Scheme of Work

Cambridge O Level

Cambridge O Level Mathematics (Syllabus D)

4024

For examination from 2018

Cambridge Secondary 2

CAMBRIDGE International Examinations

Contents

Introduction

This scheme of work has been designed to support you in your teaching and lesson planning. Making full use of this scheme of work will help you to improve both your teaching and your learners' potential. It is important to have a scheme of work in place in order for you to guarantee that the syllabus is covered fully. You can choose what approach to take and you know the nature of your institution and the levels of ability of your learners. What follows is just one possible approach you could take.

Suggestions for independent study **(I)** and formative assessment **(F)** are included in this scheme of work. Opportunities for differentiation are indicated as **Extension activities**; there is the potential for differentiation by resource, grouping, expected level of outcome, and degree of support by teacher, throughout the scheme of work. Timings for activities and feedback are left to the judgment of the teacher, according to the level of the learners and size of the class. Length of time allocated to a task is another possible area for differentiation.

Guided learning hours

Guided learning hours give an indication of the amount of contact time you need to have with your learners to deliver a course. Our syllabuses are designed around 130 hours for Cambridge IGCSE courses. The number of hours may vary depending on local practice and your learners' previous experience of the subject. We recommend you teach these units in the order they are presented, spending approximately 10% of the time available on each topic area.

Resources

The up-to-date resource list for this syllabus, including textbooks endorsed by Cambridge, is listed at **www.cie.org.uk**
Endorsed textbooks have been written to be closely aligned to the syllabus they support, and have been through a detailed quality assurance process. As such, all textbooks endorsed by Cambridge for this syllabus are the ideal resource to be used alongside this scheme of work.

Teacher Support

Teacher Support **teachers.cie.org.uk** is a secure online resource bank and community forum for Cambridge teachers, where you can download specimen and past question papers, mark schemes and other resources. We also offer online and face-to-face training; details of forthcoming training opportunities are posted online. This scheme of work is available as PDF and an editable version in Microsoft Word format; both are available on Teacher Support at **teachers.cie.org.uk** If you are unable to use Microsoft Word you can download Open Office free of charge from **www.openoffice.org.**

Websites

This scheme of work includes website links providing direct access to internet resources. Cambridge International Examinations is not responsible for the accuracy or content of information contained in these sites. The inclusion of a link to an external website should not be understood to be an endorsement of that website or the site's owners (or their products/services).

The website pages referenced in this scheme of work were selected when the scheme of work was produced. Other aspects of the sites were not checked and only the particular resources are recommended.

How to get the most out of this scheme of work – integrating syllabus content, skills and teaching strategies

We have written this scheme of work for the Cambridge O Level Mathematics (Syllabus D) syllabus and it provides some ideas and suggestions of how to cover the content of the syllabus. We have designed the following features to help guide you through your course.

Learning objectives help your learners by making it clear the knowledge they are trying to build. Pass these on to your learners by expressing them as 'We are learning to / about…'.

Suggested teaching activities give you lots of ideas about how you can present learners with new information without teacher talk or videos. Try more active methods which get your learners motivated and practising new skills.

Syllabus ref.	Learning objectives	Suggested teaching activities
12 Percentages	Calculate a given percentage of a quantity. …e …a …of	Begin by asking orally for 50% of 200g, 10% of $50, etc. and pr… percentage of a quantity. Similarly use questions such as 'Wha… expressing one quantity as a percentage of another. **(F)** Calculate percentage increase and decrease, using contexts su… (see also below). Mathematics teacher Chris Smith shared an interesting starter question. You earn £2400 a month, your boss offers you a 30% pay rise as long as you first take a 25% pay cut. Is this a good deal? https://twitter.com/aap03102/status/446997757023514624/photo/1 **(I)** **Extension activities:** Learners consider the effects of repeated percentage changes.

Past and specimen papers
Jun 15 Paper 22 Q 6a, 6b Nov 15 Paper 11 Q 2a, 4

Formative assessment (F) is on-going assessment which informs you about the progress of your learners. Don't forget to leave time to review what your learners have learnt, you could try question and answer, tests, quizzes, 'mind maps', or 'concept maps'. These kinds of activities can be found in the scheme of work.

Extension activities provide your more able learners with further challenge beyond the basic content of the course. Innovation and independent learning are the basis of these activities.

Independent study (I) gives your learners the opportunity to develop their own ideas and understanding with direct input from you.

Past Papers, **Specimen Papers** and **Mark Schemes** are available for you to download at:

https://teachers.cie.org.uk

Using these resources with your learners allows you to check their progress and give them confidence and understanding.

Unit 1: Introduction to O Level (part one)

Recommended prior knowledge

- A basic competence with number operations
- Familiarity with the ideas of using letters to represent unknown numbers
- Basic concepts of time, length, area, volume and capacity, angle measurement, mass.

Context

This is the first of two basic units for the start of the O Level course. Much of this unit will be revision for some learners. It gives an opportunity to check their prior knowledge and to move them forward at an appropriate pace. The unit covers a variety of basic skills, including calculations with integers, properties of numbers, basic algebraic manipulation, working with units of measurement and the calculation of averages.

Outline

The four rules and the order of operations are reviewed. Learners are also introduced to various properties of numbers. In algebra, the basic processes of substitution, manipulation and solving simple linear equations are introduced. Starting with integers, the unit moves on to consider powers (positive integers only at this stage) and square root, leading to introducing different types of number. The averages of mean, median and mode are met for individual data. The appropriate use of each of these averages is also taught.

Syllabus ref	Learning objectives	Suggested teaching activities
1 Number **19 Indices**	Identify and use natural numbers, integers (positive, negative and zero), prime numbers, common factors and common multiples. Understand and use the rules of indices. Use and interpret positive indices.	Define prime numbers and obtain the primes up to 100 using the Sieve of Eratosthenes method: write integers, say up to 100 in a 10 by 10 grid, then cross out 1 (1 is not a prime, primes have exactly two distinct factors). 2 is a prime, but all the multiples of 2 (except 2 itself) are not, so cross these out from the grid. Then keep 3, but cross out all the multiples of 3 except 3 itself, etc. The remaining integers are the prime numbers. Learners could extend this to obtain larger primes. Talk about multiples, factors and prime factors and use division or a factor tree method to write any integer as a product of its prime factors. This is a good point to introduce indices and ensure that learners are familiar with the conventions for writing them. Move on to common factors and common multiples, in particular to obtaining the highest common factor and lowest common multiple. Finally, introduce the idea that expressions involving indices can be simplified if the base numbers are the same, for example $2^2 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$ but

Syllabus ref	Learning objectives	Suggested teaching activities
		$3^2 \times 2^3$ cannot be simplified. Establish the general result $x^m \times x^n = x^{m+n}$. Give learners some practice in applying this. **(I)** This work may lead naturally to the conjecture that $x^m \div x^n = x^{m-n}$. This result could be introduced at this point, alternatively, the division of numbers in index form could be addressed after learners have studied fractions in Unit 2, so that they can apply their knowledge of cancelling in order to simplify statements such as: $\frac{3^5}{3^2} = \frac{3\times3\times3\times3\times3}{3\times3} = 3^3$ before going on to arrive at the general result. **Extension activities:** Solving problems involving divisibility and factors and multiples, such as those published by Don Steward. **(I)** Ask learners to consider the sum of the factors of a number (apart from the number itself). This could provide the basis for a project in which learners investigate or research abundant, deficient and perfect numbers. **(I)** **Learning resources:** Exploring prime numbers: www.utm.edu/research/primes/ Don Steward has a variety of problems involving divisibility: http://donsteward.blogspot.co.uk/2011/06/divisibility-rules.html, Don Steward also has problems involving factors and multiples: http://donsteward.blogspot.co.uk/search/label/LCM%20and%20HCF A possible starting point for investigating abundant, deficient and perfect numbers: http://donsteward.blogspot.co.uk/2007/11/serendipity.html Alternatively, go to http://donsteward.blogspot.co.uk , scroll down and use the list of labels to navigate to work on multiples, factors, primes, etc.
8 The four operations	Use the four operations for calculations with whole numbers, including correct ordering of operations, and use	Check the learners' competence in the four operations with integers, both mentally and using written methods. Teach methods such as long multiplication and long division if necessary.

Syllabus ref	Learning objectives	Suggested teaching activities
	of brackets.	Talk about the meaning of e.g. 4 + 3 × 2 and establish with learners the correct order of operations and the need to use brackets to do (4 + 3) × 2. Similarly the solution of division problems where there is more than one term in the numerator or denominator. Give the learners practice in using their calculators efficiently to solve such problems, as well as those which they should do without a calculator. **(I)** The 24 game provides practice in mental calculation. Learners can be encouraged to write their solutions as single calculations using the rules of BODMAS/BIDMAS. Instructions for the game can be found at www.24game.com/t-about-howtoplay.aspx or play online at www.mathplayground.com/make_24.html (please note - this site carries advertising) Alternatively, the game can be played by removing the picture cards from a normal pack of playing cards, shuffling the cards, then turning 4 cards over and using the numbers to try to make 24.**(I/F)** Please note – this work is extended to include fractions and decimals in unit 2. **Extension activities:** The construct 'Would you rather?' provides a basis for a variety of questions that require mathematical calculations to be used in order to justify a solution to a problem, e.g. Would you rather have a stack of quarters from the floor to the top of your head OR $225? **(I)** **Learning resources:** *Understanding the Laws of Arithmetic*, a resource from the UK Department for Education's Standards Unit, has materials that ask learners to interpret calculations in words and as area diagrams in order to develop their understanding of arithmetical rules and laws: www.nationalstemcentre.org.uk/elibrary/resource/1962/understanding-the-laws-of-arithmetic-n5 **(F)** Don Steward has a collection of mental calculation problems involving the rules of BODMAS/BIDMAS http://donsteward.blogspot.co.uk/search/label/bidmas **(I/F)** Don Steward also has a selection of long multiplication and division problems http://donsteward.blogspot.co.uk/search/label/long%20division **(I/F)** Exercises on long multiplication and division are available at section 6.4: http://www.cimt.org.uk/projects/mepres/allgcse/bka6.pdf The scores in various games, attendance at matches, etc. are good sources of data for calculations.
4 Directed numbers	Use directed numbers in practical situations (e.g.	Use a number line to show positive and negative integers and to aid addition and subtraction of negative numbers. Ask learners to perform calculations based on simple contexts, for example, changes in

<table>
<tr><th>Syllabus ref</th><th>Learning objectives</th><th>Suggested teaching activities</th></tr>
<tr><td></td><td>temperature changes or flood levels).</td><td>temperature, or differences between tide levels, or even examples involving a lift/elevator, some of which use negative numbers for floors below ground level.

Extension activities: Ask learners to use number patterns to deduce rules for multiplying and dividing positive and negative integers, e.g.
Continue and complete this pattern:
3 + 2 = …
3 + 1 = …
3 + 0 = …
3 + - 1 = …
3 + - 2 = … etc.

Learners could investigate other similar patterns, having established rules for dealing with examples such as 3 + - 2, they could investigate other patterns, e.g.
3 - 2 = …
3 - 1 = …
3 - 0 = …
3 - - 1 = …
3 - - 2 = … etc. (I)

Learning resources:
Work on directed numbers:
http://www.cimt.org.uk/projects/mepres/allgcse/bkb10.pdf

The Strange Bank Account from NRICH could provide a starting point for an interesting investigation:
http://nrich.maths.org/9923 (I/F)

Worldwide tide predictions may be found at www.ukho.gov.uk/easytide/EasyTide/index.aspx (data for the next seven days is free; there is a fee for longer term predictions)
Temperatures and other weather statistics for cities worldwide may be found at www.weatherbase.com/</td></tr>
<tr><td>3 Squares, square roots, cubes and cube roots

1 Number</td><td>Calculate squares, square roots, cubes and cube roots of numbers.

Identify and use square numbers, cube numbers, rational and irrational numbers,</td><td>Learners should be familiar already with the area of a square and the volume of a cube: drawing a sequence of squares and cubes and using their areas and volumes to obtain square and cube numbers gives meaning to the terms.

