ADDITIONAL MATHEMATICS

Paper 4037/11
Paper 11

General comments

Most candidates were able to make a reasonable attempt at the paper, although some questions proved quite difficult. Candidates should be aware of the level of accuracy to which they should work. Too many candidates were not awarded accuracy marks as they had been working to less than 3 significant figures. When this is done for several questions, the number of marks 'lost' due to inaccuracy rather than lack of knowledge can be significant and needless. It is essential that candidates are made aware of the importance of the rubric on the front of the examination paper.

Comments on specific questions

Question 1

(i) Most candidates realised that the question involved either 'completing the square' or comparing the coefficients of powers of x. Problems arose because the coefficient of x^2 was 2, leaving many candidates unable to deal with the problem correctly when attempting to 'complete the square'. Those candidates who chose to compare coefficients of powers of x tended to have more success.

(ii) Few candidates realised that their answer for b would also give them the required answer for the minimum value of y. Those that chose to use calculus to determine the minimum value usually did so with success. Attention is drawn to the fact that the mark allocation for this part of the question is 1. This implies that a relatively simple step/calculation is required.

Answers: **(i)** $a = -12$, $b = -4$ **(ii)** -4

Question 2

(i) Most candidates were able to sketch the graph of the curve of $y = \cos x$. Fewer were able to sketch the graph of the curve $y = \sin 2x$, the most common error being to give the graph an amplitude of 2 rather than 1.

(ii) Candidates were asked to write down the number of solutions to the equation $\sin 2x - \cos x = 0$. Whilst many candidates realised that all they had to do was count the number of points of intersection of the 2 curves, some candidates attempted to find these solutions, either by an attempt at solution of the trigonometric equation itself (this is outside the syllabus) or by reading off the coordinates of the points of intersection of the two curves. Attention is drawn again to the fact that not only is the mark allocation for this part of the question 1, thus implying the need for a relatively simple step/calculation, but also the word 'Hence' has been used, implying that work done in the previous part of the question is to be used.

Answer: **(ii)** 3

Question 3

This question was usually done well with most candidates expressing the two terms on the left hand side of the expression as a single fraction and thus making good use of the appropriate algebraic skills required to simplify the numerator and the appropriate trigonometric identity to simplify the denominator. Those candidates that chose to start with the right hand side of the expression were usually equally successful in showing the correct relationship.

Question 4

Many candidates were able to gain full marks for this question, showing a good knowledge of the factor theorem. Most candidates were able to 'spot' the first solution/root and went on successfully to obtain a quadratic factor. Some candidates lost marks when, rather than factorising their quadratic factor, they used the quadratic formula and gave solutions to the expression equated to zero rather than using these solutions to obtain the required factors. Candidates are advised to always check as to whether solutions to an equation, or factors of an expression are required in this type of question.

Answer: $(x+1)(x-7)(2x+1)$

Question 5

(i) Candidates were expected to give their value of *a* in terms of π. It was expected that candidates would evaluate $\sin\frac{\pi}{2}$ as part of this process.

(ii) Those candidates that realised that calculus was needed for this part of the question were usually able to obtain method marks at least. Some failed to realise that the differentiation of a product was required but this did not preclude them from obtaining later method marks associated with the gradient of a line perpendicular to the curve and the equation of this straight line. Those that attempted the question without using calculus failed to gain any marks. Answers in terms of π and also with exact terms evaluated were acceptable, the latter provided that 3 significant figures had been used.

Answers: **(i)** $\frac{4\pi}{3}$ **(ii)** $2y = \frac{19\pi}{6} - x$ or equivalent

Question 6

(i) The great majority of candidates were able to obtain full marks for this part, showing a good understanding of both the application of the Binomial Theorem and the correct evaluation of the individual terms. Some candidates made errors with the sign of their second term.

(ii) Again, most candidates were able to make a good attempt at this part of the question, realising that two terms were involved and thus identifying the necessary multiplications needed to obtain these two terms.

Answers: **(i)** $64 - 960x + 6000x^2$ **(ii)** -640

Question 7

Many candidates found it difficult to work with the idea of sets and solutions of basic trigonometric equations together and thus this question was poorly done. More practice on this topic with a variety of sets would therefore benefit candidates.

(a) (i) Those candidates that attempted this question were usually able to make a reasonable attempt at the solution of $\sin x = 0.5$.

(ii) Many candidates were unable to deal with the fact that they needed to consider the value of $\cos^{-1}(-0.5)$ and that 30° needed to be added to any solutions obtained. Most were able to deal with the union of the two sets they had obtained but many candidates gave the intersection of the sets that they had obtained rather than the union.

(b) Many candidates had problems solving the basic trigonometric equation for the elements of C initially. Too few recognised the set notation for the number of elements in the set C, choosing instead to list any solutions obtained for set C.

Answers: **(a) (i)** $30^\circ, 150^\circ$ **(ii)** $30^\circ, 150^\circ, 270^\circ$ **(b)** 4

Question 8

(i) There were different methods which were appropriate for finding the value of $\ln y$ when $\ln x = 0$. Some candidates chose to make use of the gradient of the straight line and hence find the equation of the straight line in terms of $\ln y$ and $\ln x$. This method was then useful for part **(ii)**. Others chose to use a method of ratios to obtain the value of $\ln y$ when $\ln x = 0$ with equal success.

(ii) Those candidates that attempted this part of the question were usually fairly successful, recognising that they had to rewrite $y = Ax^b$ in the form $\ln y = A - b\ln x$. Most were able to equate b to the gradient of the straight line, with the occasional sign error. Problems arose when trying to evaluate A, with some candidates thinking that they were dealing with base 10 logarithms rather than natural logarithms.

