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FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

ADDITIONAL MATHEMATICS

GCE Ordinary Level

Paper 4037/01

Paper 1

General comments

Most candidates found the paper within their grasp and there were fewer very poor scripts than in previous years. **Question 5** presented all candidates with problems and the response to **Question 11** was disappointing. Because of these two questions, there were fewer candidates than usual scoring very highly. The standard of numeracy and algebra remained high and most scripts were well presented.

Comments on specific questions

Question 1

This proved to be a successful starting question and most candidates scored 2 or more of the 3 marks available. The majority squared the left-hand-side, set the equation to zero and solved to obtain $x = 4$ and $x = 9$. Unfortunately only a small proportion then obtained the correct range; the usual offerings being either '$4 < x < 9$' or '$x < 4$ and $x < 9$'.

Answer: $x < 4$ and $x > 9$.

Question 2

Generally speaking the question yielded 0, 2 or 4 marks. There were many correct answers to part **(a)**, though many variations were offered to the simplest answers of $A' \cap B$ and $A' \cup B$. Use of the signs +, − and $^{-1}$, and misuse of brackets when three sets were given in an answer (for example $A \cap B \cup A'$) showed that many candidates had only a very shallow understanding of set theory. Part **(b)** proved to be more successful and the drawings were well presented.

Answers: **(a)(i)** $A' \cap B$, **(ii)** $A' \cup B$.

Question 3

There were two main methods for solving this question and candidates offered both methods in almost equal numbers. The first method involved eliminating y (or x) to form a quadratic equation for which '$b^2 - 4ac = 0$'. The second method involved equating the gradient of the curve with the gradient of the line, solving to find x, then y and finally c. In both cases, one solution was often missed through solving '$c^2 = 144$' as '$c = 12$' only or '$x^2 = 4$' as '$x = 2$' only and in the first method $b^2 - 4ac < 0$ and $b^2 - 4ac > 0$ were both seen.

Answer: $c = 12$ or -12.

Question 4

This proved to be a successful question that showed a confident approach to the manipulation of surds. Most candidates realised that the height was equal to the volume divided by the area of the base. Obtaining the area of the base as $7 - 4\sqrt{3}$ was accurately carried out and most candidates realised the need to multiply top and bottom by $7 + 4\sqrt{3}$. Taking the base as $2 - \sqrt{3}$ instead of $(2 - \sqrt{3})^2$ was seen in several scripts. Very few candidates used the alternative method of equating $(7 - 4\sqrt{3})(a + b\sqrt{3})$ with $2\sqrt{3} - 3$ and then solving the resulting simultaneous equations.

Answer: $3 + 2\sqrt{3}$.

Question 5

This proved to be a very difficult question and more than half of all candidates ignored it completely. Only a small proportion of the others were correct. Solutions that were seen varied between two methods. The first involved obtaining the relative displacement ($84\mathbf{i} - 7\mathbf{j}$) and the relative velocity as ($8\mathbf{i} - (k - 2)\mathbf{j}$) and then realising that one was a direct multiple of the other. The alternative was to obtain the position vector of both the fly and the spider at time t and to obtain two simultaneous equations for t and k.

Answer: $k = 2\frac{2}{3}$.

Question 6

This was generally well answered though a few candidates failed completely by linking acceleration with integration and distance with differentiation. The standard of both the differentiation and the integration was pleasing, though careless errors over the signs led to some incorrect answers in both parts. Part **(i)** was very well done. Part **(ii)** suffered through either failure to include a constant of integration or to assume that because $s = 0$ when $t = 0$, then c must automatically be 0. Only a small proportion of all attempts coped correctly with part **(ii)**.

Answers: **(i)** 2.25 ms^{-2}; **(ii)** 2.41 m.

Question 7

In part **(a)** a majority of attempts were correct and most candidates realised the need to express 2 as $\log_7 49$ (though $\log_{10} 49$ was also seen) and then to form a linear equation in y. Weaker candidates showed a complete lack of understanding by offering such statements as $\log_7(17y + 15)$ as $\log_7 17y + \log_7 15$. Part **(b)** proved to be more difficult though candidates generally showed more confidence in changing the base of a logarithm than in previous years. Most expressed $\log_p 8$ and $\log_{16} p$ in the same base, usually 2, 4, 8 or 16 (or even 10), and this proved to be more successful than changing to base p since candidates struggled to realise that $\frac{\log_p 8}{\log_p 16} = \frac{3}{4}$, (equating to $\frac{1}{2}$ was the usual offering). Use of base 16 led directly to $\log_{16} 8$ and most realised that this equated to $\frac{3}{4}$.

