ADDITIONAL MATHEMATICS

Paper 4037/11
Paper 11

Key messages

Candidates are reminded to read each question carefully and ensure that they have answered in full and to the required level of accuracy specified either in the rubric or the question itself. Candidates are advised that all of their work, apart from graphs or diagrams, should be written in pen. They should not write their solutions in pencil and then overwrite them in pen as this makes them difficult to read.

General comments

Well prepared candidates were able to show their knowledge of the syllabus and apply techniques learned both appropriately and correctly. There appeared to be no timing issues, but some candidates had issues with the amount of space for some questions. Continuing solutions on additional sheets or a blank back page is preferable to continuing solutions in spaces provided for other questions.

Comments on specific questions

Question 1

(i) This was an easy introduction to the paper for most candidates.

(ii) Most candidates used the discriminant to obtain the value of k. A few either attempted to use calculus, or complete the square, but these attempts were rarely successful. A large number of candidates assumed that $k = -27$ from part **(i)** and gained no credit.

Answers: **(i)** –27 **(ii)** $\frac{9}{8}$

Question 2

(a) This was well done by the majority of candidates, who generally expressed the equation in powers of 2 and equated the indices. A few candidates attempted to take logs, but were unable to carry the solution through to its conclusion.

(b) Generally well done.

Answers: **(a)** $\frac{10}{9}$ **(b)** $p = \frac{5}{3}$ $q = -2$

Question 3

Many candidates were unable to find the correct value of x when $y = 0$. It was also common to see the differential given as $\frac{1}{2x^2 - 7}$. Otherwise, the procedure for finding the equation of the normal was well executed. Many solutions were, however, left with fractions in the answer, and not given in the form asked for in the question.

Answer: $x + 8y - 2 = 0$

Question 4

(a) Apart from those candidates who thought that $\mathbf{A}^2 = \begin{pmatrix} 1 & 4 \\ 9 & 0 \end{pmatrix}$, this question was answered correctly by the vast majority of candidates.

(b) Most candidates were able to give the answers $x = 1$ and $y = -3$, but this was not always obtained by a valid matrix method. Those who either pre-multiplied by $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ or used an algebraic method were not awarded full marks.

Answers: $x = 1$, $y = -3$

Question 5

(i) Many candidates did not realise that the product rule was required in order to differentiate the second term, and consequently did not get an answer in the required form. It was common, for those who did use the product rule, not to bracket the two terms, leading to an answer of +4 rather than –4.

(ii) There were few correct attempts at this question. A large proportion of the candidates who did get the answer to part **(i)** in the correct form were unable to link that to part **(ii)**.

Answers: **(i)** $-4xe^{4x}$ **(ii)** $4\ln 2 - \frac{15}{16}$

Question 6

(i) Many candidates were able to state that $f(x) \leqslant 2$ but the complete answer was not often seen.

(ii) Most candidates were familiar with the technique for finding the inverse function, but marks were often lost by the incorrect squaring of $-\sqrt{x+5}$. The domain and range were often omitted and marks were also lost by the use of incorrect variables.

(iii) The vast majority of candidates performed the functions in the correct order and gained full marks.

Answers: **(i)** $2-\sqrt{5} < f(x) \leqslant 2$ **(ii)** $f^{-1}(x) = (2-x)^2 - 5$, $2-\sqrt{5} < x \leqslant 2$, $-5 \leqslant f^{-1}(x) < 0$ **(iii)** –4

Question 7

Candidates were generally unfamiliar with this part of the syllabus, and meaningful solutions were few and far between. More work on this topic would be beneficial.

Answers: **(i)** 82.1^0 **(ii)** 16.8s

Question 8

(i) Most candidates were able to find the equation of the line *AB*.

(ii) Again, most found this to be straightforward, but use of (–2, 4) was not allowed.

(iii) This was usually shown by solving the two equations simultaneously and showing that this was the same point as the mid-point of *AB*. An acceptable alternative was to show that the mid-point of *AB* was also on the perpendicular bisector.

(iv) Most candidates could apply the formula for the distance between two points.

(v) This part was usually correct, either by the determinant method or by using part **(iv)**.

Answers: **(i)** $3x + y = 10$ **(ii)** $y = 3x + 10$ **(iv)** $\sqrt{10}$ **(v)** 20

Question 9

(i) Most candidates substituted for cot x as $\frac{\cos x}{\sin x}$ and multiplied throughout by sin x. From that point onwards it was required that there should be no algebraic errors in order to get full marks, as a number of candidates fortuitously got the correct answer in spite of them.

(ii) A surprising number of candidates ignored the “hence” and tried to solve the equation without reference to part **(i)**, and consequently made little progress.

Answers: **(i)** $a = 2$, $b = 1$ **(ii)** $\frac{\pi}{3}$, $\frac{\pi}{4}$

Question 10

(i) Many candidates knew the factor theorem and gained full marks on this question. Of those who did not, the most common error was to use $f\left(\frac{1}{2}\right) = 0$ rather than $f'\left(\frac{1}{2}\right) = 0$.

(ii) Generally well done. Most candidates were able to divide through by $x + 2$.

(iii) Candidates needed to show, rather than just state, that $4x^2 – 8x + 13$ has no real roots, usually by showing that the discriminant is negative.

Answers: **(i)** $k = –3$, $p = 26$ **(ii)** $(x + 2)(4x^2 – 8x + 13)$ **(iii)** $x = – 2$ only

Question 11

(i) Many candidates did not know the meaning of “chord”.

(ii) In spite of getting part **(i)** wrong, many candidates were able to deduce this result correctly.

(iii) This was generally well done, but it was very common to see $(\theta + \sin\theta)$ differentiated as just $\cos\theta$ or, if the quotient rule was used, 10 differentiated as 1.

(iv) Most candidates appreciated that they should divide 15 by their answer to part **(iii)**.

Answers: **(i)** $2r\sin\theta$ **(iii)** –17.8 **(iv)** –0.842

ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key messages

Candidates are reminded to read each question carefully and ensure that they have answered in full and to the required level of accuracy specified either in the rubric or the question itself. Candidates should be reminded not to do their solutions in pencil and then overwrite them in ink as this can render their work difficult to read. All work, apart from graphs and diagrams, should be done in blue or black pen.

General comments

Well prepared candidates were able to show their knowledge of the syllabus and apply techniques learned both appropriately and correctly. There appeared to be no timing issues, but some candidates had issues with the amount of space for some questions. Continuing solutions on additional sheets or a blank back page is preferable to continuing solutions in spaces provided for other questions.

Comments on specific questions

Question 1

(a) This was a straightforward question designed to put candidates at ease and most were able to use appropriate set notation to complete the given statements.