Answers: **(i)** 6.8 **(ii)** $A = 898$, $b = -0.5$

Question 9

This question also caused problems amongst candidates as they misunderstood the description of the diagram in relation to the area of the cross-section of the figure. Many candidates did not realise that rates of change were involved and were hence unable to obtain many, if any, marks.

(i) Provided candidates realised that $A = x^2$, then the process of differentiation was straightforward. Many candidates misunderstood what was required and tried to involve $4x$ in their expression for A.

(ii) Those candidates that attempted this part very often attempted to use rates of change but used an incorrect equation, usually multiplying 0.003 by their $\frac{\mathrm{d}A}{\mathrm{d}x}$ rather than dividing by their $\frac{\mathrm{d}A}{\mathrm{d}x}$.

(iii) This part was usually attempted in a more successful manner, with most realising that $V = 4x^3$ and hence $\frac{\mathrm{d}V}{\mathrm{d}x} = 12x^2$. Again, rates of change were often used incorrectly with candidates mistakenly using the original value given rather than the value obtained in part **(ii)**.

Answers: **(i)** $2x$ **(ii)** 0.0003 **(iii)** 0.09

Question 10

Of the candidates that attempted this question, the great majority were able to obtain most if not all of the marks. Problems arose with premature approximation of workings thus leading to inaccurate answers.

(i) Most realised that the problem involved use of a simple trigonometric ratio to find PA, although some attempted more involved methods which were equally successful. Too many candidates however forgot to find PB.

(ii) This part was usually attempted with a good measure of success. Most candidates were able to find the correct area of the sector required, but errors occurred with finding the area of the kite. Most realised that the kite was formed by two triangles and were usually able to obtain method marks if errors were made.

(iii) Candidates appeared to have the most success with this part of the question, being able to split the perimeter into the required lengths and thus finding each of these lengths.

Answers: **(ii)** 47.7 **(iii)** 30.7

Question 11

(i) An application of the 'reverse chain rule' was made by most candidates, usually with a good deal of success. Errors usually involved an incorrect multiple and those candidates who obtained an extra term in x were penalised. The most common error was to write $\dfrac{(1+x)^{\frac{1}{2}}}{\frac{1}{2}}$ incorrectly as $\dfrac{(1+x)^{\frac{1}{2}}}{2}$.

On this occasion this was not penalised as, once a correct expression was seen, full marks were given. Although omission of the arbitrary constant was not penalised in this question, candidates should be reminded that it does form part of the solution and so should be included.

(ii) Most candidates were able to gain 3 marks for the correct differentiation of the given product. Few however were able to manipulate their correct unsimplified answer into the required form, showing the need for practice at algebraic simplification and manipulation.

(iii) Very few correct solutions to this part were seen. If a correct form of the answer for part **(ii)** was not obtained, candidates were unable to achieve success at this part of the question. The idea of differentiation being the inverse of integration is a syllabus requirement and candidates were led to this by the use of the word 'Hence' at the start of this part. In this part question, work done in part **(i)** was also required. Candidates should always check to see whether work done in earlier parts of questions of this type is able to be made use of in the latter parts of a question.

Answers: **(i)** $2(1+x)^{\frac{1}{2}}+c$ **(ii)** $A=2, B=-1$, **(iii)** $4\sqrt{1+x}-\dfrac{2x}{\sqrt{1+x}}+c$, 1

Question 12 EITHER

This question was by far the most popular option with almost all candidates attempting it, usually with a good deal of success.

(i) Most were able to integrate each term successfully. Those that did not use an arbitrary constant initially were then penalised as they did not use the given coordinate to find the equation of the curve.

(ii) Again, the great majority of candidates employed correct methods and obtained the coordinates of both A and B. Problems arose when candidates did not find both solutions of the equation $4x^2 - 9 = 0$. This then had an affect on solutions to part **(iii)** with some candidates using spurious coordinates in order to complete the question.

(iii) Most were well practiced in finding the equation of a line perpendicular to a given straight line. Marks were lost when the coordinates of the midpoint were not used, with the coordinates of either *A* or *B* being incorrectly used instead.

Answers: **(i)** $y = \frac{4}{3}x^3 - 9x - 8$ **(ii)** $\left(\frac{3}{2}, -17\right), \left(-\frac{3}{2}, 1\right)$ **(iii)** $x - 6y = 48$

Question 12 OR

This question was attempted by very few candidates.

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(iii) Correct methods were usually employed to find the nature of the stationary point. Most used the second derivative and as both exponential terms were positive, an evaluation of the second derivative was not necessary provided the candidate was able to state this.

Answers: **(i)** 40 **(ii)** $(0.231, 47.6)$, **(iii)** minimum

ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

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(iii) Correct methods were usually employed to find the nature of the stationary point. Most used the second derivative and as both exponential terms were positive, an evaluation of the second derivative was not necessary provided the candidate was able to state this.

Answers: **(i)** 40 **(ii)** $(0.231, 47.6)$, **(iii)** minimum

ADDITIONAL MATHEMATICS

Paper 4037/13
Paper 13

General Comments

Candidates generally found the paper to be one in which they had the chance to show what they had learned.