Answers: **(a)** 2; **(b)** $\frac{3}{4}$.

Question 8

Part **(i)** was generally well answered. Nearly all candidates realised the need to use the product formula and the differentiation of $\sqrt{(x - 1)}$ was accurate. The algebra needed to express the resulting expression as a single fraction in the form $\frac{kx}{\sqrt{x-1}}$ proved to be too difficult for many weaker candidates. Many candidates failed to realise (despite the word 'hence') that part **(ii)** was directly linked to part **(i)**. Of those recognising the link, about a half realised that the required integration led to $\frac{1}{k}\sqrt{(x-1)(x+2)}$, though $\sqrt{(x-1)(x+2)}$ and $k\sqrt{(x-1)(x+2)}$ were common errors.

Answers: **(i)** $k = 1\frac{1}{2}$; **(ii)** $6\frac{2}{3}$.

Question 9

(a) This proved to be accessible to most candidates and the vast majority scored at least 4 of the 5 marks available. Use of the formulae linking sine, cosine and tangent was accurate and the quadratic $3\sin^2 x + 8\sin x = 3$ was nearly always correct. Apart form a few weaker candidates who still believe that 'sin*x*(3sin*x* + 8) = 3 ⇒ sin*x* = 3 or 3sin*x* + 8 = 3', most solved the equation correctly and obtained correct values of *x* in the correct quadrants.

(b) This was poorly answered. Weaker candidates failed to realise the need to keep $\cos\left(\frac{2}{3}y\right)$ as a unit and such offerings as $\frac{2}{3}\cos y$ were depressingly common. The majority realised that $\frac{2}{3}y = \cos^{-1}\left(-\frac{1}{2}\sqrt{3}\right)$ but unfortunately errors over the minus sign and misuse of radians led to relatively few correct answers. The restriction of $4 \leqslant y \leqslant 6$ led to some weaker candidates testing only the integral values 4, 5 and 6, whilst better candidates ignored the range and offered only 3.93 as their answer.

Answers: **(a)** 19.5°, 160.5°; **(b)** 5.50 or $\frac{7\pi}{4}$.

Question 10

The question was generally well answered and proved to be a source of high marks for most candidates. Part **(i)** was usually correctly answered with most candidates obtaining $AB = BC = \sqrt{40}$. Methods varied in part **(ii)** between finding the equation of either the perpendicular bisector of *AC* or the equation of the line perpendicular to *AC* through B and then setting *y* to zero, or by obtaining and then equating expressions for *AD* and *CD* where *D* is (*d*, 0). Both methods proved equally successful in finding the coordinates of *D*. Solutions to part **(iii)** varied considerably with most candidates finding the area of each triangle, either by the matrix method or by using '$\Delta = \frac{1}{2}bh$' with *AC* as the base and *BM* or *MD*, where *M* is the mid-point of *AC*, as the height. Brighter candidates realised that the ratio of the areas was equal to the ratio *BM : MD*, others incorrectly took the ratio as $BM^2 : MD^2$ or, by assuming that the triangles were similar, as $AB^2 : AD^2$. The final answer of $\sqrt{20} : \sqrt{180}$ was also often changed to 20 : 180.

Answers: **(ii)** (10, 0); **(iii)** 1 : 3.

Question 11

This was poorly answered with most candidates showing only a sketchy understanding of the modulus function. Most realised the 'V' nature of the graph, but the lowest point was often shown on the *x*-axis. A significant number of sketches were seen in which the graph was shown as a parabola, or, through plotting integral values only, as having a horizontal line joining (1, −3) to (2, −3). Those using a table of values were successful in finding the greatest value in part **(ii)**, but failed to find the least value through not realising that this value occurred at $x = 1\frac{1}{2}$. Solutions to part **(iii)** varied considerably. Most only offered $x = 2\frac{1}{2}$, coming directly from $2x - 3 - 4 = -2$. Candidates using the algebraic method of $(2x - 3)^2 = 4$ were more successful as were many candidates who read answers directly from accurately drawn graphs in part **(i)**. Parts **(iv)** and **(v)** were poorly done, showing a lack of understanding of inverse functions. In part **(iv)**, very few solutions were seen in which the one-one property of an inverse was appreciated. Part **(v)** was rarely correct with most candidates thinking the question asked for the inverse when all that was needed was the equation of the left hand part of the 'V' shaped graph.