(b) Many candidates made use of the space below the Venn diagrams to work out the appropriate shading before using the actual diagrams themselves. This made sure that most candidates performed well with only the occasional error in shading. It was hoped that candidates would appreciate the difference that the position of a set of brackets can make.

Answer: **(a)** $Y \subset X$ or $Y \subseteq X$, $Y \cap Z = \varnothing$ or $Y \cap Z = \{\}$

Question 2

(i) This was done very well by many candidates. There were occasional slips with signs, but there was evidence of a good understanding of binomial expansions. Very few errors were made in the numerical simplification of each term.

(ii) Most candidates realised that two terms needed to be considered in order to obtain the term independent of *x*. There are still candidates who are unsure of what is meant by 'the term independent of *x*', mistakenly thinking that the term involves *x* only.

Answers: **(i)** $32 - \frac{20}{x} + \frac{5}{x^2}$ **(ii)** 16

Question 3

(i) There was misunderstanding with some candidates not being able to deal with the modulus sign correctly; it is important that they realise that $|\mathbf{b}-\mathbf{c}| \neq |\mathbf{b}|-|\mathbf{c}|$. However, most were able to gain credit for obtaining either $\sqrt{2^2+y^2}$ or $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$. Several candidates only stated the positive value of y, not realising that the question was asking for possible values, not a possible value.

(ii) There were two approaches that could be made to this question. The more common approach was to use the numerical values of the vectors given and form two equations by comparing like vectors. An equally valid method was to obtain two equations by comparing vector **b** and then vector **c** without any substitution of numerical values. Completely correct solutions were common, but some candidates made arithmetic errors when dealing with the solution of the simultaneous equations obtained.

Answers: **(i)** ± 6 **(ii)** $\mu = \frac{4}{3},\ \lambda = \frac{8}{3}$

Question 4

As candidates were asked not to use calculators in this question, it was essential that each step of working was shown clearly and in full. Many candidates were able to do this and produced well-structured and carefully thought out solutions. It was expected that candidates make use of the quadratic formula and simplify it using their knowledge of surds. In some cases, candidates reached the stage of being able to give the statement $x = \frac{3+\sqrt{5}}{8+2\sqrt{5}}$ and then used a calculator to simplify this. It was clearly evident when candidates used a calculator and this was penalised appropriately.

Answer: $\frac{7+\sqrt{5}}{22}$

Question 5

(i) As with the previous question, it was essential that candidates showed each step of their working in order to gain full marks. Many completely correct solutions were seen with most candidates adopting the usual strategy of starting with the left hand side of the given statement and working through to obtain the right hand side of the statement. The main problems occurred when candidates did not realise, or did not make use of the fact, that $\sec\theta = \frac{1}{\cos\theta}$ as further progression was not then possible.

(ii) The important word in this part of question was the word 'Hence' which is meant to indicate to candidates that they make use of previous work to help them with the current part. Quite a few candidates 'started again', often unsuccessfully. Most candidates made use of $\sin\theta = \sin\theta\tan\theta$, but too many simply divided through by $\sin\theta$ thus losing two of the possible solutions. Fully correct solutions were not common and candidates should also realise that as the range of θ was given in terms of radians, then so should possible solutions. Many candidates also gave solutions outside the range, again showing the need for the question to be read properly and in full.

Answer: **(ii)** $0,\ \frac{\pi}{4},\ \pi$

Question 6

This question was designed to test the algebraic skills of the candidates when it came to simplification. There were a pleasing number of completely correct solutions showing a good understanding of algebraic manipulation. Most candidates realised that they needed to differentiate a product and most were able to make reasonable attempts to differentiate the exponential function and make use of the chain rule. The most difficult part of the question involved writing the result as a fraction with a denominator of $(4x+1)^{\frac{1}{2}}$ and this was the point at which most errors were made, mainly due to incorrect factorisation of the numerator.

Answer: $\dfrac{e^{3x}(12x+5)}{(4x+1)^{\frac{1}{2}}}$

Question 7

(i) This part was done well by most candidates. Answers in radians or degrees were acceptable as no specific units were stated for this part.

(ii) It was intended that candidates solve this part of the question by observation and deduction rather than make use of calculus, an approach which many candidates adopted often unsuccessfully. There were two marks available, one for each of the *x* and *y* values. It was expected that candidates realise that a maximum value for *y* is obtained when $\cos 3x = -1$, thus leading to the required coordinates. Again, answers in either degrees or radians were acceptable.

(iii) It was essential that candidates realised that once calculus is used in conjunction with trigonometry, that any angles are in radians unless specified otherwise. Most candidates were able to make a reasonable attempt to integrate the given function, but often lost further available marks by substituting in for the limits in terms of degrees. Again, it was essential that candidates showed all their working as requested in the question.

Answers: **(i)** $\left(\dfrac{\pi}{9}, 0\right)$ or $(20^\circ, 0)$ **(ii)** $\left(\dfrac{\pi}{3}, 3\right)$ or $(60^\circ, 3)$ **(iii)** 1.28 or $\dfrac{2\pi}{9}+\dfrac{\sqrt{3}}{3}$

Question 8

(i) Provided candidates were able to re-write the given equation in correct logarithmic form, many were successful at finding the values of *A* and of *b*. Use of the graph to obtain the intercept and hence find an expression for $\lg A$ was usually successful. Substitution methods were usually less than successful with many candidates making incorrect use of logarithms. A small number of candidates mistakenly used natural logarithms rather than logarithms to base 10. Centres should ensure that candidates are aware of the different notations that are used. Allowances for the use of the graph were made, so there were a range of acceptable values both for *A* and for *b*.

(ii) Candidates could either make use of their results from part **(i)** or make use of the graph itself. For those candidates who did not obtain the correct values in part **(i)** there was a method mark available if they adopted a correct approach using their values from part **(i)**. Allowances for the use of the graph were made, so there were a range of acceptable values for *y*.

(iii) As in part **(ii)** candidates could either make use of their results from part **(i)** or make use of the graph itself and a method mark was again available for the correct use of their values from part **(i)**. Similarly, allowances for the use of the graph were made, so there were a range of acceptable values for *x*.

Answers: **(i)** $A = 8.71$, $b = 0.617$ **(ii)** 2.93 **(iii)** 1.74

Question 9

(i) Candidates should be very aware of the wording used in a question. In this case they were asked to find the equation of a curve. Too many found the equation of a straight line using the given derivative to find the gradient at the given point. It was expected that candidates integrate the given derivative and make use of the given point to find the value of the arbitrary constant. Of those candidates that did adopt a correct approach, many were able to produce a correct solution with errors being mainly arithmetic ones.

