CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

ADDITIONAL MATHEMATICS   **4037/13**

Paper 1   **October/November 2013**

2 hours

Candidates answer on the Question Paper.

No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \, .$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!} \, .$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The coefficient of x^2 in the expansion of $(2 + px)^6$ is 60.

    (i) Find the value of the positive constant p.   [3]

    (ii) Using your value of p, find the coefficient of x^2 in the expansion of $(3 - x)(2 + px)^6$.   [3]

2 Solve $2 \lg y - \lg(5y + 60) = 1$. [5]

3 Show that $\tan^2\theta - \sin^2\theta = \sin^4\theta\sec^2\theta$. [4]

[4]

4 A curve has equation $y = \dfrac{e^{2x}}{(x+3)^2}$.

   (i) Show that $\dfrac{dy}{dx} = \dfrac{Ae^{2x}(x+2)}{(x+3)^3}$, where A is a constant to be found. [4]

   (ii) Find the exact coordinates of the point on the curve where $\dfrac{dy}{dx} = 0$. [2]

5 For $x \in \mathbb{R}$, the functions f and g are defined by
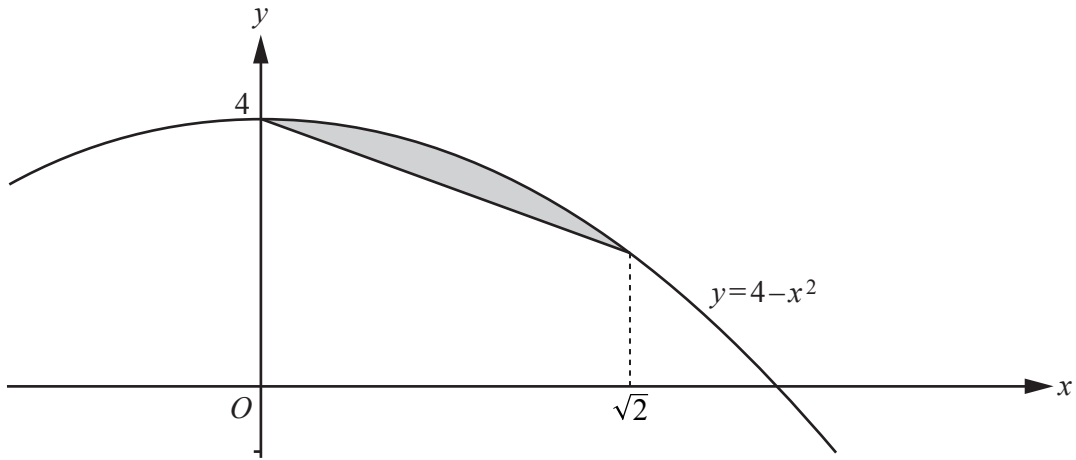
$$f(x) = 2x^3,$$

$$g(x) = 4x - 5x^2.$$

(i) Express $f^2\left(\dfrac{1}{2}\right)$ as a power of 2. [2]

(ii) Find the values of x for which f and g are increasing at the same rate with respect to x. [4]

6 **Do not use a calculator in this question.**

The diagram shows part of the curve $y = 4 - x^2$.

Show that the area of the shaded region can be written in the form $\dfrac{\sqrt{2}}{p}$, where p is an integer to be found. [6]

7   It is given that $\mathbf{A} = \begin{pmatrix} 2t & 2 \\ t^2 - t + 1 & t \end{pmatrix}$.

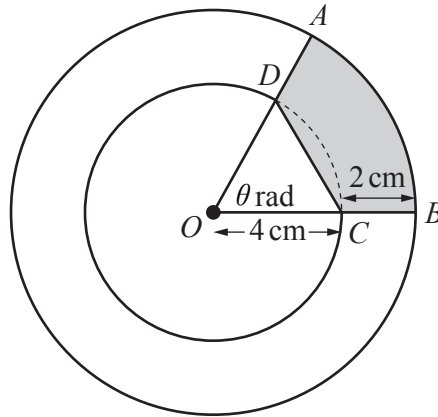   **(i)**   Find the value of t for which $\det \mathbf{A} = 1$. [3]

   (ii)   In the case when $t = 3$, find $\mathbf{A}^{-1}$ and hence solve

$$3x + y = 5,$$
$$7x + 3y = 11.$$

[5]

8 The diagram shows two concentric circles, centre O, radii $4\,cm$ and $6\,cm$. The points A and B lie on the larger circle and the points C and D lie on the smaller circle such that ODA and OCB are straight lines.

