

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2015 series

4037 ADDITIONAL MATHEMATICS

4037/12

Paper 1, maximum raw mark 80

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1	$kx^2 + (2k - 8)x + k = 0$ $b^2 - 4ac > 0 \text{ so } (2k - 8)^2 - 4k^2 (> 0)$ $4k^2 - 32k + 64 - 4k^2 (> 0)$ leading to $k < 2$ only	M1 DM1 DM1 A1	for attempt to obtain a 3 term quadratic in the form $ax^2 + bx + c = 0$, where b contains a term in k and a constant for use of $b^2 - 4ac$ for attempt to simplify and solve for k A1 must have correct sign
2	$\left(\frac{dy}{dx}\right) = -5x(+c)$ When $x = -1$, $\frac{dy}{dx} = 2$ leading to $\frac{dy}{dx} = -5x - 3$ $y = -\frac{5x^2}{2} - 3x + d$ When $x = -1$, $y = 3$ leading to $y = \frac{5}{2} - \frac{5x^2}{2} - 3x$ <p>Alternative scheme:</p> $y = ax^2 + bx + c \text{ so } \frac{dy}{dx} = 2ax + b$ When $x = -1$, $\frac{dy}{dx} = 2$ so $-2a + b = 2$ $\frac{d^2y}{dx^2} = 2a$ so $a = -\frac{5}{2}$, $b = -3$, $c = \frac{5}{2}$	M1 A1 DM1 A1 M1 A1 DM1 A1	for attempt to integrate, do not penalise omission of arbitrary constant. Must have $\frac{dy}{dx} = \dots$ for attempt to integrate <i>their</i> $\frac{dy}{dx}$, but penalise omission of arbitrary constant. for use of $y = ax^2 + bx + c$, differentiation and use of conditions to give an equation in a and b for a correct equation for a second differentiation to obtain a for a , b and c all correct

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3	$\sqrt{(\sec^2 \theta - 1)} + \sqrt{(\operatorname{cosec}^2 \theta - 1)} = \sec \theta \operatorname{cosec} \theta$ <p>LHS = $\tan \theta + \cot \theta$ $= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$ $= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$ $= \frac{1}{\sin \theta \cos \theta}$ $= \sec \theta \operatorname{cosec} \theta$</p> <p>Alternate scheme:</p> <p>LHS = $\tan \theta + \cot \theta$ $= \tan \theta + \frac{1}{\tan \theta}$ $= \frac{\tan^2 \theta + 1}{\tan \theta}$ $= \frac{\sec^2 \theta}{\tan \theta}$ $= \frac{\sec \theta}{\tan \theta} \times \sec \theta$ $= \operatorname{cosec} \theta \sec \theta$</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>may be implied by the next line</p> <p>for dealing with $\tan \theta$ and $\cot \theta$ in terms of $\sin \theta$ and $\cos \theta$</p> <p>for attempt to obtain as a single fraction</p> <p>for the use of $\sin^2 \theta + \cos^2 \theta = 1$ in correct context</p> <p>Must be convinced as AG</p> <p>may be implied by subsequent work</p> <p>for attempt to obtain as a single fraction</p> <p>for use of the correct identity</p> <p>for ‘splitting’ $\sec^2 \theta$</p> <p>Must be convinced as AG</p>
4	<p>(a) (i) 28</p> <p>(ii) 20160</p> <p>(iii) $6 \times (5 \times 4 \times 3)$ oe to give 360 $6 \times (5 \times 4 \times 3) \times 2$ $= 720$</p> <p>(b) Either ${}^{10}C_6 - {}^7C_6 = 210 - 7$ $= 203$</p> <p>Or</p> <p>1W 5M = 63 2W 4M = 105 3W 3M = 35 Total = 203</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1, B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p>	<p>for realising that the music books can be arranged amongst themselves and consideration of the other 5 books</p> <p>for the realisation that the above arrangement can be either side of the clock.</p> <p>B1 for ${}^{10}C_6$, B1 for 7C_6</p> <p>for 1 case correct, must be considering more than 1 different case, allow C notation</p> <p>for the other 2 cases, allow C notation</p> <p>for final result</p>