(ii) This part of the question posed problems for many who did not read it carefully enough. Many used a perpendicular gradient using their incorrect straight line from part **(i)**. The use of an incorrect *x* value, usually either $-\frac{5}{3}$ or $-\frac{4}{3}$ in an attempt at the equation of the normal meant that candidates were then unable to progress correctly.

Answers: **(i)** $y = \frac{2}{3}(3x + 10)^{\frac{1}{2}} - 4$ **(ii)** 5.2

Question 10

(i) Many complete and correct solutions were seen, although it was an aid to candidates that they were working towards a given answer. This meant that they were able to check for errors, usually from an incorrect statement for either the area or the perimeter, if they were unable to obtain the required result.

(ii) Having a given answer in part **(i)** meant that candidates were able to achieve full marks for this part even if they had been unsuccessful in part **(i)**. Many correct methods were seen with most obtaining a correct value for *x*. Most candidates used a second derivative method to attempt to justify that the value of *P* was a minimum, but many then omitted to find this value of *P*, thus losing an accuracy mark and also highlighting the need for candidates to ensure that they have completed all that has been asked of them.

Answer: **(ii)** $8\sqrt{5}$

Question 11

(a) (i) This part was very well done by most candidates, who realised that they needed to find the area underneath the graph. There were occasional slips in the calculations of some of the areas.

(ii) It was expected that candidates should realise that deceleration was a positive value in this part of the question. Unfortunately an answer of -1.5 was all too common.

(b) Most graphs completed were fully correct, showing a good understanding of the relationship between linear velocity and displacement.

(c) (i) It was intended that candidates used the fact that the exponential term in the equation for the velocity can never be zero, either using a simple statement or using a mathematical approach. Unfortunately, many solutions were merely statements of the fact that the velocity is never zero, with no mathematical reference at all.

(ii) Most candidates recognised the need to differentiate the equation for the velocity with respect to *x*, but too many did not give the answer in the correct exact form required. This again highlights the need for candidates to read the question carefully and check that they have answered it in the required form.

(iii) Many correct solutions were seen with candidates integrating the given equation for the velocity with respect to time and either finding the value of an arbitrary constant or by using square bracket notation and the correct limits. Some candidates used a term of $6x$ rather than $6t$. Attention should be paid to the correct use of variables in questions where x and y are not used. Some candidates gave answers of 1.6, rather than the required answer correct to 3 significant figures, thus losing the last accuracy mark.

Answers: **(a)(i)** 1275 **(ii)** 1.5 **(c)(ii)** $\frac{1}{2}\ln\frac{3}{2}$ **(iii)** 1.59

ADDITIONAL MATHEMATICS

Paper 4037/21

Paper 21

Key messages

In order to do well in this paper, candidates need to show full and clear methods in order that marks can be awarded. In questions where the answer is given, candidates are required to show that it is correct and fully explained solutions with all method steps shown are needed. In questions that state that a calculator should not be used, omitting method steps often results in full credit not being given for a solution. Showing values to several significant figures clearly indicates the use of a calculator and, in these questions, will result in credit being withheld. In questions that require a solution of several steps, clearly structured and logical solutions are more likely to gain credit. Candidates should be encouraged to write down any general formula they are using as this reduces errors and is likely to improve the accuracy of their solutions. In questions where a formula or an integral may be evaluated entirely by a calculator it is advisable to show how the values have been applied to arrive at the final answer as a slight slip without evidence will usually result in all credit being lost.

General comments

Some candidates produced high quality work, displaying wide-ranging mathematical skills, with well-presented, clearly organised answers. This meant that solutions were generally clear to follow. Other candidates produced solutions with a lot of unlinked working, often resulting in little or no credit being given.

Questions which required the knowledge of standard methods were done well. Candidates had the opportunity to demonstrate their ability with these methods in many questions. Most candidates showed some knowledge and application of technique. The majority of candidates attempted most questions, demonstrating a full range of abilities.

Some candidates need to improve their reading of questions and keep their working relevant in order to improve. Candidates should also read the question carefully to ensure that, if a question requests the answer in a particular form, they give the answers in that form. When a question demands that a specific method is used, candidates must realise that little or no credit will be given for the use of a different method. They should also be aware of the need to use the appropriate form of angle measure within a question. When a question indicates that a calculator should not be used, candidates must realise that clear and complete method steps should be shown and that the sight of values clearly found from a calculator will result in the loss of marks.

Where they were required to show a given answer, candidates needed to fully explain their reasoning. Omitting method steps in such questions resulted in a loss of marks. Candidates should take care with the accuracy of their answers. Centres are advised to remind candidates of the rubric printed on the front page of the examination paper, which clearly states the requirements for this paper. Candidates need to ensure that their working values are of a greater accuracy than is required in their final answer.

When asked for a sketch, many candidates plotted and joined coordinates, rather than making a sketch showing the key features. Candidates would improve if they realised that this often resulted in diagrams that were of the wrong shape or incomplete.

Candidates are advised not to erase work that is not replaced as this may have gained credit if it had remained readable.

Comments on specific questions

Question 1

Candidates should have begun by expanding the bracketed terms and rearranging to form a three term quadratic. The correct quadratic could then be factorised to find the critical values of –3 and 5. These first steps were successfully completed by the majority of candidates. Forming the required inequality involved identifying the outside region and this was best recognised by the use of a diagram. Some candidates needed to read the question more carefully as their solution ended with the critical values shown and no inequality. Candidates should be careful not to assume that a single range of values is always the required solution and should note that when two distinct sets of values are required it should be shown as two separate ranges.

Answer: $x < -3 \quad x > 5$

Question 2

(a) Understanding of the set notation to identify three nested 'circles' of different sizes was clearly shown by many candidates. It was important that candidates labelled these circles and labelled them correctly as the reverse statements were often shown. Candidates need to familiarise themselves with such notation and not assume that a more general diagram of three overlapping circles is required.

(b) (i) All parts of this question tested accurate knowledge and application of set notation and the symbol for membership was confused with the notation used in part **(a)** for a subset and also with '='. Candidates also needed to realise that the order of the *h* and *P* was important.

(ii) Candidates were required to rewrite the given statement and those who attempted this met with a good measure of success. Candidates who tried to interpret the statement and identify the two letters did not answer the question. The set notation expected should be precise and candidates should give careful attention to symbols and brackets.

(iii) This part did not require the answer to be given in set notation and was the part where candidates had their greatest success. Candidates who attempted to list *Q*' sometimes omitted an element.

Answers: **(b)(i)** $h \in P$ **(ii)** $n(P \cap Q) = 2$ **(iii)** $\{t, h, s\}$

Question 3

(i) Candidates who had a sound knowledge of the laws of logarithms were able to write down the correct answer without any working. Candidates who showed the two stages required were also successful. Other candidates need to remember that $\log_p p$ is equal to 1.