Candidates should be reminded to follow the instructions given on the front of the question paper concerning the degree of accuracy required in answers. A significant proportion of candidates did not and this resulted in the loss of marks especially in **Questions 7(iii)**, **10(ii)** and **11.**

Candidates should also be reminded of the need for clear presentation in their work so that Examiners can see what they are trying to achieve. Any question requiring candidates to 'show that...' will not gain full marks if candidates appear to miss out necessary steps. In a similar way, method marks, available even if an answer is incorrect, will not be awarded if the method is not clear. The most common example of this occurs if a candidate makes an error forming a quadratic equation. If he/she then factorises and solves, or shows use of the quadratic formula the method mark(s) are earned, but if only the answers are given, no method mark is possible.

Comments on specific questions

Question 1

The majority of candidates knew to rewrite sec$x$ in terms of cos$x$ and to form a single fraction. They then used the identity $\cos^2 x + \sin^2 x = 1$ and most managed to reach the desired answer. A good knowledge of trigonometric relationships was displayed by these candidates whereas others used incorrect relationships and made little or no progress (a problem which also arose in **Question 11**). Some candidates worked from the right hand side successfully.

Question 2

(i) Many candidates knew exactly how to answer this part. Others were unsure of how to approach it. Some candidates used combinations rather than permutations or simple factorials (of 4, of 7 or of 8). Some favoured the approach of successive boxes i.e. 7 choices for the first box, 6 for the second etc. and thus calculating 7 x 6 x 5 x 4.

(ii) The majority of those who dealt correctly with part **(i)** managed this part.

The box method was more popular here: 4 choices (of even number) in the first box, 6 choices in the second etc. i.e. 4 x 6 x 5 x 4. Otherwise candidates worked out ${}^6P_3$ and multiplied this by the number of different even numbers.

Answers: **(i)** 840 **(ii)** 480

Question 3

Two valid methods of solution were seen (or even a combination of both). The more popular of these was to equate the two equations to obtain a new (quadratic) equation in x (and involving m). The use of $b^2 - 4ac$, because the line was a tangent and therefore there were equal roots, was common and the solution of $b^2 - 4ac = 0$ to give values of m was well handled by most.

The other method was to differentiate the quadratic and use the result $2x + 12$ in place of m in the line equation. This gave solutions for x and substitution was necessary to find m values, a fact which some candidates overlooked.

A number of candidates incorrectly offered ranges for their answer e.g. $4 < m < 20$. A common wrong method was to solve $x^2 + 12 + 18 = 0$ and then to find m.

Answer: $m = 4, 20$

Question 4

Although some candidates wrongly evaluated f(–2) and f(1) the vast majority knew to use f(2) and f(–1). The difficulty for some candidates arose in trying to link the two remainders too early. Those who obtained the two remainders independently of each other and then considered the '5 times' relationship usually went on to a correct answer. Some candidates formed equations in k and, for example, m (remainders of m and $5m$) and found the remainders (–9 and –45) before finding k by substitution.

As usual with this type of question f(2) = 0 and f(–1) = 0 were often seen. In many cases the '=0' was subsequently ignored, but if f(2) = 0 was solved the whole question became meaningless.

Answer: $k = -10$.

Question 5

Some candidates were quite comfortable with logarithms but many would benefit from more practice on this topic. The most common error was to see $\log_3(2a - b)$ written as $\log_3 2a - \log_3 b$.

The candidates who obtained the equations $a = b^2$ and $2a - b = 3$ were usually able to solve correctly but very few realised that $a = 1$, $b = -1$ was not a possible solution.

Answer: $a = \frac{9}{4}, b = \frac{3}{2}$

Question 6

Varying methods of solution were seen, not all fully supported by working. The most common involved the use of the factor theorem to obtain one solution and subsequent work to find a quadratic factor. This latter appeared either as long division, synthetic division or as comparison of coefficients – the linear factor being multiplied by $ax^2 + bx + c$. Candidates need to read this type of question carefully to check whether they are being asked for factors or a solution. Some candidates stopped short of the answer required merely providing three linear factors.

Answer: $x = -4, -\frac{1}{3}, 2$

Question 7

(i) Because $y = 3x - 5$ cuts the *x*-axis at a non-integer value those candidates who went straight to the construction of a table of values finished up with a graph which itself never touched the *x*-axis. Those candidates who realised what was going on lightly drew the line $y = 3x - 5$ and then reflected, in the *x*-axis, the part of the original line below the axis. Again, marks were often lost due to candidates not answering what had been asked for – in this case marking the coordinates where the graph met the axes.

(ii) This was well done.

(iii) With a required degree of accuracy of 3 significant figures the intersection of the two graphs would never be accurate enough, although some credit was given for this method.

Candidates who understood the modulus function considered the two equations $8x = 3x - 5$ (giving $x = -1$) and $8x = -(3x - 5)$ leading to the correct solution $\frac{5}{11}$. Of those candidates using algebra, many omitted the second possibility thereby ignoring the modulus function altogether. As with other questions (e.g. **Questions 5** and **11**) candidates often failed to give any thought to their answer offering both $\frac{5}{11}$ and –1 even though their graphs clearly showed that there was no intersection at –1.

Answers: **(iii)** $x = -\frac{5}{11}$ (or 0.455).

Question 8

(a)(i) Many candidates seemed uncertain of how to answer this question. The mathematical language and notation used here is an important part of the syllabus, and candidates should be prepared for this type of question. Of the efforts seen some included 'differential = 0' while others involved a table of values (only of use if the least value occurs at an integral value).

(ii) Very few candidates understood what was needed here and, of course, those with no answer in part **(i)** could not attempt this. Candidates should realise that a one-one relationship was necessary for the inverse to exist. This required a domain consisting of a continuous set of numbers on one side or the other of $x = -2$. Even those candidates who had some idea let themselves down by writing, for example, $f(x) \geq 0$ instead of $x \geq 0$.

