CONTENTS

FOREWORD 1

ADDITIONAL MATHEMATICS 2

GCE Ordinary Level 2
Paper 4037/01 Paper 1 2
Paper 4037/02 Paper 2 5

FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

ADDITIONAL MATHEMATICS

GCE Ordinary Level

Paper 4037/01

Paper 1

General comments

Candidates tended to find the Paper difficult. It was very noticeable that responses to those questions of a similar style and content to those of the previous syllabus (particularly **Questions 1**, **2**, **6**, **9**, **11**) were of a much higher standard than questions on topics which were completely new to the syllabus (**Questions 5** and **10** in particular). The less stereotyped nature of **Questions 3** and **7** also presented candidates with considerable problems. In general, the standard of numeracy, algebra and presentation shown by the candidates was pleasing.

Comments on specific questions

Question 1

This proved to be a good starting question with most candidates correctly using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to obtain $\tan\theta = -\frac{3}{4}$, though $\tan\theta = +\frac{3}{4}$ or $\pm\frac{4}{3}$ were also common. Many weaker candidates used "$\sin\theta = 1 - \cos\theta$". Candidates generally coped very well with realising that there were 2 solutions within the range of 0° to 360°.

Answers: 143.1° and 323.1°.

Question 2

Most candidates, having eliminated *y* from the two equations to form a quadratic equation, then realised that the discriminant of this equation was zero if the line was a tangent to the curve. A minority differentiated both equations to obtain "$m = \frac{1}{2}x$" and then substituted into one of the original equations to obtain a quadratic for *x* or for *y*. At least a quarter of all candidates arriving at "$m^2 = k$" ignored the solution $m = -k$.

Answer: $m = \pm 3$.

Question 3

This was very badly answered with a minority of candidates realising that the question only required the solution of a quadratic inequality – i.e. $v \geqslant 5$. At least half the candidates attempted to either differentiate or integrate and made no progress. Of those setting $v = 0$, most obtained two values for *t* but then either gave decimal equivalents or failed to realise the need to subtract the two values.

Answer: Proof.

Question 4

Most candidates realised the need to integrate the equation of the given curve and to link the area obtained with the area of a rectangle to obtain the required answer. There were, however, a lot of errors, particularly with the integration, with $\int e^{-x}dx = e^{-x}$ or $\int e^{x}dx = \frac{e^{x+1}}{x+1}$ being most frequent. Use of the limits −1 to 1 was generally used, and usually accurate; others used 0 to 1 and then doubled the answer. Most correctly obtained the value of *y* at either $x = 1$ or $x = -1$ and correctly deduced the area of the rectangle. Others however took the rectangle as having an area equal to "$2 \times$ value at $x = 0$".

Answer: 1.47.

Question 5

Generally speaking this question was either completely correct or worthless. It seemed that many candidates had never encountered the topic of matrices, whilst many weaker candidates found this the easiest question on the Paper. Failure to deal with the “0” in the original matrix caused some difficulty, but most problems came from fundamental lack of knowledge of the need for matrices to be compatible for matrix multiplication.

Answer: $27 600.

Question 6

The vast majority of candidates realised the need to integrate the given equation and it was pleasing that most of these included a constant of integration. Most also realised the need to set $y = 0$ at the point where the curve crossed the x-axis. Less than a half of all attempts at the integration were correct, mainly through omission of the “$\frac{1}{2}$” and inclusion of other functions of x. Weaker candidates failed to realise that $\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$ and attempted instead to expand the denominator and to use three separate terms for integration.

Answer: (1.75, 0).

Question 7

(i) This question was badly answered with only a few completely correct attempts. Some candidates failed to recognise the need to use calculus for the first part and gave either long explanations or sketches or accurate graphs. Those who differentiated were generally correct though $\frac{d}{dx}(x^3 + x - 1) = 3x + 1$ was a common error. Most realised the need to set the differential to zero though others incorrectly set the second differential to zero. It was pleasing that most realised that there was no solution to the equation $3x^2 + 1 = 0$ and that therefore the curve had no turning points. Very few however realised that because there were no turning points, the function was always increasing and therefore was one-one – and that the inverse existed for all x.