(ii) As in part **(i)** candidates were able to arrive at the correct answer with a minimum of working. Some candidates needed to recall that lg is the concise notation for $\log_{10}$ and that $\lg 1$ is equal to 0.

(iii) The correct solution required using the subtraction/division law together with the rule for change of base. These could be applied in either order or indeed at the same time. Some candidates took less direct routes and were not always able to arrive at the required format. Care should be taken in questions like these where the full answer is not given but the format is. Candidates would improve their solutions by avoiding jumping to a final answer following insufficient steps.

(iv) This part was the most successful for the majority of candidates. When the first step was carried out correctly with $2x$ and $3x$ being multiplied, the correct answer usually followed. There was no need to combine all three log terms before anti-logging although where this was done it was completed well. Candidates were required to discard the negative root of 100 as x must be positive.

Answers: **(i)** -2 **(ii)** $-n$ **(iii)** $(\lg 5)^2$ **(iv)** $x = 10$

Question 4

(i) The correct answer in radians was stated by the majority of candidates. Answers in degrees were also given and these candidates needed to read the question more carefully.

(ii) There were many very good solutions to this part and these were usually easy to follow with each step set out separately. Candidates, in the main, found the area of a triangle and combined it with the area of three sectors. The correct triangle had a side of length 10 cm and the sectors a radius of 5 cm. There was some misunderstanding of these values most commonly candidates used a triangle of side 5 cm only. Candidates would be advised to have a separate sketch of the triangle to avoid this and to also avoid taking the height of the triangle as being equal to the side length. Candidates should have subtracted the sectors from the triangle as the diagram suggests. Where the values found made this a negative value candidates would do better to follow this as their plan and check their working, rather than reverse their calculation. There was sometimes an inaccuracy in the final answer and candidates would improve this by working with exact values and avoid combining rounded values from their earlier calculations.

Answers: **(i)** $\frac{\pi}{3}$ **(ii)** 4.03(1...) or $25\sqrt{3} - \frac{25\pi}{2}$

Question 5

Candidates were instructed not to use a calculator in this question and a clear method was therefore essential.

(a) Many fully correct solutions were seen. A correct answer with no supporting work was assumed to have been derived from a calculator and gained no credit. The method for rationalisation had clearly been well taught. Most candidates correctly stated the first step of multiplying numerator and denominator by $\sqrt{7} + \sqrt{5}$. There were some errors in manipulation.

(b) This part proved challenging for many candidates. The majority successfully expanded the right hand side but often no further progress was made. Candidates who equated the rational and irrational parts were able to find solutions giving the positive values for p and q. Candidates are advised to always consider the negative root of a value for suitability as this would have led to the remaining solutions.

Answers: **(a)** $\sqrt{14} + \sqrt{10}$ **(b)** q = 4, –4 p = 16, –16

Question 6

(i) A good knowledge of completing the square was shown by many candidates with some slips in manipulation leading to one or two incorrect values on occasion. Fewer incorrect formats were evident than in previous years.

(ii) Candidates should note that when a question asks them to state values then little if any working is required. The vertex can be found by using values from the previous part as *(–q, –r)*. The term vertex was misunderstood and many candidates solved the quadratic finding the roots instead.

(iii) Most candidates showed a graph of a modulus function. When the original quadratic was also shown, candidates made fewer errors. Candidates are advised to consider the curvature of the original quadratic and to mirror this in their modulus sketch. Candidates should read the question carefully. The question asked for coordinates on the axes to be shown. Some candidates forgot to do this. Care should also be taken not to distort the shape of a graph by excessive plotting or by assuming that the vertex lies on the *y*-axis.

Answers: **(i)** $4(x+1)^2 - 9$ **(ii)** (–1, 9)

Question 7

(i) Many candidates clearly understood what was required here. Both parts **(a)** and **(b)** were often completed correctly.

(ii) Many candidates correctly deduced that the points were collinear and used that phrase or a sufficient description. It was much less common to see justification of this statement. Some candidates would improve their answers by stating that the parallel vectors have a common point in addition to being multiples of each other.

(iii) A small number of candidates were fully successful here, finding the correct vector, magnitude and subsequent unit vector. Some candidates did not find the vector $\overrightarrow{OR}$ accurately. When this was the case, no further progress was usually made. Some candidates found the modulus of the two given vectors while others worked with **p** and **q** rather than **i** and **j**. A few candidates would have done better if they had taken a little more care as slips often spoiled an otherwise correct answer.

Answers: **(i)(a)** $\mathbf{q} - \mathbf{p}$ **(b)** $2\mathbf{q} - 2\mathbf{p}$ **(ii)** Points are collinear **(iii)** $\frac{1}{5}(4\mathbf{i} - 3\mathbf{j})$

Question 8

(a) (i) The Binomial Theorem is given on the Formulae page of the examination paper so it was expected that candidates would evaluate the coefficients and write terms in their simplest form. Most candidates carried this out successfully. A few candidates left their attempts in an unsimplified form. There were very few errors. Occasionally powers or terms were omitted.

(ii) Candidates saved themselves considerable time when they followed the instruction 'Hence' and applied their expansion from part **(i)**. Rewriting the entire expansion from scratch often led to the appropriate term not being identified. There was some uncertainty over what comprised a term independent of *x* and when the term was identified correctly bracketing often needed improvement to arrive at an accurate answer.

(b) This proved the most challenging question on the paper for many candidates who often found it difficult to proceed beyond an equation involving $\binom{10}{3}$. There were some very good and complete solutions where this was expanded, the fractions removed and a quadratic (or sometimes a cubic) equation formed and solved. Candidates using their calculators to find the value of *n* using a trial and improvement approach risked losing all the marks if they made an error.

Answers: **(a)(i)** $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ **(ii)** $\frac{24}{25}$ **(b)** 6

Question 9

(a) Candidates who were most successful in this part of the question interpreted the information given. Other candidates attributed the three given values to *a*, *b* and *c*. This often led to a correct value for *c*. Otherwise these candidates were unsuccessful.

(b) (i) Candidates who plotted points often did well. On occasion one misplaced plot completely changed the shape of the curve and many, if not all, of the expected attributes were lost. Candidates should always consider what symmetries and curvature they expect to see and check any points that seem out of place.

Many candidates sketched the curve with a good degree of success. This method allowed candidates to concentrate on the key features of a cosine curve. Candidates would have improved their mark in many cases had they given more care to the curvature of the graph at the ends of the domain. These should have shown that a turning point was being approached. Some candidates drew graphs which began and ended with straight lines or with sections of completely incorrect curvature.