Answers: **(ii)** $-4 \leqslant f(x) \leqslant 3$; **(iii)** $\frac{1}{2}$, $2\frac{1}{2}$; **(iv)** $1\frac{1}{2}$; **(v)** $-2x - 1$.

Question 12 EITHER

This was the less popular of the two alternatives. Part **(i)** was usually correctly answered with only a few candidates ignoring the instruction for a given scale. Most candidates were able to express $yx^n = a$ as $\lg y + n\lg x = \lg a$, but surprisingly the gradient was often taken as $-n$ and the intercept as a. Only a small proportion realised in part **(iii)** that $y = x^2$ could be converted to $\lg y = 2\lg x$ and represented on the graph by a line of gradient 2 through the origin. Most candidates failed to link the drawing of $y = x^2$ with the logarithmic values on the axes. Even when a correct line was drawn, only a few candidates realised that the value of the intersection was a value for $\lg x$, not for x.

Answers: **(ii)** a = 19.5 to 21.0, n = 2.45 to 2.60; **(iii)** x = 1.90 to 2.00.

Question 12 OR

A large number of candidates scored full marks on the question – others suffered through using an incorrect formula for either the area of a sector or for the area of a triangle. Use of radians was good, though some candidates converted at the start and worked throughout in degrees (usually perfectly correctly). Part **(i)** was usually correctly answered. In part **(ii)** failure to recognise that *AC* is the radius of the circle for which *CAD* was a sector, meant that further progress was limited. The length of *AC* was calculated by a variety of methods – the cosine rule, the sine rule or by splitting Δ*AOC* into two equal right angled triangles – and the calculation was usually correctly completed. Part **(iii)** presented most difficulty, with many candidates failing to realise the need to calculate the area of triangle *AOC*. The shaded area came directly from 'area of sector *COB* + area of Δ*AOC* − area of sector *CAD*'.

Answers: **(i)** 38.4 cm²; **(ii)** 52.3 cm²; **(iii)** 15.9 cm².

Paper 4037/02
Paper 2

General comments

The particular mathematical topics covered by the paper on this occasion generally produced satisfactory responses from the candidates.

Comments on specific questions

Question 1

What was intended as a simple calculation proved difficult for many candidates. A common error was to regard $\ln 1000\,e^{-kt}$ as $-kt\ln 1000e$. An issue of more concern was the large number of candidates who, presumably thinking that they were quoting to 3 significant figures, gave the value of k as 0.03; this led in turn to the incorrect value of 407 for V in part **(ii)**.

Answers: **(i)** 0.0330; **(ii)** 371 or 372.

Question 2

This proved to be the easiest question on the paper and loss of marks was usually due to carelessness rather than incorrect methods. A few candidates used the given formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to solve their quadratic in y and inadvertently found x, rather than y, to be 4 or –6; some others achieved the same result by assuming that the conventional order of quoting coordinates was (y, x) rather than (x, y). A handful of candidates first obtained the values 6 and 16 for x and then attempted to find the corresponding values of y by using $y^2 = 2x + 4$, which invariably resulted in the positive y values of 4 and 6. Some candidates found the coordinates of *A* and *B* correctly but omitted to find the mid-point of *AB*. Only a very small minority of candidates mistakenly used $\left(\dfrac{x_1 - x_2}{2}, \dfrac{y_1 - y_2}{2}\right)$ to calculate the coordinates of the mid-point.

Answer: (11, –1).