(ii) This part was very disappointing, with only a small proportion realising the need to solve the cubic equation $f(x) = 0$, and that, because $f(x)$ had no turning points, there could only be one solution i.e. $x = 2$ found by trial and improvement.

Answers: **(i)** No turning points, one-one function; **(ii)** 2.

Question 8

Responses to this question were very variable and because of the stereotyped nature of the question, some weaker candidates tended to find the question to their liking. Once candidates realised that the equation in part **(i)** reduced to a quadratic equation in e^x, solutions tended to be correct, though “$e^x(2e^x - 1) = 10 \Rightarrow e^x = 10$ or $(2e^x - 1) = 10$” was a frequent error. Many candidates took logarithms of each term and made no progress. In part **(ii)** most candidates expressed the left hand side as $\log_5\left(\frac{8y-6}{y-5}\right)$, though several attempts were seen in which $\frac{\log_5(8y-6)}{\log_5(y-5)}$ appeared. About a half of all attempts realised that $\log_4 16 = 2$ and that the equation reduced to $\frac{8y-6}{y-5} = 5^2$.

Answers: **(i)** ln2.5 or 0.916; **(ii)** 7.

Question 9

This question was very well answered by the majority of candidates and the marks obtained were high. In particular, the standard of algebra shown in the solution of the simultaneous equations to find the coordinates of P and Q was impressive. A small minority failed to realise that the perpendicular bisector passed through the mid-point of PQ, but otherwise the question posed few difficulties.

Answer: $6y + 4x + 5 = 0$.

Question 10

(i) Although this was a reasonably straight-forward vector question, most candidates seemed unfamiliar with using position and velocity vectors in this context. Less than a half of all attempts realised that the position vector was obtained from the product of the velocity vector and time and that a constant vector needed to be included. The notation of 1200 hours caused considerable problems, with t often being expressed as $t + 1200$ or $t - 1200$.

(ii) Candidates were more familiar with the numerical work though again the value of t at 1400 hours was often taken as 200, and many, unable to score in part **(i)**, obtained correct expressions for $\mathbf{r}_P$ and for $\mathbf{r}_Q$. Most realised the need to subtract to obtain $\overrightarrow{PQ}$, followed by the use of Pythagoras' Theorem. A significant number, however, made the mistake of using Pythagoras' Theorem on each position vector before subtracting.

(iii) This part was rarely correct with only a small minority realising the need to find whether there was a value of t for which $\overrightarrow{OP} = \overrightarrow{OQ}$. Equating coefficients of **i** and **j** led to different values of t. Many candidates spent considerable time in obtaining the equations of the lines followed by P and Q and deduced that these lines met at $x = 60$, $y = 80$. Unfortunately no check was made on whether P and Q arrived at this point at the same time.

Answers: **(i)** $\mathbf{r}_P = t(20\mathbf{i} + 10\mathbf{j}) + 50\mathbf{j}$, $\mathbf{r}_Q = t(-10\mathbf{i} + 30\mathbf{j}) + 80\mathbf{i} + 20\mathbf{j}$; **(ii)** 22.4 km; **(iii)** P and Q do not meet.

Question 11

This was very well answered and most candidates scored highly.

(i) Most used the correct quotient rule to show that $k = 10$ and pleasingly there were relatively few scripts in which the error "$-(2x - 6) = -2x - 6$" was seen.

(ii) The majority of attempts realised that $x = 3$ at P, though "$\frac{2x-6}{x+2} = 0 \Rightarrow x + 2 = 2x - 6$" was a common error. A large proportion of candidates evaluated the gradient of the normal at P correctly and obtained $y = 7.5$ at R. Virtually all of these then deduced that $y = -3$ at Q and found RQ correctly.

Answers: **(i)** $k = 10$; **(ii)** $RQ = 10.5$.

Question 12 EITHER

This was the less popular of the two alternatives.

(i) Only about a half of all attempts realised that, because of the symmetry of the diagram and the fact that the tangent to a circle is at 90° to the radius, trigonometry led to the result $\sin\theta = \frac{r}{R-r}$, from which the required result could be deduced.