(b)(i) This was correctly attempted by all but the weakest candidates although many, when dealing with the 2 in the denominator, forgot to multiply the 1 by this.

(ii) Some candidates could not deal correctly with gh(x), some actually finding hg(x) and others the product of g(x) and h(x). The majority, however, managed to find gh(x) correctly and, by equating to $g^{-1}(x)$, formed a quadratic equation which they solved.

Answers: **(a) (i)** Least value = –10 when x = –2 **(ii)** Any suitable domain that makes f a one-one function (e.g. $x \geq -2$) **(b) (i)** $g^{-1}(x) = 2(x+1)$ **(ii)** $x = -1$ or 6.

Question 9

(a) Candidates were expected to answer this part by expanding the bracket and then integrating term by term. A large number of those trying to do this had an x^2 term and a constant term but no x term. There were also many candidates who had a constant term of –9. Candidates would benefit from more work on expanding similar types of expression.

(b)(i) Differentiation of a product was required here and this was handled well by the majority of candidates.

(ii) This clearly stated 'Hence...'. and so candidates needed to use their result from part **(i)** in this part. Candidates who offered an answer to part **(i)** in an unsimplified form now had to attempt to reduce it to one term and to compare with the integral here. Any candidate with a wrong answer in part **(i)** would have found this impossible and thus should have realised that their previous answer was incorrect.

Answers: **(a)** $\frac{3}{5}x^{\frac{5}{3}} - \frac{9}{2}x^{\frac{4}{3}} + 9x + c$ **(b)(i)** $\sqrt{x^2+6} + x\left(\frac{2x}{2\sqrt{x^2+6}}\right)$ **(ii)** $\frac{1}{2}x\sqrt{x^2+6} + c$

Question 10

This was a demanding question, involving use of logarithms, exponentials, differentiation of product or quotient, knowledge of how s, v and a are connected and how to deal with a time interval. A few candidates could cope with the whole question but some could not make any meaningful progress at all.

(i) It was expected that something similar to $e^5 = t^2 + 1$ with subsequent work would be seen. However, this was only seen occasionally and common errors included the misuse of natural logs and omitting to bring e into the work.

(ii) This part proved to be the most difficult question on the whole paper. Candidates failed to realise that the 'third second' is that from $t = 2$ to $t = 3$. Most substituted $t = 3$ into the equation for s and stopped. Of those who did grasp the idea some erred by using $t = 3$ and $t = 4$ (the fourth second) while others worked to just 3 figures and had an inaccurate answer.

(iii) Inappropriate use of 'speed = distance/time' gave $v = 1.6094...\div 2$, close enough to 0.8 to convince these candidates that they were on the right lines. However as the body did not have uniform speed this formula did not apply. All that was needed here was differentiation and substitution of $t = 2$.

(iv) Candidates who realised that acceleration equalled $\frac{dv}{dt}$ were now faced with the differentiation of a product or a quotient depending on how they chose to work. Relatively few quoted either formula and it was often not clear what candidates were doing.

Answers: **(i)** $t = 12.1$ **(ii)** ln2 m (or 0.693 m) **(iv)** $-\frac{6}{25}\,\text{ms}^{-2}$

Question 11

Some candidates handled this question extremely well, but there were signs that others found this topic difficult. Inappropriate and incorrect substitutions were made, the most common being $\sin y = -\cos y$ in part **(ii)**. Some candidates offered only one solution per part while others offered four. Candidates often lost accuracy marks for not following the instructions on the front of the question paper. In both part **(i)** and part **(ii)** answers were often correct to 3 significant figures rather than the 1 decimal place required for angles in degrees. In each case this had no effect on the first solution but did affect the second. In part **(iii)**, candidates who gave answers as decimals often did so to 2 decimal places rather than the generally required 3 significant figures.

(i) Most candidates knew to use $\tan x$ although some made a sign error or had $\tan x = \frac{3}{4}$. The second possible solution was often wrongly given as 233°.

(ii) The usual method here was to form a quadratic equation in $\sin y$ using the identity $\cos^2 y = 1 - \sin^2 y$. Again the second solution was often given, inaccurately, as 166°.

(iii) This proved troublesome for many candidates. Dealing with the secant function was difficult for some, while others could not handle the bracket correctly. Of those who realised that $\cos\left(2z + \frac{\pi}{3}\right) = -\frac{1}{2}$ some could not decide whether to use 1.047 or 2.094 as their reference angle.

The operations of $-\frac{\pi}{3}$ then $\div 2$ were often carried out correctly but the first answer was frequently given as 0.52 and the second often omitted. Those candidates happy to work in multiples of π throughout produced, in general, short and correct solutions.

Some candidates worked the question in degrees throughout obtaining 30° and 90°, some then converting to radians, others forgetting. A few mixed degrees and radians in their working.

Answers: **(i)** x = 53.1°, 233.1° **(ii)** y = 14.5°, 165.5° **(iii)** $z = \frac{\pi}{6}, \frac{\pi}{2}$ (or 0.524, 1.57)

Question 12

12 EITHER

(i) As with **Question 11(iii)**, some candidates failed to deal with radians, treating the angles as being in degrees. Of those who made progress it was through differentation of the equation and then substituting as appropriate into the equations for y and for $\frac{dy}{dx}$. The value of A came out directly, and B by substitution into the original equation. Others were unable to find the value of A but used it correctly to find B.