Question 3

Most candidates were able to make a reasonable attempt at differentiation although the factor 2 was sometimes omitted and some were unable to cope with the constant 1, taking $\frac{dy}{dx}$ to be $1+\frac{2}{2x-3}$ or $x+\frac{2}{2x-3}$. Some of the weakest candidates thought ln(2x – 3) was identical with $\frac{1}{2x-3}$, whilst ln(2x – 3) was sometimes taken to be ln 2x – ln 3 and others thought the derivative involved $\frac{1}{\ln(2x-3)}$. Very many candidates attempted to find δy by substituting 2 + p for x and/or for δx in their expression for $\frac{dy}{dx}\times\delta x$. Having found an expression for δy, the vast majority of candidates assumed they had completed the question and the final step was rarely seen.

Answers: **(i)** $\frac{2}{2x-3}$; **(ii)** 1 + 2p.

Question 4

The amplitude was often correctly stated as 5, although 10 was given occasionally and 7 frequently occurred. The period of f was often incorrect; some candidates clearly had no idea at all and the answers 3 and 60°, presumably from 180° ÷ 3, were much in evidence. Surprisingly, some candidates who obtained full marks for the graph were unable to state the period correctly. Graphs, unless from the weakest candidates, usually earned at least some of the available marks. Common errors were to start at the origin and/or finish at (180°, 0) or to constrain the graph to pass through (60°, 0) and (120°, 0). Some candidates only produced one cycle, for $0° \leqslant x \leqslant 120°$. Many graphs showed 3 cycles for $0° \leqslant x \leqslant 180°$ or, rather unnecessarily, 6 cycles for $0° \leqslant x \leqslant 360°$. Candidates plotting values at 30° intervals usually produced the best shaped graphs, but the plotting approach was not always successful in that some candidates produced mis-shapen graphs through plotting at 45° intervals.

Answer: **(i)** 5; 120°.

Question 5

This question proved to be difficult for weaker candidates who were rarely able to cope with the algebra involved. A few candidates applied the given formula but were unable to resolve 1^{n-1} and 1^{n-2}. Others interpreted the powers of px as px^2 and px^3, rather than p^2x^2 and p^3x^3. Some failed to divide $\frac{n(n-1)}{1\times 2}p^2 = 28p^2$ by p^2 and, instead of solving n (n – 1) = 56, substituted $-\frac{12}{n}$ for p or $-\frac{12}{p}$ for n, usually becoming muddled at some point of the process. A few found a non-integer value of n or used n = –7 as well as n = 8, despite being given n > 0. The most common error was to take the coefficient of x to be 12, ignoring the negative sign and so obtaining positive values for p and q.

Answers: 8; –1.5, –189.

Question 6

Although some of the better candidates evaluated k from the transformed equation $\mathbf{A}^2 + \mathbf{I} = k\mathbf{A}$, the initial step for most candidates lay in finding $\mathbf{A}^{-1}$, a task which was performed correctly by all but the very weakest candidates. Some candidates misunderstood the left-hand side of the given equation and so multiplied **A** and $\mathbf{A}^{-1}$. Others were unable to add elements correctly, for instance taking the sum of 3 and $\frac{p}{3p-5}$ to be $\frac{3+p}{3p-5}$. Quite a number of the weaker candidates took **I** to be $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and a few clearly identified **I** with the scalar 1 in taking $k\mathbf{I}$ to be merely k. Although the simplest way to evaluate p was to equate $1-\frac{1}{3p-5}$ or $5-\frac{5}{3p-5}$ to 0, many candidates preferred to equate each of the elements of the leading diagonal of $\mathbf{A} + \mathbf{A}^{-1}$ to k, eliminate k and then solve the ensuing quadratic in p. This approach not only gave the correct values $p = 2$, $k = 5$ but also the spurious values $p = 3$, $k = 3\frac{3}{4}$ which do not satisfy the given equation and which were rarely rejected by the candidate.

Answers: 2, 5.

Question 7

Most of the errors in parts **(i)** and **(ii)** were to do with signs and directions e.g. $\overrightarrow{PQ} = \overrightarrow{OP} + \overrightarrow{OQ}$ = **p** + **q** or $\overrightarrow{PQ} = \overrightarrow{OP} - \overrightarrow{OQ}$ = **p** – **q**. A small number of candidates misread $\frac{2}{5}\overrightarrow{OQ}$ as $\frac{2}{3}\overrightarrow{OQ}$. Failure to answer the question was a frequent error in part **(ii)** where many candidates merely found $\overrightarrow{PX}$ to be $n\left(\frac{2}{5}\mathbf{q} - \mathbf{p}\right)$. Surprisingly, many of the candidates who answered parts **(i)** and **(ii)** correctly, or who connected $\overrightarrow{OX}$ and $\overrightarrow{PX}$ correctly with $\overrightarrow{OP}$, were unable to make any progress in part **(iii)**.