(ii) This part also presented candidates with difficulties. Candidates attempted to use the expressions πr^2 and $\frac{1}{2}r^2\theta$ but errors over converting 30° to radians and inability to spot that $R = 3r$ meant that a correct answer was rarely seen.

(iii) This was more to the candidates' liking, probably because it was numerical. Candidates used trigonometry or Pythagoras' Theorem to calculate the lengths of the straight parts of the perimeter and the formula $s = r\theta$ to calculate the length of the arc. Again, however, misuse of radians or failure to realise that $\theta = \frac{2\pi}{3}$ meant that an accuracy mark was often lost.

Answers: **(i)** Proof; **(ii)** $\frac{2}{3}$; **(iii)** 27.8 m.

Question 12 OR

There were many pleasing solutions to the first two parts. Candidates accurately drew the graph of $\frac{y}{x}$ against x and the majority realised that the value of A came from the gradient and B from the intercept on the y-axis. Answers were again very accurate, except for a significant number of candidates who, through their choice of starting point on the axes, failed to realise that the intercept must correspond to $x = 0$. Despite the request in part **(iii)** to draw a straight line, only about a third of all candidates realised that the equation $y = x^2$ was represented on this set of axes by the straight line $Y = X$ since $Y = \frac{y}{x}$ and $X = x$. Only a few realised in part **(iii)** that the point of intersection of the lines represented the value of x for which the rectangle became a square and in part **(iv)** that the value approached by the ratio of the two sides as x became large was equal to the gradient or the reciprocal of the gradient.

Answers: **(i)** Graph; **(ii)** $A = 0.70 \pm 0.02$, B = 39 ± 1; **(iii)** Rectangle becomes a square; **(iv)** 0.70 or 1.43.

Paper 4037/02
Paper 2

General comments

Comparisons with previous years' performances are obviously difficult in that the style and content of the Paper has changed. This unfamiliarity may be responsible, in some part, for a lessening in the attainment of candidates, as may the limited amount of choice available. On the other hand there seemed to be a considerable number of candidates inadequately prepared and lacking basic skills and knowledge.

Comments on specific questions

Question 1

The majority of candidates were able to obtain the inverse matrix correctly. Arithmetical errors were relatively few and many candidates used the inverse matrix correctly in evaluating x and y. A few candidates used $\begin{pmatrix} 7 \\ 16 \end{pmatrix}$ rather than $\begin{pmatrix} -7 \\ -16 \end{pmatrix}$, whilst others attempted to post-multiply by the inverse matrix. A considerable number of the weakest candidates ignored the instruction to use the inverse matrix and simply solved the two linear equations by elimination or substitution.

Answers: $\frac{1}{3}\begin{pmatrix} 6 & -3 \\ -7 & 4 \end{pmatrix}$; $x = 2$, $y = -5$.

Question 2

Only a small number of the very weakest candidates failed to find the first three terms in the expansion of $(2 + x)^6$, either through an inability to evaluate $\binom{6}{1}$ and/or $\binom{6}{2}$, or by taking the first term to be 1.

The subsequent part of the question provided a variety of methods with relatively few candidates replacing x by $x - x^2$. Some candidates used a similarly economical method by expanding $\{(2 + x) - x^2\}^6$. Others found it necessary to factorise $2 + x - x^2$, an approach which involved difficulties with signs for some candidates in that, in order to factorise, they had to first rearrange the expression so that it began with the term in x^2. Nevertheless, those candidates able to obtain $(2 - x)^6 (1 + x)^6$ usually went on to correctly obtain the answer 48 from $(64 \times 15) + (-192 \times 6) + (240 \times 1)$. The same answer was frequently obtained fortuitously from $(2 + x)^6 (1 - x)^6$ or from $(2 - x)^6 (1 + x)$. Weaker candidates often omitted the latter part of the question or took $(2 + x - x^2)^6$ to be $(2 + x)^6 - x^2$ or $(2 + x)^6 - x^{12}$.

Answers: $64 + 192x + 240x^2$; 48.