It has been proved that every whole number is the sum of at most 4 square numbers. Learners work in pairs, one choosing a number and the other expressing it as the sum of squares, or they could be challenged to investigate the statement by finding sums for a variety of numbers. Is there only one way to</td></tr>
</table>

Syllabus ref	Learning objectives	Suggested teaching activities
	real numbers.	do this? A basis for this activity can be found at: www.counton.org/explorer/number/square-numbers/
13 Use of an electronic calculator	Use a calculator efficiently.	Lead on to calculator use for obtaining e.g. 12.4^3 and $\sqrt{10}$. These can be used to introduce the ideas of rational and irrational numbers and real numbers. **Extension activities:** Learners apply what they have learned about prime factors in the previous section in order to simplify square or cube roots. For example, writing $\sqrt{300}$ as $\sqrt{2\times2\times3\times5\times5}$ and realising that this can be simplified to $2\times5\times\sqrt{3}$ or $10\sqrt{3}$. Alternatively, there are a number of problems which learners can work on (see resources below). For example, ask learners to compare the sum of the cubes of the natural numbers with the sum of some odd numbers: $1^3 = 1$ $1^3 + 2^3 = 1 + 3 + 5$ $1^3 + 2^3 + 3^3 = 1 + 3 + 5 + 7 + 11$ Can they explain this by writing individual cubes as the sums of odd numbers? Don Steward has a selection of problems and starting points for investigations involving squares http://donsteward.blogspot.co.uk/search/label/square%20numbers and cubes http://donsteward.blogspot.co.uk/search/label/cube%20numbers Don Steward also provides resources for the sum of cubes: http://donsteward.blogspot.co.uk/2009/03/sum-of-cubes.html

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 2, 3, 9, 24a 2018 Specimen Paper 2 Q 5a **Past papers:** Jun 13 Paper 11 Q 12 Jun 13 Paper 12 Q 1, 3, 8, 9b, 9c Jun 13 Paper 21 Q 1b Jun 14 Paper 11 Q 10a, 13a, 15 Jun 14 Paper 12 Q 1a, 2a Nov 14 Paper 11 Q 3b, 4 Nov 14 Paper 21 Q 1b, 2d Jun 15 Paper 12 Q 13 Jun 15 Paper 21 Q 15a Nov 15 Paper 11 Q 1a, 3, 10a, 10b Nov 15 Paper 12 Q 1

Syllabus ref	Learning objectives	Suggested teaching activities
17 Algebraic representation and formulae **18 Algebraic manipulation**	Use letters to represent generalised numbers and express basic arithmetic processes algebraically, substitute numbers for words and letters in formulae. Basic algebraic manipulation, simplify algebraic expressions, manipulate directed numbers.	Use word formulae representing practical situations, such as costs, e.g. a mobile phone tariff charges a certain amount for a number of texts and a different amount for a number of minutes of calls. Substitute numbers into these, then show that the same situation can be represented generally using letters to represent the variables. Move on to substituting numbers into formulae. A variety of methods can be used to practice these skills, for example you could use these two 'Top Trumps' games to practise substitution: www.tes.co.uk/ResourceDetail.aspx?storyCode=6027440 Move on to simplify expressions such as 4a + 3b − 6a + 5b, 4a × 3b. **Extension activities:** Ask learners to try more using complex algebra, for example substituting negative values into expressions involving powers, e.g. $3x^2 – x^3$ when x=-4. (Please note, negative indices are covered in unit 5) **Learning resources:**

Syllabus ref	Learning objectives	Suggested teaching activities
		Work on formulae: https://www.cimt.org.uk/projects/mepres/allgcse/bka2.pdf Work on simplifying and simple equations: https://www.cimt.org.uk/projects/mepres/allgcse/bkb10.pdf Locally available recipe books may give cooking times as word formulae (e.g. x minutes per pound plus y minutes). Mobile phone tariffs from local companies can also be useful.
20 Solutions of equations and inequalities	Solve simple linear equations in one unknown.	Puzzles such as 'I think of a number, I multiply it by 2, then add 1; the answer is 7. What number did I think of?' can be used to introduce the ideas of inverse operations. Then represent the same situation by an equation, showing how to set out the algebraic solution. Ask learners to practise solving these, setting out their working clearly. **(F/I)** *Creating and Solving Equations*, a resource from the UK Department for Education's Standards Unit, in which learners first follow simple steps to form equations where the unknown appears once, then move on to solving, is a useful way to introduce equations: www.nationalstemcentre.org.uk/elibrary/resource/1999/creating-and-solving-equations-a2 Don Steward has an 'unjumbling' activity in which learners need to sort the steps in solving various equations into order http://donsteward.blogspot.co.uk/2014/05/unjumbling.html **(F)** Formulae can be used again here, substituting a number this time for the subject of the formula and solving an equation to find the unknown variable. **Extension activities:** Learners can use contextual information to form and solve equations. For example, *'Angles and Algebra'* provides a diagrammatic context, but will require prior knowledge or research about angles (covered in unit 2 of this scheme) in order to complete the task. www.tes.co.uk/ResourceDetail.aspx?storyCode=6296832 Alternatively, Don Steward has a collection of puzzles that can be solved using algebra. Forming and solving equations: http://donsteward.blogspot.co.uk/2013/06/puzzles-that-you-could-use-algebra-to.html **Learning resources:** Work on formulae: https://www.cimt.org.uk/projects/mepres/allgcse/bka2.pdf Work on simplifying and simple equations:

Syllabus ref	Learning objectives	Suggested teaching activities
		https://www.cimt.org.uk/projects/mepres/allgcse/bkb10.pdf Some questions that involve use of brackets are included here (this is covered in Unit 2).

Past papers and specimen papers
Past papers: Jun 13 Paper 11 Q 10a Nov 13 Paper 22 Q 3a Jun 14 Paper 22 Q 2ai Nov 14 Paper 11 Q 12a Nov 14 Paper 12 Q 2 Nov 14 Paper 22 Q 8ai

Syllabus ref	Learning objectives	Suggested teaching activities
41 Categorical, numerical and grouped data	Calculate the mean, median and mode and range for individual data and discrete data and distinguish between the purposes for which they are used.	Give learners a simple set of unordered data and ask them to find the 'average' – use the term 'mean' from then onwards. Give them a set of data in context which is clearly skewed, such as 4, 4, 4, 5, 6, 6, 20 and ask them for the mean and to comment on the result. (For instance, using the context of wages may help them to see the median may be more appropriate to use as an average, whilst some may wish to use the mode). Define median and mode and give practice in obtaining all three averages. Discuss that, depending on who wishes to use the data and for what purpose, more than one average may be used. Introduce the range. Learners need to be clear that this is not an average, but is often used in conjunction with averages. For example, a sports player may have a high average score, but also a high range indicating that their results are very inconsistent. Teacher William Emeny describes using data to select a player for a cricket team in 'Who is the best England batsman? An investigation using the mean, median, mode and range.' www.greatmathsteachingideas.com/2010/06/17/who-is-the-best-england-batsman-an-investigation-using-the-mean-median-mode-and-range/ Learners can practise calculating averages and ranges using real data and use the results to make comparisons. You may wish to collect data by estimating lengths of lines and sizes of angles in order to find out who is the best estimator. This links to the topic on geometrical construction later in this unit.

Syllabus ref	Learning objectives	Suggested teaching activities
		You may wish to look at further examples of how data are used, for instance, the class could look in a newspaper to find articles where averages have been used. **Extension activities:** Solve problems that involve averages, for example finding a set of numbers with a given mean and median. Don Steward also has a selection of problems that involve averages: http://donsteward.blogspot.co.uk/2012/02/small-data-set-problems.html **(F)** and http://donsteward.blogspot.co.uk/2014/01/wipe-out.html **Learning resources:** Data suitable for use with learners: An extract from www.censusatschool.org.uk/ could allow learners to compare an 'average learner' from their own country with an 'average learner' from another country and comment on any similarities and differences. Don Steward has some simple data sets: http://donsteward.blogspot.co.uk/2012/04/best-guesser.html Two poems where learners can select the appropriate data to collect: http://donsteward.blogspot.co.uk/2012/10/two-poems.html Mean, median and mode are dealt with at: http://www.cimt.org.uk/projects/mepres/allgcse/bkb9.pdf Data collected by the class (such as their heights, or estimates of lengths of lines or angles) or data from a local newspaper could be used.

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 2 Q 2b, 2c, 2d **Past papers:** Nov 13 Paper 11 Q 16 Jun 14 Paper 12 Q 2 Nov 14 Paper 12 Q 3 Jun 15 Paper 12 Q 12 Nov 15 Paper 11 Q 12

Syllabus ref	Learning objectives	Suggested teaching activities
29 Geo-metrical construction **14 Time** **13 Use of an electronic calculator**	Measure lines and angles. Calculate time in terms of the 12-hour and 24-hour clock; read clocks, dials and timetables. Enter a range of measures including 'time', e.g. enter 2 hours 30 minutes as 2.5 hours. Interpret the calculator display appropriately, e.g. in time 3.25 means 3 hours 15 minutes.	Check that learners can use rulers and protractors competently. They could try estimating lengths of lines or sizes of angles, then check by measuring. This could provide a source of data for work with averages – who is the best estimator? This is a good point at which to check that learners are familiar with the vocabulary associated with angles and lines, e.g. acute, obtuse, parallel, vertical, etc. **(F)** Examples on measuring angles: http://www.cimt.org.uk/projects/mepres/allgcse/bka3.pdf Practice in angle estimation: The 'Banana Hunt' game https://mathslinks.net/links/banana-hunt Measuring lines: https://www.cimt.org.uk/projects/mepres/allgcse/bkb7.pdf Give the learners practice in using different measuring instruments, e.g. in measuring their heights or how much they weigh (these data could be used in unit 3) or the volume of a jug of water, as well as clocks. Give learners practice in finding information from timetables. They could be challenged to use local timetables to plan an activity or holiday or to find the maximum distance they can travel in 24 hours with a given budget. Practice in using timetables: http://www.cimt.org.uk/projects/mepres/allgcse/bkb8.pdf Calculate times, making sure that learners can convert between hours, minutes and seconds, as well as find the sum and difference of times. **(F)** Using a timeline is recommended.

Syllabus ref	Learning objectives	Suggested teaching activities
		Extension activities: Ask learners to find some examples where traditional column addition and subtraction methods do not give the correct solutions when calculating with times, then explain why this happens. **(F)** **Learning resources:** Useful images for a protractor: http://en.wikipedia.org/wiki/File:Protractor_Rapporteur_Degree_V1.jpg Images for various dials: http://commons.wikimedia.org/wiki/Category:Dial_indicators which can be used in a PowerPoint presentation or similar.
34 Measures	Use current units of mass, length, area, volume and capacity in practical situations and express quantities in terms of larger or smaller units.	Check that learners can convert competently between length units. Progress to finding the area of a rectangle in different units, and give learners practice in converting from one area unit to another, e.g. cm^2 to m^2. Similarly, apply this process to volume and capacity. Find the mass of an object in different units. **Extension activities:** Investigate connections between non-linear units, for example by drawing a variety of squares and rectangles, marking key dimensions using mm, cm and m and using these to find areas in mm, cm and m. Use their results to deduce conversion factors for these square units. Challenge learners to use graph paper to make a shape that has a million squares – this could form the basis of a classroom display. 'Viva las Colas!' could be used to generate a variety of interesting questions about volume. www.101qs.com/112-viva-las-colas **Learning resources:** Work on units: http://www.cimt.org.uk/projects/mepres/allgcse/bkb7.pdf

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 16, 22a **Past papers:** Jun 13 Paper 11 Q 3 Jun 13 Paper 12 Q 14a Nov 13 Paper 11 Q 10c Nov 13 Paper 12 Q 4a, 24a Jun 14 Paper 11 Q 10b Jun 14 Paper 12 Q 4 Nov 14 Paper 12 Q 1b Nov 14 Paper 21 Q 2a, 2b Jun 15 Paper 11 Q 20b Jun 15 Paper 12 Q 5

Unit 2: Introduction to O Level (part two)

Recommended prior knowledge

- The four rules and the order of operations with integers (see Unit 1)
- Manipulation of simple expressions and solving simple linear equations (see Unit 1)
- Calculation of mean, median and mode for individual data (see Unit 1)
- Conversion between basic units (see Unit 1)

Context

This is the second of two basic units for the start of the O level course. Some of this unit will be revision for some learners. It gives an opportunity to check their prior knowledge and to move them forward at an appropriate pace. This unit extends the work covered on integers in unit 1 to include fractions, decimals and percentages; these concepts are developed further within the context of personal and household finance. The ideas of order of operations and brackets are developed, for both number and algebra. This unit moves on from individual data to representing and calculating with simple frequency distributions for discrete data. Some basic geometrical terms are introduced and used when considering angle and symmetry properties. The work on units of measurement from unit 1 is applied when finding areas and perimeters.

Outline

The concepts of fractions, decimals and percentages and conversions between them are established. The four operations of working with fractions and decimals are used. Percentages of a quantity are found. These number skills are then applied to money and personal finance problems. Having established the order of operations, brackets are used with linear expressions and equations in algebra. This unit also develops statistical concepts, including constructing and interpreting bar charts, pie charts and pictograms. The mean of a discrete frequency distribution is calculated, in preparation for unit 3, where grouped continuous data will be used. This unit also considers lines and angles, triangles and quadrilaterals. The basic geometrical terms associated with them are introduced and unknown angles are calculated using angle properties. Triangles and quadrilaterals are classified, including the symmetry properties of different types. Areas of triangles, rectangles and parallelograms are calculated.