Answers: **(i)** $\frac{m}{3}(2\mathbf{p} + \mathbf{q})$; **(ii)** $(1 - n)\mathbf{p} + \frac{2n}{5}\mathbf{q}$; **(iii)** $\frac{2}{3}, \frac{5}{9}$.

Question 8

(a) There were many correct answers but some candidates only obtained one value correctly through errors such as $2^p = 64 \Rightarrow p = 8$ or omitting the negative sign and giving $q = 3$. Some candidates could make no progress after evaluating $\left(\frac{25}{16}\right)^{-\frac{3}{2}}$ as 0.512 or erroneously assuming $2^p \times 5^q = 10^{p+q}$.

(b) Part **(i)** was intended as a help in dealing with part **(ii)** but, in fact, many candidates, who produced e.g. $y^2 - 2y - 3 = 0$ in part **(ii)**, appeared puzzled by part **(i)** and did not entirely understand what was required e.g. offering 4^x as $(2^2)^x$ rather than $(2^x)^2$. Some candidates mistakenly took 2^{x+1} to be $2^x + 2^1$, while others attempted to take logarithms, the left-hand side of the equation becoming $x\log4 - (x + 1)\log2$. A few candidates replaced 2^x by x and finding x to be 3 or –1 stopped at that point instead of going on to deal with $2^x = 3$. Whether k was 3 or some other positive value the solution of $2^x = k$ was well understood. Occasionally 1.58496 … was taken to be 1.59 to 2 decimal places, presumably through first approximating to 1.585.

Answers: **(a)** 6; –3; **(b)(i)** $(2^x)^2 - 2(2^x) - 3 = 0$, **(ii)** 1.58.

Question 9

The vast majority of candidates applied the factor and remainder theorems successfully. Most lapses were algebraic with only occasional errors in principle, such as f(–2) = 0, f(–2) = 55 or f(2) = –55. A few candidates attempted to obtain the required equations in *a* and *b* by either long division or synthetic division, but the algebraic skill required to perform these operations successfully was usually lacking. The handling of simultaneous linear equations in two unknown was well understood. Having evaluated *a* and *b* some candidates undertook a search for a factor of their cubic expression, overlooking the fact that they had been told the expression was exactly divisible by $x - 3$. Determining the quadratic factor and solving the ensuing quadratic equation were processes with which the vast majority of candidates were familiar and proficient and, as a consequence, there were many correct solutions.

Answers: **(i)** 10, –3, **(ii)** 0.382; 2.62; 3.

Question 10

(i) Weak candidates simply ignored the possibility of constants of integration and reached $y = x^3 - x^2$. Others assumed that one constant of integration was sufficient and either introduced *c* after the second integration to obtain $y = x^3 - x^2 + c$ or introduced the same constant *c* at both stages to arrive at $y = x^3 - x^2 + cx + c$. The erroneous value of $c = -17$ was quite often in evidence, being obtained from $[3x^2 - 2x + c\,]_{x=2} = -9$.

(ii) If the candidates had been asked to find the stationary value – and its nature – of $3x^2 - 2x - 5$ it is quite possible that nearly all the candidates would have successfully completed this task. The present question posed exactly the same problem but, because of the context, a small minority of those candidates reaching $\frac{dy}{dx} = 3x^2 - 2x - 5$ earned full marks for this part of the question. Some candidates were simply unable to attempt part **(ii)**. Others put $\frac{dy}{dx}$ equal to zero and found the maximum and minimum points of the curve $y = x^3 - x^2 - 5x - 3$; a handful of candidates who used this approach then produced a convincing argument, based on symmetry, that the least value of the gradient occurred midway between the stationary points. Many candidates started with $\frac{dy}{dx}$ either equal to or not less than $-\frac{16}{3}$ and although many obtained $(3x - 1)^2$ and $x = \frac{1}{3}$ arguments were usually incomplete or vague. A few candidates successfully completed the square and arranged $3x^2 - 2x - 5$ as $3(x - \frac{1}{3})^2 - \frac{16}{3}$, but the simplest argument, $\frac{d}{dx}$(gradient) $= 6x - 2 = 0 \Rightarrow x = \frac{1}{3}$, gradient $= -\frac{16}{3}$, $\frac{d^2}{dx^2}$(gradient) $= 6 > 0 \Rightarrow$ minimum, was very rarely seen.