Question 3

Few candidates obtained full marks for this question. Part **(i)** was found to be particularly difficult with most candidates unable to rationalise the denominator of a fraction containing surds and there were many errors in the use of fractions, brackets, signs and cancellation. The instruction to express the answers in their "simplest surd form" was generally overlooked or ignored in that, firstly, many weak candidates merely substituted an approximation for $\sqrt{3}$ and proceeded to a decimal answer, and, secondly, many answers were incompletely simplified, e.g. $\frac{4 + 2\sqrt{3}}{2}, \frac{1 + 2/\sqrt{3} + 1/3}{2/3}, \frac{6}{\sqrt{3}}$. A large number of candidates who failed to score in part **(i)**, giving p as $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$, obtained full marks in part **(ii)** since this form of p enabled $p - \frac{1}{p}$ to be dealt with quite easily.

Answers: **(i)** $2 + \sqrt{3}$; **(ii)** $2\sqrt{3}$.

Question 4

Very few candidates failed to obtain $x < 3.5$ in respect of set A. Again, nearly all obtained the critical values, –1 and 2, connected with set B, although the appropriate inequalities, $x < -1$, $x > 2$, did not always follow. The universal set was sometimes ignored but a very large proportion of the candidates erred by assuming that x could only take integer values for the set $A \cap B$. Some candidates tried to combine the inequalities of A and B e.g., $7 - 2x = x^2 - x - 2$ or $8 = 2(x^2 - 2) + 1$ and then solve for values of x.

Answers: $-5 < x < -1$, $2 < x < 3.5$.

Question 5

(a) There were many correct answers. The most common errors were $\binom{5}{3} + \binom{4}{2} = 16$, $\binom{5}{3} \times \binom{4}{2} = 720$ and $\binom{5}{3} + \binom{4}{2} = 72$.

(b) Very few candidates answered this correctly. The majority failed to dissect the situation into separate cases – which can be achieved in a number of ways – and instead hoped to deal with the question as one multiple product; this resulted in the common answer 240, from $3 \times \binom{5}{2} \times 4$, or 360, from $3 \times \binom{6}{2} \times 4$. Some candidates showed a lack of common sense in offering answers of many thousands or even millions.

Answers: **(a)** 60; **(b)** 200.

Question 6

Candidates were about equally divided as to starting this question with a product of factors or with a generalised cubic polynomial. Whichever method was used, candidates found difficulty in dealing with the information that the coefficient of x^3 was -1. Thus many started with $f(x) = (x-1)(x-2)(x-k)$, which was almost invariably followed by " = 0 ". All but a few candidates found it necessary to multiply out these factors – an unnecessary process which occasionally led to error. Although some then forgot, or ignored, the fact that the coefficient of x^3 was -1, many others incorporated this information by changing the signs throughout their expression for f(x), whilst others merely changed x^3 to $-x^3$. Some tried unsuccessfully to adjust the factors e.g., $(x-1)(x-2)(-x-k)$.

Candidates starting with $f(x) = ax^3 + bx^2 + cx + d$ or $-x^3 + bx^2 + cx + d$ were usually able to obtain sufficient equations to solve but some could not begin the rather lengthy process of solving, whilst others were unable to avoid error. Having evaluated the coefficients in the cubic polynomial, some candidates then had difficulty in finding the value of k.

Many of the weaker candidates were unable to make a sensible start to the question, taking f(x) to be, for instance, $(\pm)x^3 + x^2 + x + k$ or $(\pm)x^3 + 1 \times x^2 + 2 \times x + k$.

Answers: **(i)** 7; **(ii)** 200.

Question 7

(i) The product rule was well-known and those candidates applying it were usually successful, the exceptions being those who took $\frac{d}{dx}(\sin x)$ to be $-\cos x$. It was disconcerting to see the number of candidates who proceeded from $\sin x + x\cos x$ to $\sin x + \cos x^2$.