Syllabus ref	Learning objectives	Suggested teaching activities
5 Vulgar and decimal fractions and percentages	Use the language and notation of simple vulgar and decimal fractions and percentages in appropriate contexts. Recognise equivalence and convert between these forms.	Revise the concepts of equivalence, e.g. asking how 0.8 might be expressed differently. Give learners practice in simplifying fractions and in converting between fractions, decimals and percentages. **(I)** Include the fact that all terminating and recurring decimals may be expressed as fractions and so are rational, and that non-recurring decimals are irrational. Learners investigate this by converting fractions to decimals using their calculators, identifying that some give terminating decimals and some give recurring decimals. **(I)**
1 Number	Use rational and irrational	Check the learners' level of competence in the four operations with fractions and decimals. Give

Syllabus ref	Learning objectives	Suggested teaching activities
	numbers.	appropriate teaching and practice. **(F)**
8 The four operations	Use the four operations for calculations with decimal fractions and vulgar (and mixed) fractions, including correct ordering of operations and use of brackets.	Reinforce the idea of order of operations (this was covered with integers in unit 1 of this scheme), particularly with calculations such as: $\frac{2+6}{4-2}$ **Extension activities:** Learners investigate which fractions have decimal equivalents that are terminating and which give recurring decimals. They could explain their findings using prime factors. Converting recurring decimals to fractions, for example: Let $x = 0.\dot{9}$ (i) Therefore $10x = 9.\dot{9}$ (ii) Subtracting (i) from (ii) gives $9x = 9$ Therefore $x = 1$ Since $x = 0.\dot{9}$, it follows that $0.\dot{9} = 1$ Many learners are resistant to the idea that $0.\dot{9} = 1$, even after seeing a proof. Asking them to suggest a value which would lie between $0.\dot{9}$ and 1 on the number line is a helpful way to challenge this. **Learning resources:** Exercises and worked examples for decimals; also work on using brackets and memory on a calculator at section 6.6: http://www.cimt.org.uk/projects/mepres/allgcse/bka6.pdf Work on fractions and percentages: http://www.cimt.org.uk/projects/mepres/allgcse/bkb11.pdf
12 Percentages	Calculate a given percentage of a quantity. Express one quantity as a percentage of another. Calculate percentage	Begin by asking orally for 50% of 200g, 10% of $50, etc. and progress to formal methods of finding a percentage of a quantity. Similarly use questions such as 'What fraction of 40 cm is 8 cm?' to progress to expressing one quantity as a percentage of another. **(F)** Calculate percentage increase and decrease, using contexts such as length, time or mass as well as money (see also below).

Syllabus ref	Learning objectives	Suggested teaching activities
	increase or decrease.	Mathematics teacher Chris Smith shared an interesting starter question: You earn £2400 a month; your boss offers you a 30% pay rise as long as you first take a 25% pay cut. Is this a good deal? https://twitter.com/aap03102/status/446997757023514624/photo/1 **(I/F)** Dave Gale, a mathematics teacher in the UK, poses questions about the order in which discounts should be applied: http://reflectivemaths.wordpress.com/2013/05/31/starter-body-shop-offer/ . This could form the basis of an interesting investigation for your learners. Please note – reverse calculations (e.g. finding the cost price given the selling price) will be covered in unit 5 of this scheme. **Extension activities:** Learners consider the effects of repeated percentage changes. **Learning resources:** Work on percentages: https://www.bbc.com/bitesize/guides/zcbrdxs/revision/1
15 Money **13 Use of an electronic calculator** **16 Personal and small business finance**	Solve problems involving money and convert from one currency to another. Enter a range of measures including 'time', e.g. enter 2 hours 30 minutes as 2.5 hours. Interpret the calculator display appropriately, e.g. in money 4.8 means $4.80; in time 3.25 means 3 hours 15 minutes. Use given data to solve problems on personal and small business finance involving earnings, simple interest and compound interest.	Use the context of shopping to set money problems, giving your learners practice in solving them without a calculator, as well as with a calculator when the level difficulty makes calculator use appropriate. Use conversions between currencies, using conversion graphs as well as conversion rates. Compare costs of items to find the best buy, including examples where discounts are available that will affect the final price paid or where the same item is available in two different currencies (for example sold on two different websites). Real examples are often helpful, Dave Gale, a mathematics teacher in the UK, poses questions about the price of coffee in a supermarket: http://reflectivemaths.wordpress.com/2014/06/02/price-of-coffee/ Calculate simple interest, discounts, and profit and loss (as an amount or a percentage) Move on to calculate compound interest. Learners are expected to know the formula: Value of investment = $P(1+\frac{r}{100})^n$ where P is the amount invested, r is the percentage rate of interest and

Syllabus ref	Learning objectives	Suggested teaching activities
	Extract data from tables and charts.	*n* is the number of years of compound interest. A common source of confusion is that the formula for simple interest calculates just the interest, but the formula for compound interest calculates the new total amount, i.e. principal plus interest, so this difference should be highlighted to your learners.
24 Graphs in practical situations	Interpret and use graphs in practical situations including conversion graphs.	Calculate earnings and solve other personal and household finance problems, including using tables and charts. **(I)** Use local tax rates for goods and wages and other local contexts for personal finance. Mail-order catalogues, websites and order forms are also good sources of data. Include examples where workers are paid by hourly rates (including dealing with e.g. working for 28 hours and 30 minutes), or have a basic pay and a commission. **Extension activities:** Learners compare offers, or investigate offers that do not represent good value for money. They could collect their own information from websites or local retailers, or use images – there are some suitable examples in the 'Bad Maths' Flickr group, many of which show offers in supermarkets: https://www.flickr.com/groups/badmaths/ Please note – the images in this group will change as users add to them. **It is strongly recommended** that you check the most recent content before sharing this with learners. **Learning resources:** Currency conversion tools are widely available online. One example is at: http://finance.yahoo.com/currency-converter/

Past papers and specimen papers
Specimen papers: 2018 Specimen Paper 1 Q 1, 3, 2018 Specimen Paper 2 Q 1ai, 1aii, 1aiv, 1b, 5a **Past papers:** Nov 13 Paper 11 Q 1, 2a, 3b, 8a, 9a Nov 13 Paper 12 Q 1a, 2 Nov 13 Paper 21 Q 1a Nov 13 Paper 22 Q 2bi, 6ai, 6aii, 6b Jun 14 Paper 11 Q 2, 8b Jun 14 Paper 12 Q 1, 3, 9, 16 Jun 14 Paper 21 Q 3ai, 3aii, 3aiv, 3b Jun 14 Paper 22 Q 3a, 3bi Nov 14 Paper 11 Q 2b, 3, 6a, 6b, 14 Nov 14 Paper 12 Q 1 Nov 14 Paper 21 Q 1c, 2c Nov 14 Paper 22 Q 2 Jun 15 Paper 11 Q 1, 3a Jun 15 Paper 12 Q 1, 2 Jun 15 Paper 21 Q 1ai, 1aiii, 1b Jun 15 Paper 22 Q 6a, 6b Nov 15 Paper 11 Q 2a, 4 Nov 15 Paper 12 Q 1, 2 Nov 15 Paper 21 Q 2 Nov 15 Paper 22 Q 1aia, 1aib, 1b

Syllabus ref	Learning objectives	Suggested teaching activities
18 Algebraic manipula-tion	Use brackets and extract common factors. Expand products of the form $a(bx + cy)$.	Show learners the meaning of 6(x + 2y) by expanding the brackets. Alternatively, learners investigate this using numerical examples, e.g. comparing 6(2 + 3) with 6 × 2 + 6 × 3. **(I)** Give them practice in expanding and simplifying linear expressions such as 3x + 5(x + y) − 2(5x − 4y).

Syllabus ref	Learning objectives	Suggested teaching activities
20 Solutions of equations	Factorise expressions of the form *ax + ay*. Solve simple linear equations in one unknown.	Move on to factorise expressions such as 4a + 6b by looking for common factors in each term. **(I)** Move on to solving harder linear equations than in unit 1. Include equations involving brackets such as 4(2*x* + 1) = 16 and also equations where the unknown appears on both sides, such as 4x + 3 = 2x − 5. *Creating and Solving Harder Equations*, a resource from the UK Department for Education's Standards Unit, in which learners first follow simple steps to form more complex linear equations, then move onto to solving: www.nationalstemcentre.org.uk/elibrary/resource/2001/creating-and-solving-harder-equations-a3 An interactive applet for solving equations with a balance: http://flashmaths.co.uk/viewFlash.php?id=26 Work on solving equations in section 10.5: http://www.cimt.org.uk/projects/mepres/allgcse/bkb10.pdf Please note – expanding brackets and factorising is developed further in units 4, 5 and 6. **Extension activities:** Ask learners to form equations from a context, before going on to solve them. **(I)** Learners also investigate sums of consecutive integers, expressing these algebraically. **(I)**

Past papers and specimen papers

Specimen papers:
2018 Specimen Paper 2 Q 7a
2018 Specimen Paper 1 Q 5a

Past papers:
Jun 13 Paper 11 Q 25a
Jun 13 Paper 21 Q 1a
Jun 13 Paper 22 Q 3a
Nov 13 Paper 21 Q 5bi
Jun 14 Paper 11 Q 17
Nov 14 Paper 12 Q 18a
Nov 14 Paper 21 Q 7aii
Nov 14 Paper 22 Q 8b
Jun 15 Paper 11 Q 17a
Jun 15 Paper 22 Q 9ai
Nov 15 Paper 11 Q 22a
Nov 15 Paper 21 Q 5a

Syllabus ref	Learning objectives	Suggested teaching activities
41 Categorical, numerical and grouped data	Collect, classify and tabulate statistical data. Read, interpret and draw simple inferences from tables and statistical diagrams. Calculate the mean from a table of discrete data.	Ask your learners to look at a chart of each type which has been drawn already. For each one, ask questions about the information which has been shown. Draw attention to the title, labels on axes / bars / sectors and to the key for a pictogram. Ask questions such as 'Which is the mode?' for each diagram. Ask 'How many...?' for each diagram, ensuring that learners then realise that a pie chart does not show this, just relative proportions. **(F)** Get the learners to collect some data of their own, tabulate it and represent it. For instance, in preparation for probability work in unit 4 of this scheme, they could undertake experiments with a biased die or spinner and tabulate the resulting frequencies for each number. Or you may wish them to undertake a survey: for example, your learners could collect their own data; this could be uploaded to www.censusatschool.org.uk/ to allow comparisons with learners from other countries. As well as drawing diagrams by hand, if one or more computers are available teach them to use a computer program such as GeoGebra, Excel or Autograph to represent data. **(I)**
42 Statistical diagrams	Collect, classify and tabulate discrete data. Read, interpret and draw simple inferences from tables and statistical diagrams. Construct and use bar charts, pie charts and pictograms. Find the mean, median and mode from a discrete frequency distribution.	Give the learners plenty of practice at drawing and interpreting bar charts, pie charts and pictograms. Don Steward has a variety of tasks involving pie charts: http://donsteward.blogspot.co.uk/search/label/pie%20charts Give the learners a list of data with repeats, say 30 items, and ask them to calculate the mean. Whilst they do this, put the data into a frequency distribution on the board and add an extra column. Alternatively, ask each learner to write a piece of data, for example their age or shoe size on a piece of paper. The learners hold up their sheet of paper so that everyone can see it. Ask them to find the mean; you may need to suggest that they organise themselves into groups in order to do this more efficiently. Then show them how to get their result using the frequency distribution. Then for the next example, just give the data in the form of a frequency distribution and ask them to calculate the mean. **(F)** Please note – calculating the mean from grouped data will be covered in unit 3 of this scheme. **Extension activities:** Ask learners to work out how adding or removing a piece of data would change the mean of a distribution. **(I)** Given the mean, work backwards to find a missing frequency. Give a pie chart, ask learners to invent a possible data set to fit the chart and then display it in a different way. Artist Arthur Buxton creates artwork in the form of charts. (Please note, this is not an educational website, so

Syllabus ref	Learning objectives	Suggested teaching activities
		please check the recent content before sharing with learners.) Some of his charts could be the basis of an interesting starter task. For example, can your learners identify the Van Gogh paintings represented in this piece of work: https://www.arthurbuxton.com/#/art-design/ – happily Don Steward has supplied some answers: http://donsteward.blogspot.co.uk/2012/07/pie-charts-and-van-gogh.html **Learning resources:** Work on tables and charts at: http://www.cimt.org.uk/projects/mepres/allgcse/bkb8.pdf Two worksheets on interpreting athletics data are available at: http://www.cimt.org.uk/resources/res1/dint1.htm and http://www.cimt.org.uk/resources/res1/dint2.htm , with links to the data relevant files. If you do not have printed charts that you can show, you could draw them yourself on Excel or using GeoGebra, perhaps using an extract of data from www.censusatschool.org.uk/ Global and regional statistics e.g. the populations for different districts in a country are available at: www.geohive.com/

Past papers and specimen papers

Past papers:
Jun 13 Paper 12 Q 19
Jun 13 Paper 21 Q 2b
Jun 13 Paper 22 Q 10b
Nov 13 Paper 22 Q 2
Jun 14 Paper 22 Q 7a
Nov 14 Paper 21 Q 11bi
Jun 15 Paper 22 Q 3b
Nov 15 Paper 11 Q 13

<table>
<tr><th>Syllabus ref</th><th>Learning objectives</th><th>Suggested teaching activities</th></tr>
<tr><td>28 Geometrical terms

31 Symmetry</td><td>Use and interpret the geometrical terms: point, line, plane, parallel, perpendicular, right angle, acute, obtuse and reflex angles.

Use and interpret vocabulary of triangles and special quadrilaterals.