(ii) The question asked for exact solutions and not the number of solutions as was often the answer given. The question also suggested using the graph which implied that the values could be read from the graph, as was the case. Candidates who read the question carefully usually gave some correct values. These candidates would have gained full credit had they identified the solutions from the negative root as well as the positive ones.

Answers: **(a)** $a = 2 \quad b = 4 \quad c = -2$ **(b)(ii)** $-\frac{\pi}{2}, \frac{\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6}, -\frac{\pi}{3}, \frac{\pi}{3}$

Question 10

(a) Candidates either were completely correct or incorrect in part **(i)**. Candidates who recognised that even numbers only were required fared best as they worked with 4! rather than 5!. In part **(ii)** there was a greater likelihood of success, possibly as the method was a single permutation. This was occasionally calculated as a single combination. Some candidates may have improved if they had given a simple diagram showing how many ways there are to fill each position in the 3-digit number.

(b) A few candidates gave fully correct solutions. As in part **(a)** candidates who drew sketches improved their chances of success. Candidates did better if they read the question carefully and considered how the desired objective could be achieved. In each part it was more common for a candidate to partially solve the problem.

Answers: **(a)(i)** 48 **(ii)** 60 **(b)(i)** 24 **(ii)** 18

Question 11

(i) The integration of both terms was carried out very well by the majority of candidates.

(ii) The majority of candidates substituted y as zero and found x as 9 with a small amount of work. Some candidates gave answers that were unnecessarily lengthy. These candidates risked making errors in their less direct approaches.

(iii) Verification of the given information could be achieved by substitution and did not require solution of simultaneous equations. Solution of the equations was the most popular approach and was carried out efficiently and correctly by nearly all who used this method. Candidates who did attempt to verify by substitution of $x = 3$ and $y = 9$ needed to take care to substitute these values into both equations to complete the verification.

(iv) This was a question which was best attempted as a standard solution by calculating the area of the triangle – using base and height as suggested by part **(iii)** – then calculating the area under the curve – using part **(i)** – and then subtracting the area under the curve from the area of the triangle. Candidates chose to use a very great variety of methods. These met with an equally great variety of success. For example: the triangle *OAB* was often split into two right-angles triangles and the area of each, or one, found; the area of one or both triangles was found using $\frac{1}{2}ab\sin C$; "area of triangle minus area under curve" was often found for both parts separately; the equations of line and curve were subtracted and then integrated.

Candidates would improve by avoiding using these less direct methods to answer the question. It is generally less straightforward to identify required limits when combining different parts in these ways. Candidates should be advised that when evaluating definite integrals, the substitution of the limits should always be clearly shown. Candidates are also advised to maintain accuracy throughout their calculations as the correct answer should have been exact to one decimal place.

Answers: **(i)** $\frac{3x^2}{2} - \frac{2x^{5/2}}{5} \, (+c)$ **(ii)** (9, 0) **(iv)** 16.2

Question 12

Candidates should note that on a structured question such as this where the answer to later parts relies on the answer to the previous part(s) that slips in the first part can lead to the loss of several marks overall. As much as possible, candidates are advised to follow a simple, direct method and take care with simplification.

Candidates who were careful with signs, logic and presentation often did best in this question.

(i) A good number of candidates used the simplest approach. The differentiation was best carried out by using the quotient rule for the fraction term and dealing with the $12x$ separately. The product rule was also used successfully and led to a large number of terms when finding the second derivative later. Combining the terms over a single denominator was a popular approach. This method involved more complicated algebra and a more complicated application of the quotient rule. As a result there is a greater possibility of an error appearing in the work. In all methods candidates are advised to use correct bracketing to avoid sign errors when simplifying expressions.

(ii) Candidates who had followed the more straightforward approach or had simplified their expression correctly in part **(i)** were rewarded as the constant term differentiated to zero and the chain rule could be applied to the fraction term. In each of the other approaches it was expected that simplification to a single term take place in order to reflect this.

(iii) In order to gain full credit here it was necessary for a candidate to have their derivatives correct in some form. Most candidates put their first derivative equal to zero in order to find the required x values. There were some who put y or their second derivative equal to zero. Some candidates did find the correct coordinates of the turning points, as required. Other candidates would do better if they understood that the question required them to find both the x- and the y-coordinates of the turning points. Those candidates who had proceeded correctly to this point were then able to evaluate the second derivative correctly and identify the maximum and minimum as appropriate. Almost all candidates gave the correct interpretation. Very few candidates attempted to evaluate y or $\frac{dy}{dx}$ on either side of the turning points. These approaches involved the need for more calculations and subsequently often led to more errors.

Answer: **(i)** $\frac{3}{(x-1)^2} - 12$ **(ii)** $k(x-1)^{-3}$, $k = -6$ **(iii)** minimum at (0.5, 2) maximum at (1.5, –22)

ADDITIONAL MATHEMATICS

Paper 4037/22
Paper 22

Key messages

In order to do well in this examination, candidates need to give clear and logical answers to questions, with sufficient method being shown so that marks can be awarded. Candidates need to give attention to instructions in questions such as 'Do not use a calculator in this question.' This instruction means that omitted method will result in a significant loss of marks. Whilst calculators are useful and candidates should be encouraged to use them to check their solutions throughout, efficient use of a calculator is not an assessment objective of the syllabus. Candidates who omit to show key method steps in their solution to a question through using a calculator, even when permitted, risk losing a significant number of marks should they make an error. The rubric, printed on the front page of each paper, indicates the accuracy required for non-exact answers. Candidates should ensure that their answers are given to no less than the stated accuracy in order to be credited. Attention should be given to the accuracy required for angles measured or given in degrees, which varies from those measured or given in radians.

General comments

Candidates seemed well prepared for this examination and displayed good knowledge of key concepts.

A good number of candidates gave well-presented answers. These answers showed a logical progression through the method of solution. Candidates presenting such work were often correct in their thinking and scored highly. Other candidates need to appreciate that poorly presented work is often difficult to credit as the logic of what is being attempted is often difficult to determine. Some candidates used additional paper. This helped their presentation if they had made a second attempt at a question. When candidates write answers in alternate locations within their script, indicating where they have written the continuation of their solution is helpful.

Many candidates gave full and clear solutions with all method steps shown. Some candidates could improve by showing every key step of their method. This is even more important if they make an error. Showing clear and full method is essential if a question asks candidates to 'Show that…' This instruction indicates that the answer has been given and that the marks will be awarded for showing how that answer is found. The need for this was highlighted in **Question 1(i)**, **Question 4(i)** and **Question 7(ii)** in this examination. Showing clear and full method is also very important if the use of a calculator is not allowed, such as in **Question 4**.