(ii) This was well done, with candidates realising the need to integrate, which the majority did well, and to apply limits. Most evaluated at $x = 0$ rather than assuming the value of the integral at 0 is 0.

Answers: **(i)** $B = \sqrt{2}$ (or 1.41) **(ii)** Area = 3

12 OR

The majority of candidates answered this question.

(i) Attempts at this question covered the whole mark range. Some candidates knew that the normal had a gradient which was the negative reciprocal of something but only some of them realised the need to differentiate. Some did not manage to work out the coordinates of point *A* and thus had no point to put into their equation of the normal, or put point *C* into this which led nowhere. However a reasonable number did manage to obtain the required result.

(ii) The idea of integrating for the curve was used by almost all, and carried out correctly in most cases. The use of limits was more problematical. Some candidates used an upper limit of 2.5 (presumably the y value at point *C*) rather than 2 (the x value at *B*). Others failed to grasp the logic needed and integrated with a lower limit of 0. This left them the almost impossible task of finding area *OAC*. Some did recover from this, but many did not. It was expected that candidates would find the area under the curve between $x = 1$ and $x = 2$ and add this to the area of the trapezium with parallel sides *OC* and *AD* where *D* is directly below *A* on the x-axis. Only the better candidates seemed to cope with this.

Answers: **(ii)** $\frac{49}{12}$ (or 4.08).

ADDITIONAL MATHEMATICS

Paper 4037/21

Paper 21

General comments

There was wide variation in the performance of the candidates. Candidates scored well in **Questions 2, 7** and **8**. They found **Questions 3, 4, 6** and **9** less accessible and so these are topics that would perhaps benefit from more study.

Many candidates, including some otherwise high scoring ones, lost marks through not paying sufficient attention to detail. For example, marks were lost for including answers which satisfied the algebra of the situation but not the practical side of the situation in **Questions 7** and **12 Either**.

Candidates should remember to give non-exact answers correct to 3 significant figures, as not doing this resulted in a loss of marks, particularly in **Questions 3** and **11**. In the case of **Question 3** it was evident that some candidates incorrectly thought that 0.02 was correct to 3 significant figures

Question 12 Either was chosen much more often than **Question 12 Or**.

Comments on specific questions

Question 1

Many candidates scored one mark for obtaining the answer –1.5. A number of candidates did not appear to be familiar with the use of the modulus sign. A surprising number obtained the solution $x = -8.5$ by solving $2x + 10 = -7$ but did not obtain the "easier" solution. Some candidates, aware that they needed to involve a negative sign, changed the equation to $2x - 10 = 7$. A relatively small number of candidates attempted to square the equation but very few successfully found the two correct solutions. Most did not remember to square the left hand side correctly, giving two terms only. A few chose to square both sides and of these very few managed it successfully. Common mistakes were to square one side only or to write $(2x+10)^2 = 4x^2 + 10$.

Answer: –1.5 and –8.5

Question 2

This was, in general, a well-answered question with many candidates scoring full marks. Most candidates were able to make the correct substitutions and a majority found the correct equations. In general, most candidates were able to solve simultaneous equations even if they started with incorrect equations. A few candidates attempted long division but very few of these were able to see it through to anything approaching a solution.

Answer: $a = 7$ and $b = -6$

Question 3

Candidates found this to be one of the most difficult questions. Sometimes ln appeared on both sides of the equation but sometimes it was left out. Many were confused with N and N_0, often leaving them in the answer, or mixing them up and ending up with *k* either negative or zero. This was sometimes self correcting in part **(ii)** or candidates ended with a negative number of weeks. Rounding off in part **(i)** was common and this led to an inaccurate value in part **(ii)**. After getting part **(i)** correct, part **(ii)** was sometimes attempted by using linear proportion. Some candidates differentiated. Many candidates did not attempt this question and so more practice on this topic would benefit candidates.

Answers: **(i)** 0.0204 **(ii)** 79

Question 4

This proved to be a challenging question for many candidates. A substantial number gained the first method mark for matrix multiplication using the 3×3 matrix but not all of these got a correct result. A smaller proportion got the next method mark for 1×3 with 3×1 multiplication. Very few were awarded the first mark for writing down 3 compatible matrices.

Candidates found it hard to apply matrix methods to the situation posed. Of those who did actually use matrix multiplication in an attempt to answer the question, many lost marks by misinterpreting the "loses" 2 marks. This inevitably was written as +2 instead of –2. Some interpreted it as "loses 2 of the 5" and hence used 3. Some who did come up with the correct final total mark of 30.7 lost marks as their final two matrices being multiplied were clearly a 3×1 with a 1×3 but nevertheless still ending up with a 1×1 matrix.

Answer: 30.7

Question 5

Most candidates scored quite well on this question. Indeed, many candidates only lost the final mark because they did not identify the correct set of values for *m* after finding the correct "critical values" values of *m*. Nearly all candidates realised that they needed to eliminate the *y* and the knowledge that the two equations had to be solved simultaneously and that the discriminant $b^2 - 4ac$ had to be used was appreciated by most candidates. An alternative method using calculus to find the gradients of the two tangents to the curve was used by a few candidates. This method led to the two values of *m*, but does not lead to the inequalities. Some candidates merely solved the quadratic equation $x^2 + 8x + 7 = 0$ and ignored the straight line.

Answer: $m < 2, m > 14$.

Question 6

There was a wide range of marks achieved on this question. Most candidates failed to realise that the answers to successive parts should be decreasing as they worked through the question, each being a subset of the previous part. A few were able to answer part **(i)** but correct answers to all three parts were few and far between.