Recognise rotational and line symmetry (including order of rotational symmetry) in two dimensions; and properties of triangles and quadrilaterals directly related to their symmetries.</td><td>Learners are reminded of this terminology by showing them a picture of a pattern such as a tiled floor using different shapes and asking them to identify an obtuse angle, a kite, perpendicular lines, etc. (F)

Show learners pictures of shapes and logos which have line or rotational symmetry and ask them to identify how many lines of symmetry the shape has and what its order of rotational symmetry is. Alternatively, you could also ask your learners to look for logos with interesting symmetry properties (these could be from cars or from local businesses) and then sketch or photograph them. These images could then be used as the basis for discussion. (F)

Move on to the symmetry properties of isosceles and equilateral triangles and special quadrilaterals.

Classifying Shapes, a resource from the UK Department for Education's Standards Unit, considers a variety of properties, including symmetries and regularity:
www.nationalstemcentre.org.uk/elibrary/resource/2027/classifying-shapes-ss1

The Victoria and Albert Museum has created some teachers' resources linked to the mathematical properties of Islamic designs:
www.vam.ac.uk/content/articles/t/teachers-resource-maths-and-islamic-art-and-design/

Rangoli patterns have interesting symmetry properties. This presentation has some images showing traditional patterns, followed by instructions for creating a simple pattern:
www.dropbox.com/s/ol9vpd8t4qmkhto/Rangoli%20Patterns.ppt

Extension activities: Ask learners to create a design with given symmetrical properties, by combining a set number of lines of reflection symmetry with an order of rotation symmetry. For example, draw a shape with 2-fold rotation symmetry but no reflection symmetry, or one line of reflection symmetry and 2-fold rotation symmetry. Learners go on to investigate whether all combinations are possible. (I)

Learning resources:
Useful work on symmetry as well as angles:
http://www.cimt.org.uk/projects/mepres/allgcse/bka3.pdf</td></tr>
<tr><td>32 Angle</td><td>Calculate unknown angles and give simple explanations using the following geometrical properties:
a) angles at a point</td><td>Learners investigate angle properties by drawing lines/shapes and measuring the angles. Alternatively, learners can investigate angles using straws or even spaghetti. There is a description of an angles investigation using spaghetti here: www.morethanmaths.com/teacher/2013/05/06/why-every-maths-teacher-should-keep-spaghetti-in-their-classroom/ (I)</td></tr>
</table>

Syllabus ref	Learning objectives	Suggested teaching activities
	b) angles at a point on a straight line and intersecting straight lines c) angles formed within parallel lines d) angle properties of triangles and quadrilaterals.	There is a geometric proof of the angle sum of a triangle here: www.cs.bham.ac.uk/~axs/fig/angles-segment-proof.jpg The class could make posters showing the angle and symmetry properties of special triangles and quadrilaterals. Make sure the learners know the angle properties (a) to (d) then move on to solve angle problems for each of types (a) to (d), asking the learners to give their reasons for each calculation. **(I)** The use of correct geometrical terminology when giving reasons for answers is expected, so it is important that you check on this. **(F)** **Extension activities:** Learners investigate whether all triangles and quadrilaterals tessellate; then explain their findings by considering angle sums. **Learning resources:** Work on angles: http://www.cimt.org.uk/projects/mepres/allgcse/bka3.pdf

Past papers and specimen papers

Specimen paper:
2018 Specimen Paper 1 Q 19

Past papers:
Jun 14 Paper 11 Q 1, 16
Jun 14 Paper 12 Q 6
Nov 14 Paper 11 Q 21a
Nov 14 Paper 12 Q 19
Nov 14 Paper 21 Q 8d
Jun 15 Paper 11 Q 2
Jun 15 Paper 22 Q 10b
Nov 15 Paper 12 Q 7
Nov 15 Paper 21 Q 7a, 7b

Syllabus ref	Learning objectives	Suggested teaching activities
35 Mensuration	Solve problems involving • the perimeter and area of a rectangle and triangle • the perimeter and area of a parallelogram and a trapezium • the areas of compound shapes.	It may be helpful to show learners how the area formulae for a parallelogram and a trapezium may be obtained by splitting them into two triangles. There are other ways to split these shapes; learners could be encouraged to find a different way, or ways to show that the area is given by the formula obtained using two triangles. **(F)** After this, encourage them to work with the area formulae. Give them practice in finding areas and perimeters, making sure that they give the appropriate units in their answers. Problems in context, such as finding the cost of flooring or skirting board/baseboard for a room, will provide valuable practice. **(I)** *Evaluating Statements about Length and Area*, a resource from the UK Department for Education's Standards Unit, asks learners to consider a variety of statements and respond using their own examples or counter-examples: www.nationalstemcentre.org.uk/elibrary/resource/2031/evaluating-statements-about-length-and-area-ss4 Learners need to be able to solve problems involving area, for example, finding out how many square paving slabs would cover a given area. **(F/I)** Learners also need to be able to find area and perimeters of compound shapes, for example, splitting an L-shape into two rectangles. **(F/I)** **Extension activities:** Learners complete a project in which they calculate the cost of redecorating a room, deciding what measurements they require; finding areas and using them to calculate paint or wallpaper quantities, costs of flooring, etc. Finding missing lengths from given areas can also provide challenging problems. **Learning resources:** Work on the area and perimeter of squares, rectangles and triangles at section 7.6: http://www.cimt.org.uk/projects/mepres/allgcse/bkb7.pdf

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 2 Q 4aii **Past papers:** Jun 13 Paper 11 Q 1 Jun 13 Paper 21 Q 6aii Nov 13 Paper 22 Q 1a Jun 14 Paper 11 Q 3 Jun 15 Paper 21 Q 3a Nov 15 Paper 11 Q 2 Nov 15 Paper 22 Q 2a

Unit 3: Further algebraic skills, applying Pythagoras' theorem and working to a suitable degree of accuracy

Recommended prior knowledge:

- Four rules with numerical fractions (see Unit 2)
- Conversion between fractions, decimals and percentages (see Unit 2)
- Substitution of numerical values into formulae (see Unit 1)
- Squares and square roots (see Unit 1)
- Measuring skills (see Unit 1) and key vocabulary for geometry (see Unit 2)

Context

This unit continues to develop algebraic manipulation skills and introduces graphs of straight lines. Having studied discrete data in Unit 2, learners move on to represent continuous data in classes of equal intervals and to calculating the mean of such distributions. Having learnt basic number skills for all types of real number, learners now apply these skills to ordering quantities by magnitude and to estimating and rounding. Inequality symbols are used both to compare magnitudes and when describing intervals. Learners apply both the measuring skills learnt in Unit 1 and some of the geometrical terms learnt in Unit 2 to formal geometrical constructions and scale drawings. Simple scale drawings provide an introduction to the formal study of ratio and proportion in Unit 4. Pythagoras' theorem is introduced, using the square roots learnt in Unit 1. The idea of accuracy is considered in a variety of contexts throughout this unit: in drawing graphs and diagrams, in rounding and estimating, in efficient use of calculators and in working with grouped data.

Outline

Working with algebraic fractions is introduced and learners also progress to constructing equations from given situations. Ideas of inequalities and ordering numbers are dealt with first. Then rounding skills are developed and applied to estimates and checking procedures, whilst efficient use of the calculator assists in accurate calculations. This unit includes constructing and interpreting frequency diagrams with equal intervals and identifying the modal class. It moves on to calculating the mean of a grouped frequency distribution. Formal geometrical constructions and simple scale drawings, including the use of bearings, are both studied in this unit. (If desired, scale drawings could be studied first and used as an introduction to formal constructions without a protractor.) Having found the unknown side of a right-angled triangle by construction, the method for calculating it by using Pythagoras' theorem is then introduced. Drawing and interpreting straight line graphs, including the concept of gradient, lead to finding the equation of a straight line graph. Pythagoras' theorem is applied to find the length of a line segment.

Syllabus ref	Learning objectives	Suggested teaching activities
1 Algebraic manipula-tion	Manipulate simple algebraic fractions.	Check that learners are competent at using the four rules for fractions (see Unit 2). Then introduce simple algebraic fractions with numerical denominators: simplify where possible, add and subtract them, multiply by an integer. **(I)**
20 Solutions of equations	Solve fractional linear equations with numerical denominators.	Apply these manipulating skills to solving equations containing such fractions. There are a few equations with fractions at: http://www.cimt.org.uk/projects/mepres/allgcse/bkb10.pdf **(I)**

Syllabus ref	Learning objectives	Suggested teaching activities
		Please note, at this stage, only numerical denominators or single term denominators such as *x* will be used; fractions with linear algebraic denominators are covered in unit 10. **Extension activities:** The level of challenge can be increased by asking learners to focus on more complex examples, for example, equations that are formed by adding two or more fractions. **(I)**
17 Algebraic representa-tion and formulae	Construct equations.	Move on to constructing formulae, then substituting in the subject and solving the resulting equation. Some of the formulae situations you used in Unit 1 may be suitable for the work on constructing formulae. Please note - some learners may have already constructed simple equations when attempting the challenging tasks in units 1 and 2. **(I)** Work on constructing formulae is in section 2.2 at: http://www.cimt.org.uk/projects/mepres/allgcse/bka2.pdf Don Steward has a collection of puzzles that can be solved using algebra: http://donsteward.blogspot.co.uk/2013/06/puzzles-that-you-could-use-algebra-to.html **Extension activities:** The level of challenge can be increased by asking learners to focus on more complex examples. Learners could also invent a ‘think of a number’ puzzle, for example, think of a number, add 7, double your answer, etc. Challenge them to use algebra to help them create a puzzle that always gives a certain answer. *Performing Number Magic*, a resource from the UK Department for Education’s Standards Unit, asks learners to analyse simple number 'tricks', and then use algebra to explain how they work, before going on to create their own ‘tricks’: www.nationalstemcentre.org.uk/elibrary/resource/2022/performing-number-magic-a9 **(I)**

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 18c **Past papers:** Jun 13 Paper 12 Q 20a Nov 13 Paper 12 Q 8 Nov 13 Paper 21 Q 1b Nov 13 Paper 22 Q 3d Jun 14 Paper 21 Q 5b Nov 14 Paper 21 Q 7a Nov 14 Paper 22 Q 8c Jun 15 Paper 12 Q 3 Jun 15 Paper 22 Q 1a Nov 15 Paper 11 Q 8b

Syllabus ref	Learning objectives	Suggested teaching activities
6 Ordering	Order quantities by magnitude and demonstrate familiarity with the symbols =, ≠, >, <, ⩾, ⩽.	Give a list of numbers in different forms, such as 63%, 0.54, 5/11 and 5/8. Ask the learners to write the numbers in order of size, smallest first, and discuss how this can be done (e.g. converting them all to decimals). Discuss different strategies, e.g. some learners may know that 5/8 = 62.5%. You may find *Ordering Fractions and Decimals*, a resource from the UK Department for Education's Standards Unit, to be helpful: www.nationalstemcentre.org.uk/elibrary/resource/1958/ordering-fractions-and-decimals-n1 Use the opportunity to demonstrate the equality and inequality symbols and encourage learners to use them. Futoshiki puzzles use inequality signs and require logical thinking to place integers into a grid so that every inequality is true: www.futoshiki.org/ **Extension activities:** Learners investigate sequences of numbers, such as Farey Sequences. NRICH has resources for Farey sequences: http://nrich.maths.org/2086 'Rod fractions' is an investigation using Cuisenaire rods. NRICH also has resources for 'Rod fractions' http://nrich.maths.org/4345

Syllabus ref	Learning objectives	Suggested teaching activities
9 Estimation	Make estimates of numbers, quantities and lengths, give approximations to specified numbers of significant figures and decimal places and round off answers to reasonable accuracy in the context of a given problem.	Teach learners how to round, using a number line to position the unrounded numbers, to the nearest 100, nearest 10, to 1 decimal place, etc, 3 significant figures, etc. Show the learners which are the vital digits to look at in each case. Use the convention of rounding halfway positions up, e.g. 8.5 rounding to 9 to the nearest unit. Discuss that rounding to the nearest number is not always appropriate e.g. if calculating the number of cans of paint required, an answer of 3.2 would need to round up to 4. *Rounding Numbers*, a resource from the UK Department for Education's Standards Unit, suggests an approach to teaching rounding: www.nationalstemcentre.org.uk/elibrary/resource/1960/rounding-numbers-n3 Apply rounding skills to making estimates.
13 Use of an electronic calculator	Use an electronic calculator efficiently. Apply appropriate checks of accuracy.	Consider appropriate rounding, for example, if estimating a square root, rounding to the nearest square number is usually appropriate. Model efficient use of an electronic calculator, emphasising that intermediate values should not be rounded in a multi-step calculation. The use of memory or answer functions avoids the need for premature rounding. Loss of accuracy marks as a result of premature rounding is a common error in examinations, so it is worth taking some time to address this. **Extension activities:** Investigate the effect on the final answer in a multi-step calculation if intermediate values are rounded. NRICH has some challenging problems involving estimating. The task 'Approximately Certain' asks learners to order a series of quantities: http://nrich.maths.org/6505 and the task 'Does This Sound about Right?' asks learners to check a series of calculations: http://nrich.maths.org/7418 **Learning resources:** Section 6.5 is about estimates and section 6.6 is about using a calculator efficiently: http://www.cimt.org.uk/projects/mepres/allgcse/bka6.pdf