Many candidates followed any specific instructions they were given and demonstrated their ability to use the methods of solution required. Other candidates need to give more attention to key words in a question such as 'Hence' or 'exact'. The need for this was highlighted in **Question 2(ii)**, **Question 9(b)(ii)** and **Question 10(b)** in this paper. When 'Hence' is used, candidates are expected to apply what they have just previously found to form the solution to the part of the question they are now answering. When an 'exact' answer is required, decimals are not generally acceptable.

When a question demands that candidates 'Explain why' something is valid or correct, it is important that the explanation is supported with a valid reason. Comments made without justification are often just restatements of the information given in the question. This was evident in **Question 6(i)** and **Question 11(b)** in this paper.

All candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

The question was a good start to the paper for nearly all candidates.

(i) A very high proportion of candidates found a correct expression for the discriminant. Most of these knew the correct condition to apply for the given equation to have no real roots. A small number of candidates would have improved if they had been a little more careful as they mixed x with k or made sign or rearrangement errors or omitted brackets.

(ii) Most candidates factorised correctly to find the correct critical values, 1 and 3. The majority of these stated the correct inequality in k. A small number of candidates gave their answer as $k < 1$ and $k < 3$ or gave the solutions for $k^2 - 4k + 3 > 0$. Those candidates who found the critical values and made a simple sketch were almost always successful.

Answer: **(ii)** $1 < k < 3$

Question 2

(i) This question was also well answered by most candidates. The majority applied the quotient rule and did so correctly. There were a few candidates who, again, would have done better if they had been more careful with brackets and signs as some good solutions were spoiled by sign errors. Among those who did not score full marks, it was common for the numerator of the final answer to be given as 16, for example. Candidates who quoted the general form of the quotient rule and the components required to complete it, before substitution, were usually successful. A few candidates used the product rule. This was acceptable, although the simplification that was required was not always completed successfully in these cases. Almost all candidates attempted to simplify, as required. It was not necessary to expand the denominator, although some candidates did so.

(ii) It was a requirement that candidates used their derivative from part **(i)** to answer this part of the question. This was indicated by the word 'Hence'. A good number of candidates did find that $x = 2$ when $y = 9$ and successfully completed their solution using $\frac{0.07}{\delta x} \approx \left.\frac{dy}{dx}\right|_{x=2}$. A few candidates found $\frac{dx}{dy}$ in terms of y and used $y = 9$. These were given full credit if their solution was completely correct. Some candidates found x when $y = 9$ and when $y = 9.07$ and worked out the difference. This was not an acceptable method of solution as a calculus-based method was being assessed. Many candidates would have benefited, perhaps, from reading the question more carefully as often candidates used $x = 9$ in their $\frac{dy}{dx}$ and multiplied this by 0.07. Other candidates were unsure of how to use their answer to part **(i)**. These usually attempted to find the value of x that arose from setting $\frac{dy}{dx} = 9$.

Answers: **(i)** $\frac{dy}{dx} = \frac{14}{(3-x)^2}$ **(ii)** 0.005

Question 3

Many candidates scored full marks for this question. Some candidates would improve if they knew how to use their calculator efficiently to evaluate combinations. Other candidates only gave one part of the sum of differences required to find the answer. These candidates could have done better if they had listed the possible outcomes required i.e. that they should have 3 women and 0 men or 2 women and 1 man to satisfy the criterion given. A small number of candidates used permutations rather than the necessary combinations. Some candidates combined their combinations using incorrect operations – commonly adding when they should have been multiplying.

Answer: 161

Question 4

Candidates were instructed that calculators were not to be used in this question. It is a syllabus requirement that candidates are able to factorise polynomials. A high proportion of candidates demonstrated their mastery of this skill. Full credit was given to candidates who showed the correct factorisation of the cubic and gave the full set of roots from that factorisation. Candidates who used their calculator to find the roots and work back were not given full credit.

(i) Most candidates successfully used the factor theorem. This was the simplest method of showing what was required. Some candidates used synthetic division. This resulted in some interpretation errors. Other candidates chose formal long division. Again, this was more prone to error.

(ii) A high proportion of candidates found or stated the correct quadratic factor. Most of these successfully factorised and gave the roots from their factors. Candidates who chose to use the method of synthetic division needed to make sure that they interpreted their workings. They needed to state the relevant quadratic factor that resulted. It was clear that some candidates were using their calculators to find roots and work back to state factors. The question required the factorisation to be done first and 'hence' the roots be found. As $x - 2$ was given as a factor in part **(i)**, candidates needed only to factorise the correct quadratic factor here. Those who concentrated their efforts in doing this were almost always rewarded. A few candidates would have done better if they had read the question more carefully. Some factorised successfully but did not state the roots. Some used the quadratic factor rather than factorising. This was not credited.

Answer: **(ii)** $(x - 2)(2x - 7)(x + 4)$ leading to $x = 2$, $x = -4$, $x = 3.5$

Question 5

(i) Very well answered – almost universally correct.

(ii) Some candidates used the relationship between the gradient of *AB* and the gradient of *CM* and found the value of *p* directly. Others found the equation of the line *CM* and then used that to find the value of *p*. Very many candidates found the correct value. Occasional arithmetic slips were made in calculating the gradient of *AB*. The few who did not score at all would likely have done better if they had taken more care over the points used to form their gradient or equation as many of these used the coordinates of *A* or *B* rather than *C*. A simple sketch may have helped.

(iii) A good number of candidates earned all three marks here. Some candidates earned two marks only as they had rounded their working values prematurely and their final answer was insufficiently accurate. Some candidates drew diagrams which were helpful. Other candidates would have done better if they had plotted the points they were working with in the approximately correct positions, so that the lengths they were working with were clear. Some candidates, with poorly orientated diagrams, stated the lengths of *MB* and *CM* as 4 and 8 – their diagrams misleading them into subtracting *x*- and *y*-coordinates respectively, as their orientation gave the impression of vertical and horizontal differences, which was not the case. Some candidates realised that the tangent of the angle could be equated to the gradient and this used to find the answer. The simplest method of solution was to find the length of *MB* or *CM* and use that with simple trigonometry in a right-angled triangle. Many good solutions were seen. Some chose to use the cosine rule. This was fine, but did introduce more opportunities for errors, such as rounding errors.

Answers: **(i)** (2, 8) **(ii)** $p = 6$ **(iii)** 26.6°

Question 6

It is possible that more careful attention to the terminology used in circular measure questions – such as sector, segment, arc length etc. would have helped some candidates in this question. Some unnecessary conversion to degrees resulted in rounding errors.