(ii) Weaker candidates had little idea and made no connection with part **(i)**. The candidates who appreciated that integration is the reverse process to differentiation were frequently let down by a failure to correctly complete a mathematical statement, in that $\int (x\cos x + \sin x)\,dx = x\sin x$ was commonly written as $\int x\cos x + \sin x = x\sin x$ leading to $\int x\cos x = x\sin x - \sin x$. A few candidates clearly differentiated $x\cos x$. Evaluations were usually correct although some candidates used 90 for $\frac{\pi}{2}$ or used $\frac{\pi}{2}$ degrees.

Answers: **(i)** $\sin x + x\cos x$; **(ii)** 0.571 .

Question 8

(i) There were many incorrect or incomplete graphs. The incorrect versions included curves through the origin and graphs of $y = e^x$ and $y = \frac{1}{x}$. By far the most common omission was to show the graph of $y = \ln x$ only for $x \geqslant 1$.

(ii) This was found to be difficult by weaker candidates and so was frequently omitted. Some candidates attempted to use the calculus whilst others tried to find some sort of solution of $y = \ln x$ and $x^2e^{x-2} = 1$. Those candidates applying natural logarithms frequently had difficulty with the 1, either leaving it unchanged, replacing it by ln e, or failing to evaluate ln 1, whilst others were unable to simplify $\ln x^2$ or $\ln e^{x-2}$.

Answer: **(ii)** $y = 1 - \frac{x}{2}$.

Question 9

(a) Some of the weakest candidates did not understand, or ignored, the word "product" and so attempted to simplify each of the given expressions. Others assumed that, for instance, $a^{1/3} + b^{2/3}$ was $(a + b^2)^{1/3}$. Most of the candidates correctly added powers on multiplication, with only a minority multiplying powers, e.g., taking $a^{1/3} \times a^{2/3}$ to be $a^{2/9}$. Some had difficulty with products involving $a^{1/3}b^{2/3}$ so that e.g., $a^{1/3} \times a^{1/3}b^{2/3}$ became $a^{2/3} \times a^{1/3}b^{2/3}$. Sign errors sometimes led to the inclusion of $\pm 2a^{2/3}b^{2/3}$ or $\pm 2a^{1/3}b^{4/3}$ in the final answer.

(b) Cardinal errors such as $2^{2x+2} \times 5^{x-1} = 10^{3x+1}$ or $\lg(2^{2x+2} \times 5^{x-1}) = (2x + 2)\lg 2 \times (x - 1)\lg 5$ were made by many of the weaker candidates. Many candidates thought it was necessary to evaluate x and frequently achieved a correct answer by taking logs and solving. Others obtained two values for x by comparing powers of 2, to give $2x + 2 = 3x$, and of 5, to give $x - 1 = 2x$; although $10^x = 100$ or $\frac{1}{10}$ usually followed from the two values of x, i.e., 2 and – 1, these were sometimes used to give a fortuitous answer via $2^2 \times 5^{-1}$. Some candidates correctly reached $2^{2-x} = 5^{x+1}$ but were unable to proceed further. A rather neat solution was offered by those candidates expressing the left hand side as $\frac{4}{5} \times 20^x$ and the right hand side as 200^x, leading immediately to $10^x = \frac{4}{5}$.

Answers: **(a)** $a + b^2$; **(b)** 0.8 .

Question 10

This question proved to be a reasonable source of marks for most candidates. Incorrect expressions for $\overrightarrow{AP}$ and $\overrightarrow{OM}$ were usually due to errors in sign e.g., $\overrightarrow{AO}$ = **a**, $\overrightarrow{OM}$ = **a** + ½(**a** – **b**) = 1½**a** – ½**b**, although some candidates took $\overrightarrow{OM}$ to be ½$\overrightarrow{AB}$.

(i) A fairly common error was to take $^1/_3$**b** – **a**, found for $\overrightarrow{AP}$, to be $^1/_3$ (**b** – **a**) when used in subsequent work.

(iii) Many candidates clearly failed to read the demand, producing an expression for $\overrightarrow{AQ}$ but not for the requested $\overrightarrow{OQ}$.

(iv) Only a small proportion of candidates failed to make a sensible attempt at this part.

Answers: **(i)** $^1/_3$**b** – **a**, ½(**a** + **b**); **(ii)** $\frac{\lambda}{2}$(**a** + **b**); **(iii) a** + μ($^1/_3$**b** – **a**); **(iv)** $\frac{1}{2}$, $\frac{3}{4}$.