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 11,24b **Past papers:** Jun 13 Paper 11 Q 4a Jun 13 Paper 12 Q 9 Nov 13 Paper 11 Q 3, 7 Nov 13 Paper 12 Q 4b, 9 Jun 14 Paper 11 Q 8a Jun 14 Paper 12 Q 10 Nov 14 Paper 11 Q 1 Nov 14 Paper 12 Q 5 Jun 15 Paper 11 Q 3, 4 Jun 15 Paper 12 Q 3, 6 Nov 15 Paper 11 Q 6 Nov 15 Paper 12 Q 3, 8

Syllabus ref	Learning objectives	Suggested teaching activities
41 Categorical, numerical and grouped data **42 Statistical diagrams**	Collect, classify and tabulate continuous data into classes of equal intervals. Calculate an estimate of the mean for grouped and continuous data. Identify the modal class from a grouped frequency distribution. Construct and interpret bar graphs, simple frequency distributions, frequency	Give the learners a list of raw continuous data, such as their heights from Unit 1, and ask them how the data could be organised and displayed. Introduce the idea of using equal class intervals to enable some comparisons. Show them how to describe a class interval formally, e.g. $170 < h \leqslant 175$ for a height of h cm and how to find the mid-point of a class. Give practice in constructing simple frequency distributions. Show them how to draw bar graphs (histograms of equal width where the vertical scale is the frequency) and frequency polygons (where the data is plotted at the mid-point of the class). Identify the modal class. Give the learners practice at constructing and interpreting these types of diagram. *Maths Is Fun* has work on simple histograms: www.mathsisfun.com/data/histograms.html (alternatively, go to www.mathsisfun.com , click data and navigate to histograms). **(I)** Give the learners a grouped frequency distribution and discuss with them how the mean can be found (you may wish to revise first finding the mean of a discrete frequency distribution as studied in Unit 2). Emphasise that for a grouped frequency distribution, the calculation gives only an estimate of the mean. **(I)**

Syllabus ref	Learning objectives	Suggested teaching activities
	polygons and histograms with equal intervals.	Ask learners to write down an estimate of the length, in metres, of their classroom. Collect these, ask learners to group and tabulate them and calculate the mean. The room can then be measured and the results compared. A poster could be made of the results of this activity, showing also diagrams representing the data. **Extension activities:** Ask learners to find the mean from a list of values, and then put the same data into a grouped frequency distribution in order to estimate the mean. Comment on the results, especially on any differences. **(I)** Learners could also investigate the effect on the estimated mean of adding, removing or changing some of the values in the data set. **(I)** **Learning resources:** Work on statistical diagrams at: https://www.cimt.org.uk/projects/mepres/book9/bk9_8.pdf Raw data may be extracted from sites such as http://censusatschool.org.uk/ , sites giving athletic records, etc.

Past papers and specimen papers

Past papers:
Jun 13 Paper 21 Q 4b, 12a
Nov 13 Paper 21 Q 4a, 4b
Jun 14 Paper 12 Q 24
Jun 14 Paper 21 Q 8a
Jun 14 Paper 22 Q 7bi
Nov 14 Paper 21 Q 4bi
Jun 15 Paper 11 Q 7
Jun 15 Paper 21 Q 10b
Jun 15 Paper 22 Q 3a

Syllabus ref	Learning objectives	Suggested teaching activities
29 Geometrical	Construct a triangle given the three sides, using ruler and pair	Re-visit the symmetry properties of a rhombus and a kite and show that these are the basis for constructing perpendicular and angle bisectors.

Syllabus ref	Learning objectives	Suggested teaching activities
construc-tions **11 Proportion** **36Trigon-ometry**	of compasses only. Construct other simple geometrical figures from given data, using a ruler and protractor s necessary. Construct angle bisectors and perpendicular bisectors using a pair of compasses as necessary. Read and make scale drawings. Use scales in practical situations. Interpret and use three-figure bearings.	Give learners practice in constructing triangles from different data, given three sides, a side and two angles, or two sides and an angle. Include also construction of some other geometrical figures, such as some quadrilaterals. **(I)** (Polygons are studied in Unit 4.) Learners could also construct diagrams in preparation for their work on Pythagoras' theorem. Apply the construction skills to making scale drawings, using simple scales only (harder scales will be met in Unit 4, when ratio and proportion are studied). Include drawings giving the position of places, using three-figure bearings measured clockwise from the north (i.e.000° − 360°). There is some work on bearings and scale drawings at section 3.7: http://www.cimt.org.uk/projects/mepres/allgcse/bka3.pdf You could use local maps, or a map of the school or college, or imagery from a mapping website such as Google Maps, and ask your learners to plan a route of a given length using bearings. If mathematical problems are placed at key points, this could form the basis of a mathematical treasure hunt – this would make a good revision activity. **Extension activities:** Ask learners to deduce how to use perpendicular and angle bisectors in order to construct the circumcircle and the inscribed circle of triangles. Learners could also construct nets of shapes, for example, Platonic solids – these could be made into gift boxes or decorations to be used in local festivals. **(I)** **Further resources:** Links for teachers about constructions, giving background and ideas: www.mathforum.org/library/topics/constructions
36 Trigon-ometry	Apply Pythagoras' theorem to the calculation of a side of a right-angled triangle.	Constructing right-angled triangles can be used as an introduction to Pythagoras' theorem. Another approach would be to provide images of right-angled triangles drawn on squared paper, with squares drawn on each edge; then ask your learners to find the areas of the squares and look for a connection. Alternatively, learners could research the theorem for themselves in preparation for this unit of work. **(I)** You may wish to use a dissection method to show learners that the two squares on the smaller sides can be pieced together to form the square on the hypotenuse. (A formal proof is not required but may be shown to learners after similarity has been studied in Unit 6.) Learners could practise their construction skills by drawing an accurate diagram to illustrate a dissection method for showing that the theorem is true. Give practice to the learners in finding the hypotenuse and another side, using triangles in different

Syllabus ref	Learning objectives	Suggested teaching activities
		orientations. Apply the theorem also to questions in context, such as how far up a wall a ladder reaches. **(I)** Pythagoras' theorem will be used in the next section to find lengths of line segments. **Extension activities:** Learners investigate Pythagorean triples, or research proofs of the theorem. Pythagoras proofs on NRICH: http://nrich.maths.org/6553 Alternatively, they could apply Pythagoras' theorem to 3-dimensional problems, for example, calculating whether a pencil would fit in a box by finding the longest diameter, or designing a pyramid-shaped gift box with a given height and base.

Past papers and specimen papers

Specimen paper:
2018 Specimen Paper 1 Q 22b

Past papers:
Jun 14 Paper 11 Q 3, 11a, 11b
Jun 14 Paper 12 Q 22
Jun 14 Paper 22 Q 1a, 1b, 1c
Nov 14 Paper 11 Q 3
Nov 14 Paper 12 Q 24a
Nov 14 Paper 21 Q 8ai
Nov 14 Paper 22 Q 6
Jun 15 Paper 11 Q 3, 22a
Jun 15 Paper 12 Q 14
Jun 15 Paper 22 Q 8a. 8di
Nov 15 Paper 11 Q 19
Nov 15 Paper 12 Q 21a
Nov 15 Paper 21 Q 2ai
Nov 15 Paper 22 Q 5a

Syllabus ref	Learning objectives	Suggested teaching activities
25 Graphs of functions	Construct tables of values and draw graphs of functions of the form $y = ax + b$. Interpret graphs of linear functions.	Show the learners how to construct tables of values and use them to draw a straight line graph. It is worth going through the process of finding values and converting these to coordinates to model the process for your learners. A surprisingly common error is to plot the points, but then fail to draw the line, so remind learners about this. Another common problem is a lack of accuracy, so remind learners that points must be plotted accurately and that lines should pass through all points. Graphical packages can be used to support introductory work; they will allow you to plot points when modelling the process for your students, before entering the function to draw the final line.
27 Coordinate geometry	Demonstrate familiarity with Cartesian coordinates in two dimensions. Find the gradient of a straight line. Calculate the gradient of a straight line from the coordinates of two points on it. Interpret and obtain the equation of a straight line graph in the form $y = mx + c$. Calculate the length and the coordinates of the midpoint of a line segment from the coordinates of its end points.	Ask groups of learners to draw families of graphs and compare the results e.g. one group to draw $y = x$, $y = 2x$, $y = 3x$, etc, one to draw $y = x$, $y = x + 1$, $y = x - 2$, one to draw $y = 2x$, $y = -0.5x$, $y = -4x$, $y = 0.25x$, etc. as a lead-in to work on the equation of a straight line, gradient, intercept, and parallel (and perpendicular) lines. If possible, use computers for this activity, using graph-drawing programs such as GeoGebra, Autograph or Desmos. There is a free online graphical calculator at: www.desmos.com/ . GeoGebra software for graphing and constructions may be downloaded from: www.geogebra.org/ . Autograph is a powerful piece of software, but a licence fee is payable. A 30-day trial version can be downloaded from www.autograph-maths.com/download/ Finally, apply Pythagoras' theorem to line segments given the end-points; including also how to find the mid-point of a line segment. **(I)** **Extension activities:** Learners attempt to use graphing software such as GeoGebra, or the online graphical calculator at www.desmos.com/, to build up a picture from line segments. If this is not possible, a similar task could be attempted using pencil and paper methods. **Learning resources:** Work on straight line graphs: http://www.cimt.org.uk/projects/mepres/book8/bk8_14.pdf *Connecting Perpendicular Lines*, a resource from the UK Department for Education's Standards Unit, asks learners to use graphical calculators or graphing software to investigate perpendicular lines: www.nationalstemcentre.org.uk/elibrary/resource/2008/connecting-perpendicular-lines-a10

Past papers and specimen papers
Past papers: Jun 13 Paper 11 Q 23ai, 23aiii Jun 13 Paper 12 Q 4 Nov 13 Paper 12 Q 3 Nov 13 Paper 21 Q 3a, 3b, 3c, 3e Jun 14 Paper 11 Q 21 Jun 14 Paper 21 Q 7ci Nov 14 Paper 11 Q 16a, 16b Nov 14 Paper 12 Q 25 Nov 14 Paper 21 Q 9a, 9c Jun 15 Paper 21 Q 2a, 2c Jun 15 Paper 22 Q 1b Nov 15 Paper 12 Q 22a, 22b, 22ci Nov 15 Paper 22 Q 2a, 2b, 2c

Unit 4: Ratio, inequalities, polygons, circles and probability

Recommended prior knowledge

- Solve linear equations (see Units 1–3) and have a basic understanding of inequality symbols (see Unit 3)
- Work with directed numbers (see Unit 1)
- A basic familiarity with fractions, decimals and percentages is expected (see Unit 2)
- Recognise line and rotational symmetries (see Unit 2)
- Know how to use Pythagoras' theorem (see Unit 3)
- Be familiar with drawing and interpreting linear graphs (see Unit 3)

Context

This unit builds on earlier work on manipulative algebra, particularly on expanding brackets and solving linear equations, introducing further concepts. The concepts of probability are introduced. Earlier work done on money conversion and simple scale drawings is followed here by more work on ratio and proportion and rates, including speed. Earlier work on triangles and quadrilaterals is extended to work on polygons in general. Circles are introduced.

Outline

From solving linear equations, the learners move on to solving linear inequalities. Then simultaneous linear equations are introduced. Finally, the learners meet quadratic expressions, learning to expand the product of two brackets. Ideas of probability used in everyday life, including experimental and theoretical probabilities, are used to establish the idea of probability being on a scale from 0 to 1, expressed as a fraction or decimal. The probabilities of a single event are calculated. Basic ideas of proportion are dealt with first, followed by formal work on ratio. Speed is introduced and then applied numerically and in travel graphs. Other rates are also dealt with numerically and by conversion graphs. The vocabulary and angle and symmetry properties of polygons are met and used. The focus then moves to circles, introducing the necessary terminology and looking at symmetry properties before establishing the formulae for the circumference and area of a circle.

Syllabus ref	Learning objectives	Suggested teaching activities
20 Solutions of equations and inequalities	Solve simple linear inequalities.	Revise solving linear equations and then move on to replacing an equation with an inequality, teaching the learners to use the inequality throughout their solution. Using a number line to illustrate that solutions may take any value in a given range is useful. **(I)** Practise solving linear inequalities. **(I)** Move on to consider cases where the solution requires both sides to be multiplied or divided by a negative number. The learners could investigate this for themselves, by starting with numerical inequalities, such as $3 < 5$, then applying the same operation to both sides and deducing whether the inequality statement is still true. **(I)** Creating a card matching activity or jigsaw using Tarsia software can provide variation in the type of task and

Syllabus ref	Learning objectives	Suggested teaching activities
		an opportunity for formative assessment **(F)**. Tarsia software may be downloaded free of charge from www.mmlsoft.com/index.php/products/tarsia or by going to the homepage www.mmlsoft.com/ and choosing either free software, or downloads. **Extension activities:** Extend to find solutions that fit a second constraint in the form x * a, where * is an inequality symbol and a is an integer. For example, solve $3x + 4 > 10$, then find integers that satisfy both $x < 6$ and $3x + 4 > 10$. **Learning resources:** Work on inequalities: http://www.cimt.org.uk/projects/mepres/allgcse/bkc16.pdf
20 Solutions of equations	Solve simultaneous linear equations in two unknowns.	Introduce simultaneous equations by giving the learners a puzzle such as '3 bananas and 2 apples cost *, one banana and one apple cost **. Find the cost of a banana' using local fruits and prices. Usually some learners are able to solve this logically, by doubling the second set of information then subtracting. Move on to express the information algebraically and show them how to set out the solution, using the same processes. Don Steward has a variety of questions and problems that use simultaneous equations: http://donsteward.blogspot.co.uk/search/label/simultaneous%20equations **(I/F)** Continue then with further work on simultaneous equations, including those where both equations have to be multiplied and those where addition is needed rather than subtraction. **Extension activities:** To give further practice in constructing and solving equations, learners work in pairs, making up similar word problems leading to simultaneous equations and then solving them algebraically for each other. Keep your learners' ideas for future years, to build up your bank of resources! Learners could also be challenged to apply their skills to solve three variable problems. Don Steward also has some three variable problems: http://donsteward.blogspot.co.uk/2013/08/three-variable-simultaneous-equations.html **Learning resources:** Work on simultaneous equations in section 10.8 of: http://www.cimt.org.uk/projects/mepres/allgcse/bkb10.pdf
18 Algebraic manipula-	Expand products of algebraic expressions.	Revise work on expanding and simplifying such as $3x(x + 4)$ and then $2(x + 5) - 3(4x - 1)$.