(i) The expressions ${}^7C_4$ and ${}^7P_4$ were seen often but the meaning of these expressions was often confused.

(ii) Some candidates realised that only two numbers could be used as the first digit but few were able to get any further.

(iii) Similarly, few candidates were able to break the question down well enough to find the correct number. An answer of $2\times a\times b\times 2$ was sometimes seen, but with the wrong digits for *a* and *b*.

Answers: **(i)** 840 **(ii)** 240 **(iii)** 80

Question 7

Candidates found the second part of this question more accessible than the first. There were many strange expressions seen, such as 'the volume of the rectangle' and 'the volume of the two squares'. There was also a great deal of contrived algebraic manipulation in an attempt to arrive at the required expression. Some candidates omitted brackets in their expressions but demonstrated an understanding of the situation.

Part **(ii)** was generally started correctly, although weaker candidates divided the expression for V by x and then applied the quadratic formula. Many candidates who correctly solved $\frac{dV}{dx} = 0$, failed to eliminate the value $x = 45$, even though the wording of the question clearly indicated that only one value of x was expected.

Answer: **(ii)** $x = 10$

Question 8

This was one of the better answered questions with many candidates scoring full marks on both parts. In part **(a)** the main error was not appreciating that the 1 needed to be replaced with log 10 before using the laws of logs. The 2log3 was handled much better becoming log 9. Some candidates who correctly dealt with 2log3 and 1 merely deleted the logarithms instead of using the laws of logarithms correctly.

Part **(b)** was answered even better than part **(a)**. The first mark was usually obtained, even by candidates who scored very low marks on the paper as a whole. A number failed to achieve the second mark by writing $\frac{4y}{(7-y)} = \frac{(4y+3)}{(3y-6)}$. Some who applied the rules of indices correctly lost marks through carelessness with brackets and signs.

Answers: **(i)** $x = 6$ **(ii)** $y = 4$

Question 9

This proved to be the most challenging question on the paper. Many candidates did not attempt it. Few seemed to appreciate the need for a clear diagram to represent the situation. Consequently the use of the sine rule or cosine rule to find necessary angles or distances was rarely seen. Some attempted to use components but this involved using two variables, the velocity of the plane and the angle steered by the plane, most used the velocity of the plane and an angle of 60°. Part marks could be awarded where candidates included the angle of 60° between the 80 and 250 vectors.

Answer: 107 minutes

Question 10

Many candidates found this a challenging question. Large numbers only scored the first mark for the gradient of *AB*. The next section proved more problematic. Stronger candidates did go on successfully, but a significant number found the equation of *AB*, realising it was parallel to *CD*, and were confused about where to go next. Quite a number of candidates seemed to guess (7, 0), others gave (6, 0) although *C* was obviously not vertically below *B*. Some assumed that the midpoints of *AC* and *BD* would be the same point. Some candidates who were confident of the principles of coordinate geometry and obtained the equation of *CD* stalled when they failed to realize that the *x*-coordinate of *D* was 1. Many candidates thought that the equation of *CD* was the same as the equation of *AB*.

The most common method of finding the area was as a trapezium and a number of candidates were successful at both finding the lengths and using the formula. The array method was also popular and many achieved a method mark, although some forgot to halve the difference of the products.

Answers: **(i)** $5y = x - 7$ **(ii)** 28.6

Question 11

Again there were many good answers to this question. A few candidates had little idea how to solve trigonometrical equations. Most candidates who made good attempts understood the need to find all solutions in the given domain and usually did so correctly. In part **(iii)** candidates unnecessarily lost marks through premature approximation of the answers.

(i) Most candidates divided by cos x, but a minority failed to get any marks due to mathematical errors, particularly giving $\tan x = \frac{3}{5}$ or $\tan x = \frac{5}{3}$. It was generally well done by those that attempted it, and very few marks were lost due to rounding.

(ii) Most used $\cos^2 y = 1 - \sin^2 y$, but some made careless errors in the substitution, such as replacing $2\cos^2 y$ with $2 - \sin^2 y$. Others had difficulties with signs when rearranging the quadratic equation, particularly with correctly rearranging the equation so that $\sin^2 y$ had a positive coefficient. The solution to $\sin y = -1$ was frequently given as 90°, 270° or just discarded.

(iii) Common errors were replacing $\sec z$ with $\frac{1}{\tan z}$, or sometimes working out $\cos z$ as $\frac{1}{30}$. The radian measure was usually well handled.

Answers: **(i)** 31 and 211 **(ii)** 30, 150 and 270 **(iii)** 1.27 and 5.02

Question 12 EITHER

This option was generally well answered and a number of candidates scored full marks. In part **(i)** those candidates who chose to combine the functions algebraically tended to be less successful than those who found g(9) and then substituted their answer to this into the function f.

In part **(ii)** candidates seemed more familiar with the techniques needed to find the inverse of g, the inverse of f causing more problems despite it being technically the easier to find. A small number of those who knew how to find the inverse of f correctly left the answer with $\pm$ before the square root.

Most candidates knew what was required in part **(iii)** but many made slips in rearranging the quadratic, often ending up with a two term expression. Of those who did rearrange and solve correctly a significant number did not pick up the domain restriction, including -1 as an answer.

Answers: **(i)** 21 **(ii)** $\mathrm{f}^{-1}(x) = \sqrt{x+4} - 1$, $g^{-1}(x) = \frac{x+5}{x-3}$ **(iii)** 5

Question 12 OR

The candidates who attempted this option generally knew that acceleration was the differential of velocity and that they needed to integrate the velocity function to obtain displacement. Despite this, there were very few who scored highly. The use of degrees instead of radians lost marks for almost all of those who did choose this option and very few knew that the 5th second was from $t = 4$ to $t = 5$.

