 45$

$3x = 45°, 315°, \cancel{405°, 675°, 765°}$ $3x = 135°, 225°$

$x = 15°, 105°$ $x = 45°, 75°$

$\cancel{x = 15°, 105°, 135°, 225°, 255°}$

$\boxed{x = 15°, 45°, 75°, 105°}$

(b) Solve $3\operatorname{cosec}^2 y + 5\cot y - 5 = 0$ for $0° \leqslant y \leqslant 360°$. [5]

$3\left(\dfrac{1}{\sin^2 y}\right) + 5\left(\dfrac{\cos y}{\sin y}\right) = 5$

$\dfrac{3 + 5\cos y \sin y}{\sin^2 y} = 5$

$\cancel{\dfrac{3\sin y + 5\cos y \sin^2 y}{\sin^3 y} = 5}$

$\cancel{\sin y\,(3 + 5\cos y \sin y) = 5\sin^3 y}$

$\cancel{5\sin^2 y = 3 + 5\cos y \sin y}$

$\cancel{5\sin^2 y - 5\cos y \sin y = 3}$

$\cancel{5\sin y(\sin y - \cos y) = 3}$

$\cancel{\sin^2 y - \sin y \cos y = \dfrac{3}{5}}$

$\cancel{1 - \cos^2 y - \sin y \cos y = \dfrac{2}{3}}$

$3 + 5\cos y \sin y = 5\sin^2 y$

$5\cos y \sin y = 5\sin^2 y\,3$

$\cancel{\cos y \sin y = \dfrac{2}{5}}$

$5\sin y(\sin y + \cos y) = 3$

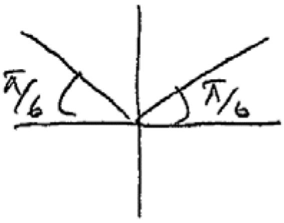
$\sin y + \cos y = \dfrac{3}{5\sin y}$

Example candidate response – middle, continued

(c) Solve $2\sin\left(z+\frac{\pi}{3}\right)=1$ for $0 \leqslant z \leqslant 2\pi$ radians. [4]

$$\sin\left(z+\frac{\pi}{3}\right)=\frac{1}{2}$$

$$\alpha=\frac{1}{6}\pi$$



$$z+\frac{\pi}{3}=\cancel{\frac{1}{6}\pi},\ \frac{5}{6}\pi,\ \frac{13}{6}\pi$$

$$z=\frac{1}{2}\pi,\ \frac{11}{6}\pi$$

Examiner comment – middle

(a) A completely correct method dealing with sec 3*x* correctly was used by the candidate. The candidate considered both the positive and negative roots of the resulting equation and as a result gave all the possible solutions (M1, A!, A1).

Marks awarded = **(a) 3/3**

(b) The candidate attempted to express all the equation in terms of sin *y* and cos *y*. Although this is an approach which is possible, with a lot of extra work involved to gain the final solutions, the candidate was unable to progress very far and as a result was unable to obtain any of the available marks (M0, M0, M0, A0, A0). It had been intended that candidates make use of the trigonometric identity $\operatorname{cosec}^2 y=\cot^2 y+1$.

Marks awarded = **(b) 0/5**

(c) A clearly well thought out approach, showing all the relevant steps and correct work concerning the necessary range of values for *z* in radians, gained the candidate full marks (M1, A1, M1, A1).

Marks awarded = **(c) 4/4**

Total marks awarded = 7 out of 12

Example candidate response – low

10 **(a)** Solve $2\cos 3x = \sec 3x$ for $0° \leqslant x \leqslant 120°$. [3]

$$2\cos 3x = \frac{1}{\cos 3x}$$

$$2\cos 3x \times \cos 3x = 1$$

$$3\cos 3x = 1$$

$$\cos 3x = \frac{1}{3}$$

$$\cancel{\alpha = 23.5°}$$

$$3x = 70.5$$

$$\cancel{x = 23.5°}$$

$$0° \leqslant x \leqslant 120°$$

$$0° \leqslant 3x \leqslant 360°$$

$3x = 70.5$ and

$3x = 289.5$

$$\boxed{\therefore\ x = 23.5° \text{ or } x = 96.5°}$$

(b) Solve $3\,\text{cosec}^2 y + 5\cot y - 5 = 0$ for $0° \leqslant y \leqslant 360°$. [5]

$$3 + 3\cot^2 y + 5\cot y - 5 = 0$$

$$3\cot^2 y + 5\cot y - 2 = 0$$

$\cot y = \frac{1}{3}$ or $\cot y = -2$

$\alpha = 71.6°$ $\boxed{\text{or } y = -63.4°}$ N.A

$$\boxed{y = 71.6°,\ 251.6°}$$

Example candidate response – low, continued

(c) Solve $2\sin\left(z+\frac{\pi}{3}\right)=1$ for $0 \leqslant z \leqslant 2\pi$ radians. [4]

$$2\sin z + \frac{2}{\cancel{3}}\sin\frac{\pi}{3} = 1$$

$$2\sin z + 1.73 = 1$$

$$2\sin z = -0.73$$

$$\sin z = -0.365$$

$$z = -\sin^{-1}(0.365)$$

$$d = +21.4$$

$$\boxed{z = 21.4°, 158.6°}$$

Examiner comment – low

(a) The candidate initially dealt with sec 3*x* correctly. When attempting to simplify the equation prior to solving it, the candidate added the cosine terms rather than multiplying them. This was considered an incorrect method (M0, A0, A0).

Marks awarded = **(a) 0/3**

(b) Use of the correct trigonometric identity together with a correct solution of the resulting quadratic equation in cot *y* and the correct method of dealing with cot *y* gained the candidate all the available method marks (M1, M1, M1). Only the solutions for $\cot y = \frac{1}{3}$ were given correctly, with no real attempt at the solutions of cot *y* = -2 within the required range being attempted (M1, M1, M1, A1, A0).

Marks awarded = **(b) 4/5**

(c) A completely incorrect method of dealing with the compound angle meant that the candidate was unable to gain any marks for this part (M0, M0, A0, A0).

Marks awarded = **(c) 0/4**

Total marks awarded = 4 out of 12

Cambridge International Examinations
1 Hills Road, Cambridge, CB1 2EU, United Kingdom
tel: +44 1223 553554 fax: +44 1223 553558
email: info@cie.org.uk www.cie.org.uk