Syllabus ref	Learning objectives	Suggested teaching activities
tion		Then move on to introducing expansion and simplifying quadratic expressions such as $(x + 5)(x - 3)$ and then $(3x - 4)(2x - 7)$. **(I)** Work on multiplying out brackets is in section 10.7 of: http://www.cimt.org.uk/projects/mepres/allgcse/bkb10.pdf Creating a card matching activity or jigsaw using Tarsia software can provide variation in the type of task and an opportunity for formative assessment **(F)**. Tarsia software may be downloaded free of charge from www.mmlsoft.com/index.php/products/tarsia or by going to the homepage www.mmlsoft.com/ and choosing either free software, or downloads. **Extension activities:** Learners work with brackets containing more than two terms. Alternatively, learners could begin to work on factorisation. If given some simple quadratic expressions, can they work out what the factorised form is, by considering the problem as the reverse of multiplying out brackets?

Past papers and specimen papers

Specimen papers:
2018 Specimen Paper 1 Q 23
2018 Specimen Paper 2 Q 7b

Past papers:
Jun 14 Paper 11 Q 5
Jun 14 Paper 12 Q 23a, 25
Jun 14 Paper 21 Q 1b, 5a
Jun 14 Paper 22 Q 2b
Nov 14 Paper 11 Q 7
Jun 15 Paper 11 Q 8
Jun 15 Paper 12 Q 23
Jun 15 Paper 21 Q 6a, 6c
Nov 15 Paper 11 Q 10c, 23
Nov 15 Paper 22 Q 6b, 6d

<table>
<tr><th>Syllabus ref</th><th>Learning objectives</th><th>Suggested teaching activities</th></tr>
<tr>
<td>40 Probability</td>
<td>Calculate the probability of a single event as either a fraction or a decimal.

Understand that the probability of an event occurring = 1 – the probability of the event not occurring.

Understand relative frequency as an estimate of probability, e.g. use results of experiments with a spinner to estimate the probability of a given outcome; use probability to estimate from a population.</td>
<td>Introduce elementary ideas of probability using familiar contexts, for example the probability of obtaining a red ball when a ball is chosen from a bag without looking, or a weather forecaster saying there is a 20% chance of rain today. Introduce the idea of ‘at random’, and that probabilities are on a scale from 0 to 1. Emphasise that probabilities should be given as fractions or decimals (never as ratios).

Include some examples where information is given in tables or graphs, for example a student is to be selected from this group:

<table>
<tr><th></th><th>Can swim</th><th>Can’t swim</th></tr>
<tr><td>Girls</td><td>14</td><td>3</td></tr>
<tr><td>Boys</td><td>10</td><td>5</td></tr>
</table>

If the student is picked at random, what is the probability that they are a girl / can swim / a boy who can swim, etc? If you pick a boy at random, what is the probability that he can swim?

To help the learners to grasp the concepts, some probability experiments are a good idea, to demonstrate the difference between theoretical and experimental probability, and short and long-term trends. For instance, learners could work in pairs, with several pairs doing the same experiment, to compare their results and then pool them to obtain larger samples. Possible experiments include taking balls or coloured pencils out of a bag, throwing dice, choosing cards numbered on one side, tossing coins, dropping a drawing pin and seeing whether it lands point down or point up, etc. There are a number of games which could be used as a basis for a probability experiment.

Alternatively, learners could construct a spinner or a dice and then test it to see whether it is fair.

Discuss when an experiment is needed to establish probabilities (e.g. if a die is biased, or in a situation where equally likely outcomes are not guaranteed) and show how to use the data from the experiment to estimate probabilities from relative frequencies.

Go on to calculate the theoretical probability of a single event in different contexts, e.g. rolling a die, pulling counters of a bag, etc.

Establish also and use the fact that the sum of probabilities of all the outcomes is 1; use this to find probabilities of events not occurring.</td>
</tr>
</table>

Syllabus ref	Learning objectives	Suggested teaching activities
		Use probabilities to work out expected values, e.g. given the probability that a machine produces a faulty item, calculate the number of faulty items that would be expected in a batch of 1000. **Extension activities:** Place some counters in a bag. You will need a mixture of colours, for example, 1 red, 3 blue, 6 green and 10 yellow counters would work well. Ask learners to pick out one counter at a time without looking and then replace it. The aim is to identify what is in the bag – ask the learners to consider how many trials they would need to be confident about their answer. **Learning resources:** Work on probability, with sections 5.1 and 5.2 suitable for this unit (combined or compound events are covered in Unit 8 of this scheme): https://www.cimt.org.uk/projects/mepres/allgcse/bka5.pdf An introductory lesson on probability with a quiz to check understanding: www.mathgoodies.com/lessons/vol6/intro_probability.html A game involving simple probabilities: https://math.stackexchange.com/questions/109614/odds-of-each-hand-size-in-a-game-of-go-fish Interactive dice and spinners: http://nrich.maths.org/6717 Mathematics Activities from Diverse Cultures has descriptions of various games from a variety of cultures which could be used as a basis for probability work: https://www.uccs.edu/Documents/pipes/mccoyprob-games.pdf A variety of materials may be useful for experiments, including dice, spinners and counters. Biased dice are also useful for experiments.

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 21 **Past papers:** Jun 14 Paper 22 Q 4a Nov 14 Paper 21 Q 11bii Jun 15 Paper 12 Q 4 Jun 15 Paper 21 Q 10ai Jun 15 Paper 22 Q 11ai, 11aii Nov 15 Paper 12 Q 23a

Syllabus ref	Learning objectives	Suggested teaching activities
11 Ratio, proportion, rate	Demonstrate an understanding of ratio and proportion; solve problems using ratios, including dividing a quantity in a given ratio. Use scales in practical situations. Increase and decrease a quantity by a given ratio. Use common measures of rate. Solve problems involving average speed.	Introduce proportion by using a recipe with quantities for 4 people, asking learners for the quantities needed for 8 people, 6 people, etc. This could be a traditional local recipe, or if you have looked at data from another country using the Census at School website, you could use a recipe from that region. Recipes are widely available on websites. For instance, there are links to over 500 sites giving Asian recipes at: www.cbel.com/asian_recipes/ . Do further work on proportion such as how many cans of paint are needed to paint a fence, knowing the area which can be painted using 1 litre. Include some inverse proportion, such as the fixed cost of hiring a bus finding the cost per person if 50 people go in the bus or only 40 do. Move on to comparing quantities as ratios, using ratio notation. Include work on scales and scale drawings, following on from the work in Unit 3, using harder scales, including scales written in the form 1:n. Local maps with a variety of scales will be useful for this. Do work on solving problems with ratios, including simplifying ratios and dividing a quantity in a given ratio. **(F)** Move on to consider increasing and decreasing a quantity by a given ratio. Explain that this means interpreting the ratio as old quantity:new quantity, e.g. decrease $240 in the ratio 5:3 means treating $240 as 5 parts, then finding the corresponding value of three parts. This can also be done using multipliers:

<table>
<tr><th>Syllabus ref</th><th>Learning objectives</th><th>Suggested teaching activities</th></tr>
<tr><td></td><td></td><td>
<table>
<tr><td>old quantity</td><td>:</td><td>new quantity</td></tr>
<tr><td>5</td><td>:</td><td>3</td></tr>
<tr><td>× 48</td><td></td><td>× 48</td></tr>
<tr><td>$240</td><td>:</td><td>$144</td></tr>
</table>

There are some further examples at www.mathsteacher.com.au/year8/ch06_ratios/03_inc/dec.htm

Formally define speed and use it to solve problems involving constant speeds. (This may be combined with the work on travel graphs in the next section.)

Use problems involving other rates, for example, the volume of water per minute that is flowing through a tap.

Please note – there is further work on rates, including finding acceleration, in Unit 8 of this scheme.

Extension activities: Learners investigate the ‘Golden ratio’.

Learning resources:
An introductory lesson on ratio: www.mathsisfun.com/numbers/ratio.html

Work on ratio and proportion and scale drawings:
http://www.cimt.org.uk/projects/mepres/allgcse/bkc15.pdf

Developing Proportional Reasoning, a resource from the UK Department for Education’s Standards Unit, asks learners to create and solve a variety of problems:
www.nationalstemcentre.org.uk/elibrary/resource/1963/developing-proportional-reasoning-n6</td></tr>
<tr><td>27 Coordinate geometry

24 Graphs in practical situations</td><td>Demonstrate familiarity with Cartesian coordinates in two dimensions.

Interpret and use graphs in practical situations including travel graphs and conversion graphs.

Draw graphs from given data.</td><td>One starting point would be to review the money conversion graphs met in Unit 2 before moving on to use other conversion graphs such as miles / kilometres and Fahrenheit / Celsius temperatures or other conversions with which your learners are familiar. Include drawing these graphs as well as reading off them and interpreting them to find conversion rates.

Dan Meyer’s ‘Graphing Stories’ activity provides a meaningful way to develop learners’ understanding of real life graphs and the features that these have: http://blog.mrmeyer.com/2007/graphing-stories/ (F)
Another teacher, Sean Sweeney, has created a similar video with two athletes:
www.youtube.com/watch?v=FGC6usaESpg

Alternatively, use a travel graph of someone’s journey to school, perhaps stopping on the way, then going straight home at the end of the day, plotting distance from home against time. You can use this for reading</td></tr>
</table>

Syllabus ref	Learning objectives	Suggested teaching activities
		off, calculating speed and interpreting zero and negative gradients, etc. relating back to the work on gradients done in Unit 2. Then use other travel graphs, including drawing two journeys on the same graph and interpreting where they cross. **Extension activities:** Card sorting tasks, such as the one in the 'Interpreting distance time graphs' from the Standards Unit, can be offered at a varying level of challenge: learners could initially be given just set A and set B (match statements to graphs), then move on to include tables of values and statements about speeds and acceleration. The 'Interpreting distance time graphs' from the Standards Unit can be found at: www.stem.org.uk/elibrary/resource/28924/interpreting-distance-%E2%80%93-time-graphs-a6 Learners could draw on earlier work with estimation, by working out whether top sprinters, such as Usain Bolt, would be breaking local speed limits. Alternatively, they could time themselves running 100m, then work out how far behind Usain Bolt they would be when he crossed the finish line.