Answers: **(i)** 4 **(ii)** 10.4 **(iii)** 14 **(iv)** 23.9

ADDITIONAL MATHEMATICS

Paper 4037/22

Paper 22

General comments

There was wide variation in the performance of the candidates. Candidates scored well in **Questions 2, 7** and **8**. They found **Questions 3, 4, 6** and **9** less accessible and so these are topics that would perhaps benefit from more study.

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Answers: **(i)** 4 **(ii)** 10.4 **(iii)** 14 **(iv)** 23.9

ADDITIONAL MATHEMATICS

Paper 4037/23
Paper 23

General comments

Some candidates produced high quality work displaying wide ranging mathematical skills, with well presented clearly organised answers. Presentation of answers was generally very clear to follow. There were some weak performances but most candidates seemed able to find some questions on which they could demonstrate knowledge and technique. Nearly all candidates attempted the majority of questions. Candidates need to ensure that they read the questions carefully so that the work they do is relevant. Some candidates wasted time by producing several pages of working that had little relevance to the question they were answering. Candidates should take care in the accuracy of plotting points on graphs and be wary of approximating values during working leading to inaccuracies in their final answers.

Candidates found that **Questions 5, 7** and **9** were the most straightforward, and most likely to yield full marks. The use of calculators was evident on **Question 8** at times despite the clear instruction not to do so. Candidates should be reminded that use of a calculator in such questions will result in a score of zero. Candidates would also be advised to use radians when instructed as in **Question 9** where the use of appropriate circular measure formulae was more successful than finding proportions of the full circle.

Comments on specific questions

Question 1

(i) The most successful answers were those who rearranged and differentiated using a negative index. Progress was often made using the quotient rule but candidates did not see the need to simplify by cancelling appropriate brackets and left an unsimplified answer or gave the derivative of 10 as 10.

(ii) Candidates needed to substitute 6 into their derivative and then multiply by p to gain credit. Answers without p gained no credit as did answers which substituted 6 and 6 + p into y.

Answers: **(i)** $-30(x + 4)$ **(ii)** $-0.003p$

Question 2

Many candidates knew that (4, 22) had to be substituted to find a constant. There were many candidates that mistakenly tried to find a linear equation, rather than an equation of a curve. For those that realised the need to integrate, this was generally done well, although a significant number tried to integrate without expanding brackets first.

Answers: $y = x^3 - 3x^2 + 6$

Question 3

This question tested knowledge and the ability to interpret graphs and their equations rather than methodology. Part **(b)** was generally attempted better than part **(a)**. Finding the period confused some who realised the significance of the 2 but gave this rather than π as the answer. This contrasted with finding B in part **(a)(ii)** which was often the only correct part.

Answers: **(a)(i)** 4 **(ii)** 3 **(iii)** 5 **(b)(i)** π or 180 **(ii)** 6

Question 4

(i) This was usually completely correct with the requirement to simplify coefficients being well understood by most. Additional terms were sometimes included and the occasional misread led to terms in descending order.

(ii) Candidates need to realise the significance of the word 'hence' in the question. Marks are unlikely to be awarded if work from previous parts is not used in these questions. Those who did usually made the correct substitution, if not in all terms, and attempted to multiply out brackets. The correct solution required identifying two appropriate terms from their expansion and adding these. The common error was to only find the term from the cubed term in part **(i)**.

Answers: **(i)** $1+6x+15x^2+20x^3$ **(ii)** -10

Question 5

Almost all candidates were able to gain some if not all marks on this matrix question and, in the case of some very weak candidates, this question contributed significantly to their total. Candidates are advised to take care when combining matrices, as marks were lost through careless arithmetical slips. Some candidates thought that it was not possible to calculate the product **AB** asked for in part **(a)**. It was also important to form a row rather than a column matrix in this part. The inverse of matrix **C** was usually found correctly, but the product $\mathbf{DC}^{-1}$ was often attempted.

Answers: **(a)** $\begin{pmatrix}4 & -15\end{pmatrix}$ **(b)(i)** $\frac{1}{2}\begin{pmatrix}4 & 5\\ -2 & -3\end{pmatrix}$ **(ii)** $\begin{pmatrix}17 & 12\\ -9 & -8\end{pmatrix}$

Question 6

(a) Some candidates did not include the region outside both *A* and *B* and the intersection of the two sets and so were not awarded this mark.

(b) Many candidates gained a mark by representing one or other of the required terms but few managed both. The question asked for one diagram showing all three sets and partial answers gained no credit. *P* and *Q* should have had no intersection and writing the empty set symbol in the intersection contradicts this.

(c) Most candidates formed an equation and solved it but only the minority had the correct equation. Using a Venn diagram was most useful in order to identify the four regions which summed to 50 and not the 4 values stated in the question. It was also important to realise that *x* was not the required answer and that this needed to be multiplied by 3.

Answers: **(c)** 12

Question 7

There were many good attempts at this question and many completely correct solutions to a question which was not broken down to give guidance on the method. Elimination of one variable, most frequently *x*, was attempted by virtually all candidates with very few complete omissions. Accuracy was an issue for some, forming an incorrect quadratic – those finding a linear equation should have noted that two points of intersection were required. Using the quadratic solutions to find the other coordinates was also frequently correct and the distance formula/Pythagoras was usually stated and applied accurately although it was possible to see the correct final value from erroneous earlier work.