(i) The question required an explanation of why it had to be greater than 1 radian, rather than a demonstration that it was actually greater than 1. The simplest explanation was to state that the arc length was greater than the radius. Some candidates observed that the angle was not equal to 1 radian as the arc length was not 5. These candidates would have scored if they had completed their arguments. Many candidates showed or demonstrated that θ was greater than 1 radian using calculations. Calculations such as $5 \times 1 \neq 7$ or statements such as '5 must be multiplied by something more than 1 for the answer to be 7' were common. Calculations such as these **demonstrated** the result but did not **explain** or **justify why** this was the case. Other candidates thought that the sector would not exist if the angle were less than 1 radian.

(ii) Most candidates used the relationship between the arc length, radius and angle in radians to find θ correctly. Some candidates misinterpreted the information in the question and thought that the chord *AB* was 7 cm. These candidates used the cosine rule in this part to find the angle. Some candidates found the angle in degrees. These were not credited as it was clear in the question that the angle, θ, was an angle in radians not in degrees.

(iii) Many candidates successfully found the sector area. Some candidates found the area of the triangle rather than the sector. A few candidates worked in degrees and were not accurate enough to score.

(iv) A good number of candidates arrived at the correct answer here. Even candidates who omitted to score in part **(iii)** often made a recovery here and calculated a correct difference and answer.

Answers: **(ii)** 1.4 radians **(iii)** 17.5 **(iv)** 5.18

Question 7

(i) Very well answered – almost universally correct.

(ii) A good number of candidates gave fully correct solutions. These solutions detailed the matrix **AB**, the calculations and determinants for **AB**, **A** and **B** and compared them. Some candidates omitted to show full details of how they had found their determinants. Evidence of independent calculation of each determinant was required for all three determinants for full marks to be awarded. A small number of candidates did not conclude the argument, which was also required for full credit to be given. Other candidates would have improved if they had fully understood the determinant as an important value in its own right. Many candidates were clearly confused between the determinant and its reciprocal as a required component of the inverse matrix. Calculation of the determinant of a non-singular 2×2 matrix is a syllabus requirement. Some candidates were confused between the modulus notation for absolute value and the notation used for determinant. These candidates gave det **AB** as $|-70| = 70$, for example. This was not condoned. It may have helped these candidates if this notation had been avoided.

(iii) Many good solutions were seen. Candidates were able to state both components of the inverse correctly and few errors were seen. Some candidates found the matrix **AB** in this part, having not realised its usefulness in part **(ii)**. Candidates were clearly well drilled in this skill.

Answers: **(i)** $\begin{pmatrix} 16 & 17 \\ 10 & 9 \end{pmatrix}$ **(iii)** $-\frac{1}{70}\begin{pmatrix} 12 & -23 \\ -14 & 21 \end{pmatrix}$

Question 8

This was a standard simultaneous equations question which was answered very well by almost all candidates. Although a few candidates were careless with their initial substitution or expansion of brackets, errors were rare and fully correct answers common. Most candidates chose the simpler option of eliminating *y* to give an equation in terms of *x*. Some candidates did not simplify to a quadratic equation and had clearly used their calculator solver function to find solutions. This is not recommended. If the solutions found are incorrect or incomplete, many marks may be lost. Also, if such a question requires all working to be shown, as is often the case, full marks will not be awarded for truncated working of this form. Whilst most solutions were well presented, occasionally solutions were poorly presented with 'scribbled out' sections or alterations where candidates had over-written figures making it difficult to determine whether the work was actually correct.

Answer: $\left(-\frac{1}{2}, \frac{5}{2}\right)$, $(-1, -5)$

Question 9

(a) A good number of candidates understood the need to simplify the expression to the sum of three terms, each a power of *x*, prior to integrating. When this was the case, the integration was usually carried out correctly. After correct simplification, occasionally, candidates gave $\int \frac{1}{x^2}\,dx = \frac{3}{x^3}$. The other two terms were usually integrated correctly. Many candidates had clearly been well taught and understood the need to include a constant of integration as part of their solution. Other candidates would improve if they had understood that it is not valid to integrate a product 'term-by-term'. Many candidates who did attempt to simplify the rational expression prior to integrating did so incorrectly. It was common to see $\int (x^3 + x^2 + 1 + x^{-2})\,dx$ attempted, for example. Some candidates also treated $\frac{x^2}{x^2}$ as being zero and consequently only integrated 2 terms. It was also common to see $\int (x^3 + x^2 + 1 \times x^{-2})\,dx = \frac{x^4}{4} + \frac{x^3}{3} + x \times \frac{x^{-1}}{-1}$ or $\int \frac{x^3 + x^2 + 1}{x^2}\,dx = \frac{\frac{x^4}{4} + \frac{x^3}{3} + x}{\frac{x^3}{3}}$

Some candidates attempted the quotient rule for differentiation here.

(b) (i) Many good and fully correct solutions were seen here. Some candidates unnecessarily rewrote the expression given using the identity for the sin(*A* + *B*). This was not wrong, but was more prone to error. Some candidates may have improved had they realised that 'raising the power by 1 and dividing by the new power' only applies to certain powers of *x* and is not a universal rule when integrating. These candidates would likely have benefited from a deeper understanding of what they were trying to do. Some candidates need to take more care as they differentiated the expression, rather than integrating it. Some candidates would have done better if they had taken more care with brackets. A small number of candidates treated π as a variable.

(ii) Those who had answered part **(i)** correctly almost always earned two marks here. Candidates were required to use their answer from part **(b)(i)**. This was indicated by the use of the word 'Hence' in the question. Candidates who gave the correct answer without evidence of the substitution of the limits were not credited. Candidates should not rely on the numerical integration facility of their scientific calculators and method should always be shown. Some numerical errors were seen in the evaluation of the integral. These may have arisen from approximating the value of π. Candidates should take care with such things as using approximate values for π will generally not result in full marks being awarded at this level. Some candidates reversed the order of substitution of the limits. These candidates may have improved if they had had more practice in evaluating definite integrals where one or both of the limits were negative. Some candidates ignored the upper limit. It is likely that these candidates assumed that, as the upper limit was 0, the expression would also be 0. These candidates would possibly have done better if they had had more practice with situations where this is not the case – situations similar to this one, with cosines – or with the exponential function, for example.

Answers: **(a)** $\frac{x^2}{2}+x-\frac{1}{x}+c$ **(b)(i)** $\frac{-\cos(5x+\pi)}{5}+c$ **(ii)** 0.4

Question 10

(a) This question was very well answered by most candidates. Candidates understood the correct method of solution was to substitute the points given into the equation given thus forming a pair of simultaneous equations to be solved. Usually, these equations were solved correctly. Those candidates not scoring full marks made errors such as thinking $4^0 = 4$ or $4^{2\times 0} = 16$. Others found a gradient using the two points given and attempted to proceed as if they were dealing with a straight line, rather than a curve.