Past papers and specimen papers

Specimen paper:
2018 Specimen Paper 1 Q 6

Past papers:
Jun 13 Paper 21 Q 7a
Jun 13 Paper 22 Q 8a, 8b, 8c
Nov 13 Paper 11 Q 3a, 11a
Nov 13 Paper 12 Q 1b, 25a, 25b
Jun 14 Paper 11 Q 4
Jun 14 Paper 22 Q 3bii
Nov 14 Paper 11 Q 6
Nov 14 Paper 12 Q 7
Nov 14 Paper 21 Q 11
Nov 14 Paper 22 Q 10
Jun 15 Paper 11 Q 23
Jun 15 Paper 12 Q 22
Jun 15 Paper 22 Q 2a, 2b
Nov 15 Paper 12 Q 14

Syllabus ref	Learning objectives	Suggested teaching activities
28 Geometrical terms **32 Angle**	Use and interpret the geometrical terms interior and exterior angles. Use and interpret vocabulary of polygons, including: • regular and irregular polygons • pentagons • hexagons • octagons • decagons. Calculate unknown angles and give simple explanations using the following geometrical properties: (e) angle properties of regular and irregular polygons	Islamic designs are a rich source of examples of use of polygons, as are tiled floors in some public buildings. Use a picture of a tiled design including triangles, quadrilaterals and other polygons to ask learners what shapes they can identify. **(F)** Move on to introducing the names of different polygons and identifying line and rotational symmetries (met in Unit 2) of regular and some irregular polygons. The Victoria and Albert Museum has created some teachers' resources linked to the mathematical properties of Islamic designs: www.vam.ac.uk/content/articles/t/teachers-resource-maths-and-islamic-art-and-design/ Many architectural designs incorporate polygons or tiling patterns. Learners could be encouraged to look for examples in the local area and photograph or sketch these. **(F)** Show learners how to construct regular polygons with their vertices on circles by dividing the angle at the centre by the number of sides and constructing radii. A poster can be made showing different regular polygons constructed in this way and they can be used as a lead-in to the work on angle properties of polygons. Learners should be made familiar with the terms exterior and interior angles. Asking them to consider a walk around the edge of a polygon leads naturally to the conclusion that the sum of exterior angles is 360°. One approach to considering interior angles is to divide the polygon into triangles, then consider the sum of the angles in all of the triangles. Having deduced key ideas about exterior and interior angles, learners should apply these to finding missing angles in both regular and irregular polygons. **(I)** **Extension activities:** A challenging problem involving two polygons from NRICH: http://nrich.maths.org/5642 Learners consider which regular polygons will tessellate, based on the sizes of their interior angles. An interactive tessellation tool from NRICH: http://nrich.maths.org/6069 An investigation into regular and semi-regular tessellations from NRICH: http://nrich.maths.org/4832 Learners could also investigate convex polygons, for example considering how many acute angles are possible. Convex polygons from NRICH: http://nrich.maths.org/1972 **Learning resources:** Work on polygons in section 3.5: http://www.cimt.org.uk/projects/mepres/allgcse/bka3.pdf

Past papers and specimen papers
Past papers: Jun 13 Paper 21 Q 2a Nov 13 Paper 11 Q 18 Jun 14 Paper 12 Q 18 Nov 14 Paper 11 Q 21 Nov 15 Paper 12 Q 19

Syllabus ref	Learning objectives	Suggested teaching activities
28 Geometrical terms	Use and interpret the vocabulary of circles.	Having used circles and their radii in the previous section of this unit, draw a circle and extend the learners' vocabulary to include diameter, tangent, chord, circumference, arc, sector, segment, showing these on the drawing.
31 Symmetry	Use the following symmetry properties of circles: (a) equal chords are equidistant from the centre (b) the perpendicular bisector of a chord passes through the centre (c) tangents from an external point are equal in length.	Ask learners to draw a circle with two tangents meeting at a point. Join this point to the centre and draw in the radii to the points of contact of the tangent, and the chord joining these points. Discuss the symmetries of this diagram and hence obtain and explain the required symmetry and angle facts. Then solve angle and length problems using these. For instance, Pythagoras' theorem (from Unit 3) may be used to find the length of a chord, given the radius and its distance from the centre. Some of the tasks in 'Discovering circle theorems', an activity from the National Centre for Excellence in Teaching Mathematics (NCETM) are relevant to this unit: www.ncetm.org.uk/public/files/280228/NCETM_Mathematics_Department_Workshops_Circle_Theorems_Resource_Sheet_HT1.CIR.2.pdf Please note, further angle properties of circles are covered in Unit 7.
32 Angle	Calculate unknown angles and give simple explanations using the following geometrical properties: (g) angle between tangent and radius of a circle.	Ask learners to measure the diameter and circumference (use string or a flexible tape for the latter) of various circular objects in the classroom or at home, such as cans or flowerpots. Get them to draw a graph of their results with a line of best fit and to use the work done in Unit 3 to obtain the equation of this line. This gives them an estimate for $C = \pi d$. A practical demonstration of the area formula may be obtained by cutting a circle into, say, 16 sectors and putting these together alternately to form an approximate rectangle, one sector being cut in half for the ends. This rectangle has width r and length = half the circumference, so leading to $A = \frac{1}{2} Cr$ and hence $A = \pi r^2$.

Syllabus ref	Learning objectives	Suggested teaching activities
35 Mensuration	Solve problems involving the circumference and area of a circle.	Then solve circumference and area problems involving the whole circle. Learners should be able to give answers in terms of pi. Where tasks are in context, answers should be to a suitable degree of accuracy. Articles giving more background about the formulae for area and circumference, and π, may be found, for example at: https://www.exploratorium.edu/pi/history-of-pi https://www.historytoday.com/archive/feature/man-who-invented-pi https://sciencing.com/origins-perimeter-circumference-7815683.html Please note – the area of sectors, etc. is covered in Unit 7. **Extension activities:** Learners find areas and perimeters of simple fractions of circles, such as quarter circles, or semicircles. These ideas will be developed further in Unit 7. **Learning resources:** Work on circles in section 7.7 of: http://www.cimt.org.uk/projects/mepres/allgcse/bkb7.pdf

Past papers and specimen papers

Specimen paper:
2018 Specimen Paper 1 Q 25a

Past papers:
Jun 13 Paper 11 Q 5, 14a, 14b
Jun 13 Paper 12 Q 12a
Nov 13 Paper 12 Q 22a
Nov 13 Paper 22 Q 4a, 4bii
Jun 14 Paper 12 Q 8
Nov 14 Paper 11 Q 9
Nov 14 Paper 22 Q 5a, 5b

Unit 5: Quadratics and other formulae; further percentages; volume, surface area, transformations and loci

Recommended prior knowledge

- Algebraic manipulation, especially as applied to solving linear equations (see Units 1 – 3)
- Multiplying pairs of brackets (see Unit 4)
- Basic understanding of ratio and proportion (see Unit 4)
- Calculate percentage changes (see Unit 2)
- Work with simple indices (see Unit 1)
- Basic constructions (see Unit 3)

Context

Algebraic manipulation skills are extended to transforming formulae and factorising quadratic expressions. Graphically, learners move on from straight line graphs to the graphs of quadratic functions. The work on percentages and positive indices in earlier units is extended in this unit. Some three dimensional work is introduced, dealing with prisms. Objects changing position are dealt with in two ways, by considering transformations and by work on loci.

Outline

Earlier work on algebraic manipulation and solving equations is extended to transforming simple formulae and to using algebra for direct and inverse variation. After expanding quadratic expressions in Unit 4, in this unit quadratic expressions are factorised and the graphs of quadratic functions drawn. Finding a percentage increase or decrease is revisited and extended to the harder case of doing reverse percentage calculations. Whilst the calculator is being used, the opportunity is taken to emphasise efficient use of a calculator. Work on indices is extended to include negative and zero indices, their use and interpretation. Mensuration is extended from two-dimensional objects to finding the volume and surface area of prisms. Informal ideas of symmetry are extended to prisms and then to formal work on transformations from a geometrical point of view (the introduction of matrices comes later in the course in Units 9 and 10). Finally, the concepts of loci are established and formal geometrical constructions used to make accurate drawings of loci.

Syllabus ref	Learning objectives	Suggested teaching activities
17 Algebraic representation and formulae	Transform simple formulae.	One way to begin this is to start with solving an equation such as $6x + 1 = 7$, then ask the learners how they would find an expression for x when $6x + 1 = y$. Alternatively, learners could transform a numerical statement, describing the steps needed. Move on to asking the learners for simple formulae they know, such as $C = \pi d$, or $P = 2(l + w)$ and ask them to transform them to make another variable the subject. If learners have used the forming and solving equations tasks from the UK Department for Education's Standards Unit in earlier work on solving equations, they can be reminded of this process and encouraged to set their work out in a similar way. Give practice with a range of suitable simple formulae. **(I)**

Syllabus ref	Learning objectives	Suggested teaching activities
		Don Steward has materials for transforming a statement: http://donsteward.blogspot.co.uk/2013/03/transforming-statement.html **(F)** Don Steward also has work on linear equations in which formulae need to be transposed to deduce properties of straight line graphs before placing equations in the correct place on a simple Venn diagram. This would provide useful revision of early work: http://donsteward.blogspot.co.uk/2012/01/linear-equations-and-mr-venn.html **(F)** **Extension activities:** Learners can be challenged by asking them to focus on more difficult formulae, where there are several steps to the rearrangement. **Learning resources:** Work on transforming formulae: http://www.cimt.org.uk/projects/mepres/allgcse/bka2.pdf
18 Algebraic manipula-tion **25 Graphs of functions**	Factorise, where possible, expressions of the form $x^2 + bx + c$. Construct tables of values and draw graphs for functions of the form $y = ax^n$ where $n = 0$, 1, 2 and simple sums of these. Interpret graphs of quadratic functions. Solve associated equations approximately by graphical methods.	Start by practising a few examples of expanding brackets and simplifying (done in Unit 4) such as $(x + 3)(x + 5)$, $(x - 2)(x + 1)$, etc. Alternatively, mathematics teacher William Emeny has created a 'Sum Products' activity for use as a starter: www.greatmathsteachingideas.com/2012/07/01/sum-products-preparation-for-factorising-quadratic-expressions/ Then progress to examples such as $(x + 2)(x \quad) = x^2 + 5x + 6$, asking them to fill in the gap. Finally, give them $(x \quad)(x \quad) = x^2 + 2x – 8$, etc. and ask them to fill in the gaps. Some learners can usually do this without further prompting and can explain how they have worked, which forms the basis for a good discussion on the principles needed to factorise quadratics. Practice is then needed in this skill. Factorising quadratic expressions is in section 10.10: http://www.cimt.org.uk/projects/mepres/allgcse/bkb10.pdf Some online factorising exercises: https://worksheetmath.com/Algebra/DoubleBrackets/Factorising **(F)** Move on to show learners how to construct a table of values for a graph such as $y = x^2 - 3$, and how to draw the graph, paying attention to obtaining a smooth curve which is of the correct shape at the vertex. Show the learners how to use the graph to solve equations such as $x^2 - 3 = 0$ or $x^2 - 3 = 14$. Then show them how to construct tables of values for a more complex quadratic function such as $y = 2x^2 + 3x - 2$, similarly going on to draw and use the graph.

Syllabus ref	Learning objectives	Suggested teaching activities
		Questions about plotting quadratic graphs as well as other curves (see Unit 8) are in section 13.4: https://www.cimt.org.uk/projects/mepres/allgcse/bkc13.pdf This work can be used further in Unit 6 when quadratic equations are solved by factorising. Many graphing programs accept inputs using function notation. This could be used to introduce learners to the idea of function notation during graphical work in preparation for further work in Unit 10. **Extension activities:** Ask learners to draw graphs of $y = ax^2 + bx + c$, perhaps using graphing software such as GeoGebra, Autograph or the online calculator at Desmos.com. Ask learners to investigate what effect varying the values chosen for a, b and c will have on the graphs.

Past papers and specimen papers
Past papers: Jun 13 Paper 11 Q 10b Jun 14 Paper 22 Q 2a Nov 14 Paper 11 Q 12 Nov 14 Paper 21 Q 3a, 3b Nov 14 Paper 22 Q 8a Jun 15 Paper 12 Q 15 Jun 15 Paper 22 Q 7b Nov 15 Paper 21 Q 8b, 8c

Syllabus ref	Learning objectives	Suggested teaching activities
23 Variation	Express direct and inverse variation in algebraic terms and use this form of expression to find unknown quantities.	Begin work on variation by using an example such as the price of a circular cloth being proportional to its area, and then moving on to the price being proportional to the radius squared. Show the learners how to form the formula connecting the variables and to use the information given to find the constant of proportionality, then to use the equation they have found; and similarly for inverse variation. Throughout this topic, use examples in context as well as purely algebraic examples. Work on variation:

Syllabus ref	Learning objectives	Suggested teaching activities
		http://www.cimt.org.uk/projects/mepres/allgcse/bkc15.pdf Mathematics teacher William Emeny has a card-sorting activity for proportionality formulae: www.greatmathsteachingideas.com/2012/03/29/proportionality-formulae-conceptual-card-sort/ **(F)** Don Steward has a collection of materials, including examples in context: http://donsteward.blogspot.co.uk/search/label/proportion **Extension activities:** Ask learners to consider the shapes of the graphs of various formulae and to describe their key features. They should already be familiar with linear and quadratic graphs and should recognise these shapes. Learners should be to apply their knowledge of graphs to plot other types, either on paper or using graphing software. A card sorting activity that links proportionality statements to equations and graphs would be a useful follow up activity to check the learners' understanding. **(F)** (Further work on graphs of cubics, etc. appears in later units).

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 15a, b **Past papers:** Jun 13 Paper 11 Q 21a, 21b Jun 13 Paper 12 Q 14 Nov 13 Paper 12 Q 6 Jun 14 Paper 12 Q 5 Jun 14 Paper 21 Q 3 Nov 14 Paper 11 Q 13 Nov 14 Paper 12 Q 10 Jun 15 Paper 11 Q 9 Jun 15 Paper 12 Q 7 Jun 15 Paper 22 Q 6c Nov 15 Paper 11 Q 9 Nov 15 Paper 12 Q 3

Syllabus ref	Learning objectives	Suggested teaching activities
12 Percentages	Carry out calculations involving reverse percentages, e.g. finding the cost price given the selling price and the percentage profit.	Sale prices and percentages in local shops or newspaper advertisements or local tax rates can all provide useful contexts for introducing this topic. Ask the learners to find the price of an item after a 5% increase, and to tell you how they did this. For those who found 5% and added it on, discuss the result of: Price after increase = 105% of price before = 1.05 × price before, as a calculator efficient method. This leads to: Price before increase = Price after ÷ 1.05, but first give them an example of a price after such an increase and see if they can work it out for themselves. **(I)** Similarly, do examples with price decreases, finding prices after and before increases. Apply the reverse percentage method to other contexts such as prices including or excluding tax, or selling price and cost price. Work on reverse percentages is at section 11.9: http://www.cimt.org.uk/projects/mepres/allgcse/bkb11.pdf *Using Percentages to Increase Quantities*, a resource from the UK Department for Education's Standards Unit, introduces the idea of multipliers: www.nationalstemcentre.org.uk/elibrary/resource/1964/ Remind learners that methods such as inverse operations can be used to check their results – they should work with the price before increase that they have found and see if it gives them the correct price after increase. **Extension activities:** Learners consider how to reverse a repeated percentage change.