Answer: 29.1

Question 8

Candidates are reminded that where an answer is given in the question it is important to give a full justification of their arrival at that value or term.

(i) This mark was gained by virtually all candidates with clear indication of the expansion components which summed to 43.

(ii) This was far less successful with only the more able candidates acknowledging the hence and using part **(i)**. Many squared rather than rooting or tried to solve simultaneous equations.

(iii) Similar problems arose as in part **(ii)** but more candidates arrived at the correct answer realising the sign link with part **(i)**.

(iv) Those who rationalised the given term first made progress, usually carrying out this process correctly but forgetting the need to find the square root. Those who realised the denominator as the given answer from part **(i)** were more likely to gain full credit successfully rationalising their initial fraction.

Answers: **(ii)** $6+5\sqrt{2}$ **(iii)** $5-3\sqrt{2}$ **(iv)** $\frac{5-3\sqrt{2}}{7}$

Question 9

There were many different correct methods available to candidates for most parts of this question.

Part **(i)** was well answered by the majority of candidates. Part **(ii)** was less successful with common errors including rounding to 2.2 radians without showing a more accurate value first, leaving the answer in degrees, or only calculating angle *AOD* or equivalent. In parts **(iii)** and **(iv)**, many candidates had a complete correct plan. A common error in part **(iii)** was to add an extra '30' and in part **(iv)** to include the entire rectangle in the answer. Some of the candidates who lost a mark due to their angle being in degrees in part **(ii)** managed to recover to gain full credit in parts **(iii)** and **(iv)** through application of the appropriate formulae using a fraction of a circle.

Answers: **(i)** 17 **(ii)** 2.16 **(iii)** 82.7 **(iv)** 432

Question 10

As with **Question 7** it was pleasing to see so many good attempts at an unstructured question. Some candidates omitted this question or stopped having attempted only the first step. Candidates should remember to work to a greater degree of accuracy than that required in the answer, since marks were lost due to inaccuracy caused by rounding interim values. Not all candidates realised the need to apply the product rule and of those that did there were errors in differentiating one or other of the terms. Some candidates were too eager to find the equation of the normal and did not find point *A* despite correctly finding the gradient of the tangent. Care must also be taken when substituting values into denominators – e quite often started there but migrated to the numerator on the next line. There were several methods of finding the area correctly but in some cases this was not found. Fewer than usual used the array method, realising that they were dealing with a right-angled triangle.

Answers: $3e^3+e$ or 63(.0) or 63.1

Question 11

Parts **(i)** and **(ii)** were well attempted by a large number of candidates although the sign of a vector needs to be carefully addressed. Candidates also need to consider using brackets when multiplying vectors as the sense is almost always lost without them. Part **(iii)** created greater problems and only the higher scoring candidates were totally successful. When candidates equated coefficients of **a** and **b** there was always some credit gained and but for slips in number work or the use of signs, there was often full credit. When candidates tried other routes such as forming equations to find **a** and **b** first then little if anything was gained.

Answers: **(i)** $\mathbf{a} + \mu(3\mathbf{b} - \mathbf{a})$ **(ii)** $\mathbf{b} + \lambda(2\mathbf{a} - \mathbf{b})$ **(iii)** $\mu = 0.2$, $\lambda = 0.4$

Question 12

While the **OR** option was the most popular there were a large number of attempts at both. Most candidates felt able to plot some points and virtually all realised that their graph should be linear. More care needs to be taken in the accuracy of the plotting which ranged, in both alternatives, from very erratic to trying to be too emphatic in displaying points to within 0.1 mm. The algebraic rearrangement parts of each question lacked depth with answers arrived at too quickly with insufficient evidence or omitted altogether.

Question 12 EITHER

(i) Points were usually plotted with some degree of accuracy. Some graphs were spoilt by one poor point leading to a line of best fit which caused accuracy problems later.

(ii) Not all candidates rewrote the original equation in a linear form to match their graph and therefore used tabulated points and simultaneous equations rather than reading values from their graph. Those using the graph were usually more accurate in finding a and b. Those substituting into a non-linear equation failed to score as a result.

(iii) Attempts at rearrangement were usually correct. On a number of scripts an incorrect equation was given with no evidence of working.

(iv) In order to gain credit in this part it was necessary to have correctly rearranged in part **(iii)**. Simply stating how a and b might be determined without reason was insufficient.

Answers: **(ii)** b = 10.2 to 10.3, a = 4 to 4.5 **(iii)** pv **(iv)** Gradient is a, y intercept is b

Question 12 OR

(i) Points were usually plotted with some degree of accuracy. Some graphs were spoilt by one poor point leading to a line of best fit which caused accuracy problems later.

(ii) The accuracy depended greatly on the quality of the graph with candidates often avoiding reading errors by using tabulated values. The better candidate often jumped in one step from an equation linking $\ln r$ to $\ln t$ to a correct equation involving r and t. Some candidates stopped after the first step and need to reflect on the wording which implies an answer of the form $r = \mathrm{f}(t)$ was required. A significant number incorrectly wrote an answer in this form which should have been the ln form.

The intention of the final part was misunderstood by many candidates. The best answers simply followed on from values found in part **(ii)**. A large number of candidates in effect began again and tried to recalculate points for the graph they had been instructed not to draw and often arrived at the values they had not found earlier due to careless plotting.

Answers: **(ii)** Gradient is 1.3, Intercept is 2.2, $r = 9t^{1.3}$ Gradient is 1.3, Intercept is 0.95