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5	(i)	$\frac{dy}{dx} = (x-3) \frac{4x}{2x^2+1} + \ln(2x^2+1)$ <p>when $x=2$, $\frac{dy}{dx} = -\frac{8}{9} + \ln 9$ oe or 1.31 or better</p>	B1 M1 A1 A1	for correct differentiation of ln function for attempt to differentiate a product for correct product, terms must be bracketed where appropriate for correct final answer
	(ii)	$\partial y \approx (\text{answer to (i)}) \times 0.03$ $= 0.0393$, allow awrt 0.039	M1 A1FT	for attempt to use small changes follow through on <i>their</i> numerical answer to (i) allow to 2 sf or better
6	(i)	$A \cap B = \{3\}$	B1	
	(ii)	$A \cup C = \{1, 3, 5, 6, 7, 9, 11, 12\}$	B1	
	(iii)	$A' \cap C = \{1, 5, 7, 11\}$	B1	
	(iv)	$(D \cup B)' = \{1, 9\}$	B1	
	(v)	Any set containing up to 5 positive even numbers ≤ 12	B1	
7	(i)	$\text{Gradient} = \frac{0.2}{0.8} = 0.25$ $b = 0.25$ <p>Either $6 = 0.25(2.2) + c$ Or $5.8 = 0.25(1.4) + c$ leading to $A = 233$ or $e^{5.45}$</p> <p>Alternative schemes:</p> <p>Either $6 = b(2.2) + c$ Or $e^6 = A(e^{2.2})^b$ $5.8 = b(1.4) + c$ $e^{5.8} = A(e^{1.4})^b$</p> <p>Leading to $A = 233$ or $e^{5.45}$ and $b = 0.25$</p>	M1 A1 M1 A1 M1 DM1 A1, A1	for attempt to find the gradient for a correct substitution of values from either point and attempt to obtain c or solution by simultaneous equations dealing with $c = \ln A$ for 2 simultaneous equations as shown for attempt to solve to get at least one solution for one unknown A1 for each
	(ii)	<p>Either $y = 233 \times 5^{0.25}$ Or $\ln y = 0.25 \ln 5 + \ln 233$ leading to $y = 348$</p>	M1 A1	for correct use of either equation in attempt to obtain y using <i>their</i> value of A and of b found in (i)

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8	$\frac{dy}{dx} = \frac{2(x^2 + 5)^{\frac{1}{2}} - \frac{1}{2}(2x)(x^2 + 5)^{-\frac{1}{2}}(2x - 1)}{x^2 + 5}$ <p>or</p> $\frac{dy}{dx} = 2(x^2 + 5)^{-\frac{1}{2}} - \frac{1}{2}(2x)(x^2 + 5)^{-\frac{3}{2}}(2x - 1)$ <p>When $x = 2, y = 1$ and $\frac{dy}{dx} = \frac{4}{9}$ (allow 0.444 or 0.44)</p> <p>Equation of tangent: $y - 1 = \frac{4}{9}(x - 2)$ ($9y = 4x + 1$)</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1, B1</p> <p>M1</p> <p>A1</p>	<p>for $\frac{1}{2}(2x)(x^2 + 5)^{-\frac{1}{2}}$ for a quotient or $-\frac{1}{2}(2x)(x^2 + 5)^{-\frac{3}{2}}$ for a product allow if either seen in separate working</p> <p>for attempt to differentiate a quotient or a correct product for all correct, allow unsimplified</p> <p>B1 for each</p> <p>for attempt at straight line, must be tangent using <i>their</i> gradient and y allow unsimplified.</p>
9	<p>(i) $\frac{2}{3}(4+x)^{\frac{3}{2}} (+c)$</p> <p>(ii) Area of trapezium $= \left(\frac{1}{2} \times 5 \times 5\right)$ $= 12.5$</p> <p>Area $= \left[\frac{2}{3}(4+x)^{\frac{3}{2}}\right]_0^5 - \left(\frac{1}{2} \times 5 \times 5\right)$ $= \left(\frac{2}{3} \times 27\right) - \frac{16}{3} - \frac{25}{2}$ $= \frac{1}{6}$ or awrt 0.17</p> <p>Alternative scheme: Equation of AB $y = \frac{1}{5}x + 2$</p> <p>Area $= \int_0^5 \sqrt{4+x} - \left(\frac{1}{5}x + 2\right) dx$ $= \left[\frac{2}{3}(4+x)^{\frac{3}{2}} - \frac{x^2}{10} - 2x\right]_0^5$ $= \left(\frac{2}{3} \times 27\right) - \frac{16}{3} - \frac{25}{2}$ $= \frac{1}{6}$ or awrt 0.17</p>	<p>B1, B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p>	<p>B1 for $k(4+x)^{\frac{3}{2}}$ only, B1 for $\frac{2}{3}(4+x)^{\frac{3}{2}}$ only Condone omission of c</p> <p>for attempt to find the area of the trapezium</p> <p>for correct use of limits using $k(4+x)^{\frac{3}{2}}$ only (must be using 5 and 0)</p> <p>for $18 - \frac{16}{3}$ or equivalent</p> <p>for a correct attempt to find the equation of AB</p> <p>for correct use of limits using $k(4+x)^{\frac{3}{2}}$ only (must be using 5 and 0)</p> <p>for $18 - \frac{16}{3}$ or equivalent</p> <p>for 12.5 or equivalent</p>