Answer: **(i)** $y = x^3 - x^2 - 5x - 3$.

Question 11

(a) Many candidates earned both available marks. The most common error lay in using ${}^{10}P_3$ and ${}^{9}P_2$ rather than ${}^{10}C_3$ and ${}^{9}C_2$. Some very weak candidates showed a lack of common sense, both here and in part **(b)**, by giving an answer for the restricted case which was larger than their answer for the unrestricted case.

(b) The first part of this question was generally well done with many candidates obtaining the correct answer of 96. On the other hand relatively few candidates obtained full marks for the final part and, in fact, many failed to score. Some candidates thought the answer could be obtained in one operation so that 72, from 4 × 3! × 3, was a common answer. Others assumed that, of the 96 5-digit numbers, half would be odd and half even. In order to answer this question candidates must be able to analyse the situation and, most importantly, explain what they are attempting to do in a clear and logical fashion. Unfortunately many answers consisted of a jumble of numbers and an incorrect answer, making it impossible to give any credit.

Answers: **(a)(i)** 120, **(ii)** 36; **(b)** 96; 60.

Question 12 EITHER

Not only was this the more popular option, it also rewarded the vast majority of candidates with a reasonable number of marks, in that the first two parts proved to be easy. Few candidates approached the coordinates of *P* by the more direct route of equating the gradient of the curve, $2x - 10$, to -4, and the preferred method was to find the equation of the tangent *ST* and solve this with the equation of the curve. The ideas required to formulate the equation of the normal at *P* and to obtain the coordinates of *R* were well known, although it was surprising that, despite the given diagram clearly indicating that a positive *x*-coordinate of *R* must be incorrect, those candidates obtaining such a value were not deterred from using this in part **(iii)**. The difficulty posed by part **(iii)** did not lie in integrating $x^2 - 10x + 24$, which was almost always performed correctly, but in conceiving a correct plan to evaluate the required area. Many candidates calculated $\int_{3.75}^{4}\left(x^2 - 10x + 24\right)dx$ and added this to the area of triangle *RPT*. This latter area was very often obtained by the laborious method of $\frac{1}{2}$*RP* × *PT*, rather than the much more direct $\frac{1}{2}$ base × height with *RT* as the base. Another erroneous method involved the area of the triangle *RPT*, or of the right-angled triangle with *RP* as hypotenuse, added to $\int_{3}^{4}\left\{\left(x^2 - 10x + 24\right) - \left(15 - 4x\right)\right\}dx$.

Answers: **(i)** (3, 3); **(ii)** (– 9, 0); **(iii)** $19\frac{1}{3}$ units^2.

Question 12 OR

(i) Errors in sign were frequent, as was the omission of the factor 2 arising from the differentiation of the double angle.

(ii) Some candidates thought they were being asked to investigate a stationary point of $y = 2\sin x \cos x$. The replacement of $\sin 2x$ by $2\sin x \cos x$ was not always successfully accomplished; in some cases $2\sin 2x$ became merely $2\sin x \cos x$ and in others $4\sin x\ 2\cos x$. The given domain was quite often overlooked so that the *x*-coordinate of the stationary point was taken to be 0, from $\sin x = 0$. The process of determining the nature of the stationary point was clearly understood by almost all candidates.

(iii) Correct answers were in a minority. Some candidates did not integrate at all, merely attempting to evaluate $\left[2\cos x - 2\cos 2x\right]_{\pi/3}^{\pi/2}$, others clearly differentiated rather than integrated and some of the weakest candidates took $\int y\,dx$ to be $\frac{1}{2}y^2$.

Answers: **(i)** $2\sin 2x - 2\sin x$, $4\cos 2x - 2\cos x$; **(ii)** $\frac{\pi}{3}$, maximum; **(iii)** 0.701.