(i) Given that the area of triangle OCD is $7.5\,cm^2$, show that $\theta = 1.215$ radians, to 3 decimal places. [2]

(ii) Find the perimeter of the shaded region. [4]

(iii) Find the area of the shaded region. [3]

4037/13/O/N/13 **[Turn over**

9 **(a)** **(i)** Solve $6\sin^2 x = 5 + \cos x$ for $0° < x < 180°$. [4]

(ii) Hence, or otherwise, solve $6\cos^2 y = 5 + \sin y$ for $0° < y < 180°$. [3]

(b) Solve $\quad 4\cot^2 z - 3\cot z = 0 \quad$ for $\quad 0 < z < \pi$ radians. $\hspace{1cm}$ [4]

---

10 The variables *s* and *t* are related by the equation $t = ks^n$, where *k* and *n* are constants. The table below shows values of variables *s* and *t*.

s	2	4	6	8
t	25.00	6.25	2.78	1.56

(i) A straight line graph is to be drawn for this information with lg *t* plotted on the vertical axis. State the variable which must be plotted on the horizontal axis. [1]

(ii) Draw this straight line graph on the grid below. [3]
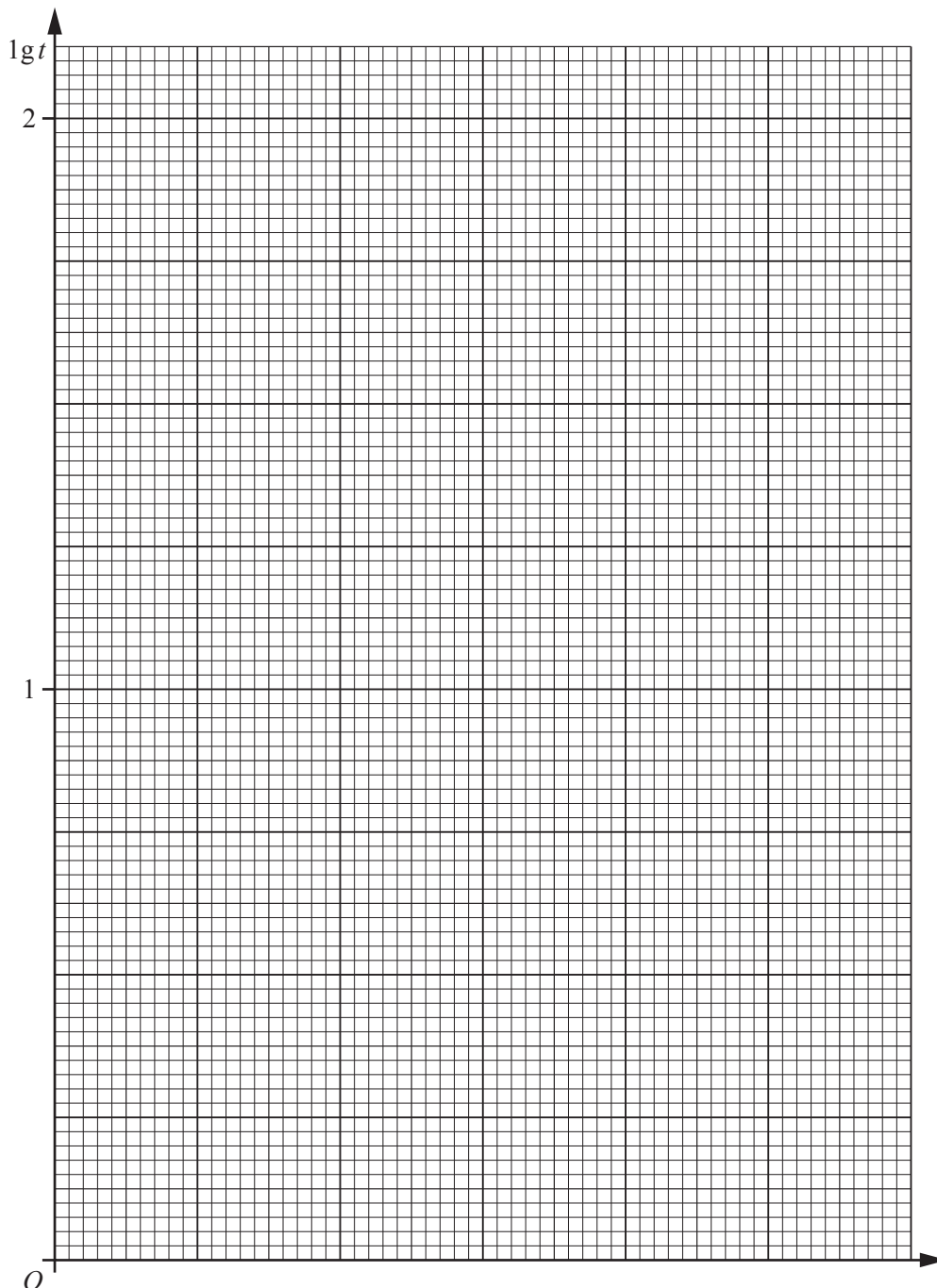
(iii) Use your graph to find the value of k and of n. [4]

(iv) Estimate the value of s when $t = 4$. [2]

Question 11 is printed on the next page.

11 (i) Given that $\displaystyle\int_0^k \left(2e^{2x} - \frac{5}{2}e^{-2x}\right) dx = \frac{3}{4}$, where k is a constant, show that

$$4e^{4k} - 12e^{2k} + 5 = 0.$$ [5]

(ii) Using a substitution of $y = e^{2k}$, or otherwise, find the possible values of k. [4]