(b) Most candidates used the approach suggested in the question and correctly rewrote the equation as $u^2-2u-24=0$. Usually, this was factorised correctly and the solutions $10^x=6$ and $10^x=-4$ found. The question required the exact answer to be given. Very many candidates would have done better if they had reread the question before giving their final answer. Many candidates gave their answers as decimals – and often these were correct to 3 significant figures. These candidates would have done better if they had realised that, in a question involving logarithms, an exact answer is likely to be in the form of a logarithm.

(c) Some very good solutions were given to this question and many candidates scored full marks. These candidates showed good knowledge of the laws of logarithms and applied them correctly. Many candidates were clearly well prepared for questions of this type. A few candidates misapplied the subtraction/division rule and started their solution with $\frac{\log(x+1)}{\log x}=3$ or misapplied the addition/multiplication rule and started with $\log_2 x.\log_2 1-\log_2 x=3$. Very occasionally, candidates anti-logged giving $\frac{x+1}{x}=3^2$ rather than $\frac{x+1}{x}=2^3$

Answers: **(a)** $p = 5$, $q = 3$ **(b)** $x=\lg 6$ **(c)** $x=\frac{1}{7}$

Question 11

(a) Some very neat and precise solutions were seen. Candidates who used calculus to find the turning point or who used $x = -\frac{b}{2a}$ almost always successful. These methods of solution were simplest and most who used them scored full marks. Occasionally candidates omitted to find the greatest value of the function, but this was rare. Those candidates who chose to complete the square often made sign errors. This resulted in many candidates not having a valid method for finding the value of *x* at which the greatest value arose. Candidates who chose to find the roots rarely proceeded to use the symmetry of the curve and find the value of *x* from that. Often these candidates simply stated that the maximum value was 0 or 1 and it arose when *x* was 1 or 0.

(b) When asked to 'Explain why' something is valid, the explanation given should always be fully justified. Some very neat and well-justified explanations were seen. Candidates who justified the function being one-one on the domain given because the graph had already turned were rewarded. Also candidates who said it was one-one on this domain and sketched the function to demonstrate this were also rewarded. Candidates who observed that the gradient of the function was decreasing after this point had also fully justified their answer. Very many candidates simply stated that the function was one-one, without any supporting evidence or justification. This meant that their explanations were incomplete and insufficient. These candidates may have improved if they had made a simple sketch of the function or made some comment in justification. Some candidates made reference to the inverse function and its domain. Many said that the inverse function would have a square root so the domain could not be negative. Most of these solutions were too vague to be credited.

(c) (i) Candidates performed well when answering this question. Almost all candidates recognised that they need to form a composite function and did so in the correct order. A few candidates thought that, as one of the functions involved a square root, there would be two solutions. This was not the case. Candidates should understand that if the negative form of the square root is intended, there will be a minus sign in front of the radical to make the distinction. A very small number of candidates attempted to find a product of the two functions, rather than forming the composite. A few candidates would have improved if they had been more careful with brackets – bracketing errors occasionally resulted in an incorrect final answer or an incorrect general expression for hk(x).

(ii) A few candidates were careful enough in reading the question and with their use of notation to earn all five marks. A small number of candidates only found the inverse function and did not attempt to find the domain or range. A few other candidates miscopied the function k as $5+\sqrt{x+1}$ even though they had used the function correctly in part **(c)(i)**. A few candidates made algebraic errors when finding the inverse – such as square rooting rather than squaring or separating $\sqrt{x-1}$ into $\sqrt{x}-\sqrt{1}$. Some candidates would improve if they took more care with the notation used for domain and range. An inequality representing a domain should be in terms of x, if the independent variable is x for that function. Inequalities in y or k etc. for the domain are not given credit. Similarly, the range for the inverse function should be in an appropriate notation for a dependent variable. In this question, that would be $k^{-1}(x)$ or y, although other notations that implied 'output' were allowed. Some candidates would have done better if they had paid more attention to the inequality signs in the domain of k. It was common for the domain to be given as $x \geqslant 5$ and the range as $y \geqslant 1$. It was clear that many candidates had missed the fundamental connections between the domain and range of a function and its inverse. These candidates often gave the domain of the inverse as the domain of the function and the range as $17 \leqslant y \leqslant 9217$.

Answers: **(a)** Greatest value is $\frac{1}{4}$ when $x = \frac{1}{2}$ **(c)(i)** 1 **(ii)** $k^{-1}(x) = (x-5)^2 + 1$, $5 < x < 15$, $1 < k^{-1}(x) < 101$

Question 12

Some candidates gave excellently-presented answers and made it clear which angles were part of their final answer and which angles were 'base angles'. Other candidates would have done better if they had been a little more careful with their presentation and the description of what they had found. A reasonable number of candidates earned full marks in both parts. Some candidates would have done better if they had followed the guidance given in the rubric on the front page of the examination paper. Angles in degrees should be given to 1 decimal place and other, non-exact answers, should be given to 3 significant figures. Premature approximation in questions of this type results in a loss of marks.

(i) Almost all candidates used the correct substitution $\sin^2 A = 1 - \cos^2 A$ and found a correct 3-term quadratic in $\cos A$. Some candidates used the relationship $\cos^{-1}(-0.25) = 180 - \cos(0.25)$ successfully. On occasion, this approach resulted in an error as some candidates included the angle 75.5 as a solution. Candidates should try, where possible, to use the most direct method of solution, as this is likely to produce the simplest solution. Some candidates also included 120 as a solution. Some candidates would have done better if they had been more careful with signs as, on occasion, errors were made in converting $-8\cos^2 A + 2\cos A + 1 = 0$ to $8\cos^2 A - 2\cos A - 1 = 0$ or in factorising the correct quadratic.

(ii) Many candidates started this question successfully – realising the correct relationship between cosec(3*B*+1) and sin(3*B*+1). Many of these then applied the correct order of operations and found at least one correct angle. A reasonable number of candidates earned full marks. Those candidates scoring three marks usually gave 0.58 as the first angle without stating a more accurate answer. Some candidates would have done better if they had attempted to look for angles of greater magnitude. These candidates stopped working once they had found the first angle in range. Some candidates converted to degrees. Some of these then converted back to radians for their final answers. These had often made approximation errors because of this unnecessary conversion. A small number of candidates would have improved if they had understood the correct order of operations. For example, $\sin(3B + 1) = \sin(3B) + \sin(1)$ was seen on occasion. Also, a few candidates found $\sin^{-1}\left(\frac{0.4-1}{3}\right)$. Another common issue was an initial step of $\frac{1}{\sin}(3B+1) = 0.4$ followed by $(3B+1) = \sin 0.4$ or $(3B+1) = \sin^{-1} 0.4$.

Answers: **(i)** 60°, 104.5° **(ii)** 0.577, 1.90, 2.67 radians