Question 11

Most candidates realised that integration was required in parts **(i)** and **(ii)** and only the weakest candidates attempted to apply the constant acceleration formulae. Integration in part **(i)** was usually correct and although a constant of integration was almost always included this was frequently followed by "when $t = 0$, $v = 0$ so $c = 0$" though subsequently the self-same candidates started their graph at the point $t = 0$, $v = 20$. Taking c to be 0 led, in part **(i)**, to $v = -12$ and, in part **(ii)**, to $AB = -32$. Neither of these results appeared to prompt any misgivings and both quantities usually became positive without further ado. Part **(iii)** was frequently incorrect in that candidates, including some of those who had dealt successfully with parts **(i)** and **(ii)**, obtained a value of $5^1/_3$ s by substituting $a = 2$ in the expression $a = \frac{3t}{2} - 6$. Others fortuitously obtained the answer of 6 s by assuming that $v = u + at$ held for the interval from $t = 0$, $u = 0$ to $v = 20$ and then subtracting 4 s from the resulting 10 s. The graph usually consisted of two straight lines.

Answers: **(i)** 8 ms^{-1}; **(ii)** 48 m; **(iii)** 6 s.

Question 12 EITHER

There were many correct solutions and also a considerable number of candidates who just failed to obtain full marks by omitting to "Find this value of V". The weakest candidates were unable to cope with the mensuration involved in the first part of the question. Those taking A, the area of sheeting, to be $\pi r^2 + \pi rl$ had little difficulty in arriving at the given expression for V. A common error was to include the base area, rl, in the expression for A and some candidates fortuitously arrived at the expression for V by taking $A = \pi r^2 + 2\pi rl$ and then substituting for l in $V = \pi r^2 l$.

A few of the weakest candidates thought that the required value of r could be obtained by solving $V = 0$. Most candidates realised that it was necessary to solve $\frac{dV}{dr} = 0$ and only careless errors in differentiation, manipulation or evaluation prevented the correct value of r being obtained; an error seen several times was $r^2 = \frac{40}{\pi}$ followed by $r \approx 2.01$, obtained from $r = \frac{\sqrt{40}}{\pi}$. Candidates who applied the calculus usually knew how to determine the nature of the stationary value, almost invariably by considering the sign of the second derivative, although a few thought that the sign of V was the criterion. Only rarely was the wrong inference drawn from the sign of $\frac{d^2V^2}{dr^2}$. However, $\frac{d^2V^2}{dr^2}$ was quite often found to be positive, either through ignoring the negative sign when differentiating $60 - \frac{3\pi r^2}{2}$ or through differentiating a rearrangement e.g., $\frac{3\pi r^2}{2} - 60 = 0$, obtained as a step in solving $\frac{dV}{dr} = 0$.

Answers: 3.57; 143, maximum.

Question 12 OR

Although this was the slightly less popular alternative, it seems to be selected mainly by able candidates, confident in dealing with the product rule and with $\frac{d}{dx}(\ln x)$, who had little difficulty in completing the question successfully. Some of the errors seen from weaker candidates were failure to understand that $\ln x = 0 \Rightarrow x = 1$, failure to appreciate that $\frac{dy}{dx} = 0$ at P, taking $\ln x$ to be identical with $\frac{1}{x}$, and taking $x + 2x \ln x$ to be $x + 2\ln x^2$. In part **(ii)** the argument $\ln x = -½ \Rightarrow x = e^{-½} = \frac{1}{\sqrt{e}}$ was often forthcoming but there were also many who, having obtained $\ln x = -½$, used the calculator to find $x \approx 0.6065$ and then compared this with the value of $\frac{1}{\sqrt{e}}$. In part **(iii)** some candidates, concentrating on again applying the product rule to the differentiation of $2x \ln x$, overlooked the other term, x, in the expression for $\frac{dy}{dx}$, arriving at $2 + 2\ln x$ and hence the value 1.

Answers: **(i)** 1; **(iii)** 2.