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10	(i)	All sides are equal to the radii of the circles which are also equal	B1	for a convincing argument
	(ii)	Angle $CBE = \frac{2\pi}{3}$	B1	must be in terms of π , allow 0.667π , or better
	(iii)	$DE = 10\sqrt{3}$	M1	for correct attempt to find DE using <i>their</i> angle CBE
		Arc $CE = 10 \times \frac{2\pi}{3}$ Perimeter $= 20 + 10\sqrt{3} + \frac{20\pi}{3}$ $= 58.3$ or 58.2	A1	for correct DE , allow 17.3 or better
(iv)	Area of sector: $\frac{1}{2} \times 10^2 \times \frac{2\pi}{3} = \frac{100\pi}{3}$	M1	for attempt to find arc length with <i>their</i> angle CBE (20.94)	
	Area of triangle: $\frac{1}{2} \times 10^2 \times \sin \frac{2\pi}{3} = 25\sqrt{3}$	M1	for $10 + 10 + DE +$ an arc length	
	Area $= \frac{100\pi}{3} + 25\sqrt{3}$ or awrt 148	A1	allow unsimplified	
			M1	for sector area using <i>their</i> angle CBE allow unsimplified, may be implied
			M1	for triangle area using <i>their</i> angle DBE which must be the same as <i>their</i> angle CBE , allow unsimplified, may be implied
			A1	allow in either form

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11	(a) (i)	$(x+3)^2 - 5$	B1, B1	B1 for 3, B1 for -5
	(ii)	$y \geq 4$ or $f \geq 4$	B1	Correct notation or statement must be used
	(iii)	$y = \sqrt{x+5} - 3$	M1	for a correct attempt to find the inverse function
		Domain $x \geq 4$	A1	must be in the correct form and positive root only
	(b)	$h^2g(x) = h^2(e^x)$ $= h(5e^x + 2)$ $= 25e^x + 12$ $25e^x + 12 = 37,$ leading to $x = 0$	B1FT	Follow through on <i>their</i> answer to (ii), must be using x
			M1	for correct order
			M1	for dealing with h^2
			DM1	for solution of equation (dependent on both previous M marks)
			A1	
		Alternative scheme 1: $hg(x) = h^{-1}(37)$ $h^{-1}(37) = 7$ $5e^x + 2 = 7,$ leading to $x = 0$	M1	for correct order
M1			for dealing with $h^{-1}(37)$	
DM1			for solution of equation (dependent on both previous M marks)	
A1				
Alternative scheme 2: $g(x) = h^{-2}(37)$ $h^{-2}(37) = 1$ $e^x = 1,$ leading to $x = 0$			M1	for correct order
	M1	for dealing with $h^{-2}(37)$		
	DM1	for solution of equation (dependent on both previous M marks)		
	A1			

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<p>12</p> <p>$x^2 + 6x - 16 = 0$ or $y^2 + 10y - 75 = 0$ leading to $(x + 8)(x - 2) = 0$ or $(y - 5)(y + 15) = 0$</p> <p>so $x = 2, y = 5$ and $x = -8, y = -15$</p> <p>Midpoint $(-3, -5)$</p> <p>Gradient = 2, so perpendicular gradient = $-\frac{1}{2}$</p> <p>Perpendicular bisector: $y + 5 = -\frac{1}{2}(x + 3)$ $(2y + x + 13 = 0)$</p> <p>Point C $(-13, 0)$</p> <p>Area = $\frac{1}{2} \begin{vmatrix} -13 & 2 & -8 & -13 \\ 0 & 5 & -15 & 0 \end{vmatrix}$</p> <p>= 125</p> <p>Alternative method for area: $CM^2 = 125, AB^2 = 500$ Area = $\frac{1}{2} \times \sqrt{125} \times \sqrt{500}$ = 125</p>	<p>M1</p> <p>DM1</p> <p>A1, A1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>for attempt to obtain a 3 term quadratic in terms of one variable only</p> <p>for attempt to solve quadratic equation</p> <p>A1 for each 'pair' of values.</p> <p>for attempt at straight line equation, must be using midpoint and perpendicular gradient</p> <p>for use of $y = 0$ in <i>their</i> line equation (but not $2x - y + 1 = 0$)</p> <p>for correct attempt to find area, may be using <i>their</i> values for A, B and C (C must lie on the x-axis)</p> <p>for correct attempt to find area may be using <i>their</i> values for A, B and C</p>
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