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 2 Q 1 **Past papers:** Jun 13 Paper 11 Q 9b Jun 13 Paper 22 Q 5b Nov 13 Paper 11 Q 9 Nov 13 Paper 21 Q 5d Nov 13 Paper 22 Q 6 Nov 14 Paper 21 Q 1c Jun 15 Paper 21 Q 1 Nov 15 Paper 21 Q 1a

Syllabus ref	Learning objectives	Suggested teaching activities
19 Indices	Use and interpret positive, negative and zero indices.	Recap the basic index laws $x^m \times x^n = x^{m+n}$ and $x^m \div x^n = x^{m-n}$ from Unit 1. Move on to consider cases such as $(2^3)^4$. Learners could write this out as (2 × 2 × 2) × (2 × 2 × 2) × (2 × 2 × 2) × (2 × 2 × 2) in order to reach the conclusion that $(2^3)^4 = 2^{12}$. Repeat for other examples in order to establish the general result $(x^m)^n = x^{mn}$. Give learners some practice in applying this. Move on to consider sequences. Start with $2^4 = 16$, $2^3 = 8$, $2^2 = 4$ and continue the pattern, noting that each step uses a division by 2, to obtain $2^0 = 1$, $2^{-1} = ½$, etc. generalising to $2^{-n} = 1/2^n$ and $2^0 = 1$; and similarly for other numbers. Negative indices can also be demonstrated using index laws: $\frac{5^3}{5^5} = 5^{-2}$, but by simplifying, $\frac{5^3}{5^5} = \frac{1}{5^2}$, so $\frac{1}{5^2} = 5^{-2}$. Show the learners how to use their calculator to evaluate negative and zero indices and that their calculator results are consistent with the patterns found. Then practice evaluating expressions with negative and zero indices, both with and without a calculator. Learners should also be able to apply these ideas to algebraic expressions, for example recognising that $a^{-n} = 1/a^n$. Ask learners to apply the index laws to solve simple equations, such as $\frac{5^t}{5^3} = 5^2$. They also need to be able to rewrite expressions in terms of a different base number, for example, write 8^3 in the form 2^k. **Extension activities:** Ask learners to explain why 0^0 is defined, perhaps using a sequence of powers, as above. Learners could also create questions for others to solve. **Learning resources:** Work on indices is in section 1.6 (including fractional indices, which are in Unit 7): http://www.cimt.org.uk/projects/mepres/allgcse/bka1.pdf

Syllabus ref	Learning objectives	Suggested teaching activities
		Online worksheets, including one about negative indices: https://mathsmadeeasy.co.uk/gcse-maths-revision/rules-indices-gcse-maths-revision-worksheets/ **(I/F)**

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 18a **Past papers:** Jun 13 Paper 11 Q 20a Jun 14 Paper 12 Q 19a Nov 14 Paper 11 Q 8a, 8b Nov 14 Paper 12 Q 17b Jun 15 Paper 12 Q 16a Nov 15 Paper 12 Q 11a, 11c

Syllabus ref	Learning objectives	Suggested teaching activities
28 Geometrical terms	Use and interpret vocabulary of simple solid figures: cube, cuboid, prism and cylinder.	Draw the nets of some prisms and construct the prisms. If card is available, these could be decorated (easiest to do at the net stage) and made into gift or storage boxes, perhaps with separate lids. This activity leads naturally into calculations of surface area and volume. (It could be set as a task to design a storage box taking these elements into consideration.) **(I)**
35 Mensuration	Solve problems involving the surface area and volume of a cuboid, cylinder, and prism. Solve problems involving the areas and volumes of compound shapes.	*Boxes and bottles* from Cre8ate Maths has some suggestions for some straightforward practical starter tasks. Resources from Cre8ate Maths are free, but registration is required. Go to the Cre8ate Maths homepage, www.cre8atemaths.org.uk/, choose resources, then food and drink then boxes and bottles. Area and volume problems are at sections 7.7 and 7.8: http://www.cimt.org.uk/projects/mepres/allgcse/bkb7.pdf
31 Symmetry	Recognise symmetry properties of the prism	Discuss also the symmetry properties − having the prisms constructed also enables learners to see the symmetry properties more easily. Symmetry properties of prisms are at section 3.6: http://www.cimt.org.uk/projects/mepres/allgcse/bka3.pdf

Syllabus ref	Learning objectives	Suggested teaching activities
	(including cylinder).	Include examples where learners have to work with compound shapes, for example, two cuboids put together to make a prism with an ‘L’ shaped cross section. **Extension activities:** Ask learners to create a set of designs for a container with a certain volume. Which design uses the least card? **(F)**

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 20 **Past papers:** Jun 13 Paper 21 Q 9a Jun 13 Paper 22 Q 12a Jun 14 Paper 21 Q 6a Nov 14 Paper 11 Q 17a Nov 14 Paper 22 Q 4a Nov 15 Paper 21 Q 11b

Syllabus ref	Learning objectives	Suggested teaching activities
39 Trans-formations **37 Vectors in two dimensions**	Identify and give precise descriptions of transformations connecting different figures; describe transformations. Describe a translation by using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$ or $\overrightarrow{AB}$ or $\boldsymbol{a}$	Learners have met reflection and rotation symmetry, so you can use these as an introduction to reflecting and rotating non-symmetrical objects. Use graph grids to enable descriptions and plain paper for them to create designs using reflections and rotations. Tracing paper is very useful when teaching reflections, rotations (especially for considering the centre of rotation) and translations. If ICT facilities are available for you to use with your learners, you could use LOGO or Scratch and ask learners to program a series of simple transformations. Alternatively, the drawing facilities in specialised programs or in programs such as Word could be used to create simple transformations. **(I)** An online version of LOGO is available at: https://turtleacademy.com Scratch software can be downloaded from: http://scratch.mit.edu/

Syllabus ref	Learning objectives	Suggested teaching activities
		Move on to introducing enlargement and translation – both drawing the images after transformations and describing a transformation given an object and its image. For translations, describe them using a column vector. *Maths is Fun* has some interactive resources for transformations: www.mathsisfun.com/geometry/transformations.html Vector snakes and ladders provides an introduction to column vectors and practice in their use: www.tes.co.uk/teaching-resource/Vector-snakes-and-ladders-Vector-game-6030422/ Use some simple combinations of the above transformations to give learners practice in both drawing and describing as a single transformation the result of a combined transformation. **Extension activities:** Ask learners to investigate traditional designs which use transformations, or create some designs of their own. **Learning resources:** Work on loci and transformations: http://www.cimt.org.uk/projects/mepres/allgcse/bkc14.pdf

Past papers and specimen papers

Specimen paper:
2018 Specimen Paper 2 Q 3a

Past papers:
Jun 13 Paper 11 Q 22a, 22b
Jun 13 Paper 12 Q 6
Nov 13 Paper 11 Q 25a, 25b
Nov 13 Paper 12 Q 5
Nov 13 Paper 22 Q 12bi, 12bii
Jun 14 Paper 12 Q 17a, 17c
Nov 14 Paper 12 Q 16
Nov 14 Paper 21 Q 9a, 9b, 9c, 9d, 0e
Nov 14 Paper 22 Q x
Jun 15 Paper 11 Q 15, 24a
Jun 15 Paper 22 Q 10ai, 10aii, 10aiii
Nov 15 Paper 21 Q 7ci, 7cii, 7cii
Nov 15 Paper 22 Q 11b

Syllabus ref	Learning objectives	Suggested teaching activities
33 Loci	Use the following loci and the method of intersecting loci for sets of points in two dimensions which are: (a) at a given distance from a given point, (b) at a given distance from a given straight line, (c) equidistant from two given points (d) equidistant from two given intersecting straight lines.	Introduce the idea of locus by using examples in the classroom. 'I want to walk so that I stay 1.5m from this wall. Where can I go?' 'I want to stay 1 m from this chair. Where can I go?' 'Where can the spider be if it is 30 cm from this light bulb?' Alternatively, ask a group of your learners to stand in places so that they meet certain conditions, for example 1m from a certain point, or an equal distance from two points. If a suitable outdoor space is available, learners could construct loci, using chalk, string and a metre ruler to mark out a locus on the ground. Progress to using pencil and paper to draw accurate scale diagrams to represent loci in two dimensions, revising geometrical constructions from unit S3 as necessary. Include examples of intersecting loci, for example, given a diagram showing the positions of villages A and B: 'Shona lives less than 4 km from village A. She lives nearer to village B than to village A. Shade the region where Shona lives.' **(I)** Work on loci and transformations: http://www.cimt.org.uk/projects/mepres/allgcse/bkc14.pdf **Extension activities:** Learners consider more complex loci. For example, the locus of a pedal on a moving bicycle, or of the hat on someone who is hopping along a path.

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 22 **Past papers:** Jun 13 Paper 11 Q 15 Jun 13 Paper 12 Q 12 Nov 13 Paper 12 Q 24 Nov 13 Paper 21 Q 2 Jun 14 Paper 11 Q 22 Jun 14 Paper 22 Q 1 Nov 14 Paper 12 Q 24 Nov 14 Paper 21 Q 8a Jun 15 Paper 11 Q 22 Jun 15 Paper 22 Q 8d Nov 15 Paper 12 Q 21

Unit 6: Further quadratics, graphing inequalities, cumulative frequency, standard form and trigonometry

Recommended prior knowledge

- Factorising quadratics in the form $x^2 + bx + c$ (see Unit 5)
- Drawing straight line graphs (see Unit 3) and drawing graphs of quadratic functions (see Unit 5)
- Basic use of indices and index laws (see Units 1 and 5)
- Pythagoras' theorem (see Unit 3)
- Use of simple ratios and scales (see Unit 4) and enlargements (see Unit 5)

Context

Work on quadratic functions is expanded, leading to solving a quadratic equation by factorisation. Graphical work is extended to graphing linear inequalities. Work on grouped data studied in Unit 3 is developed further to include the use and interpretation of cumulative frequency diagrams. Knowledge about indices is applied to using standard form. Work on triangles is developed to include similar and congruent triangles and an introduction to trigonometry. Concepts of enlargement and ratios already met assist in understanding the use of both trigonometrical and similarity ratios.

Outline

Having factorised simple quadratics in Unit 5, learners learn to factorise the remaining range of quadratic functions, where possible, and then apply their knowledge to the solution of quadratic equations. Work done in Unit 3 on drawing and interpreting straight line graphs is now applied to representing linear inequalities graphically. The concept of cumulative frequency is introduced, leading on to representing this information in a diagram. The information given by this diagram is then discussed and used, leading on to comparing distributions. The use of standard form to write very large and very small numbers is taught. Learners learn to convert numbers to and from this form and to use it in calculations, both with and without a calculator. First, trigonometrical ratios are introduced, leading on to solving right-angled triangle problems including trigonometry and/or Pythagoras' theorem. The focus then moves to similarity, developing an understanding of the ratios there and applying them. Finally, the conditions for congruence for triangles are established.

Syllabus ref	Learning objectives	Suggested teaching activities
18 Algebraic manipulation	Factorise expressions of the form $ax + bx + kay + kby$; $a^2x^2 - b^2y^2$; $a^2 + 2ab + b^2$ $ax^2 + bx + c$.	First revise the work done in Unit 5 on factorising quadratic expressions with 1 as the coefficient of x^2. Then, in a similar way to the work that was done there in introducing factorisation, first expand and simplify some quadratic expressions such as $(3x - 1)(2x + 5)$, before establishing a method for factorising such quadratics. You may wish to combine drawing the graph of a function such as $y = x^2 - 4x + 3$ with solving $x^2 - 4x + 3 = 0$ graphically and factorising $x^2 - 4x + 3$ as a way in to the case of solving the equation algebraically compared with drawing the graph. Show the learners also that the product of two factors being zero means that one of the factors must be zero, so that they see the reasoning behind the method of solution by factorisation.
20 Solutions of equations	Solve quadratic equations by factorisation.	Give the learners practice so that they become proficient in the algebraic manipulation required. Factorising is covered in section 10.10 and solving by factorising in section 10.11: http://www.cimt.org.uk/projects/mepres/allgcse/bkb10.pdf *Maths is Fun* has an explanation of factorising and includes graphs of each function that is considered: www.mathsisfun.com/algebra/factoring-quadratics.html Online worksheets: https://mathsmadeeasy.co.uk/gcse-maths-revision/solving-quadratics-factoring/ Move on to consider some cases where an equation must be formed from information given in context, then go onto solve the equation and use the solution to solve the original problem. Don Steward has some problems that can be solved by forming and solving a quadratic equation: http://donsteward.blogspot.co.uk/2013/02/form-and-solve-quadratic.html **Extension activities:** Use graphing software such as GeoGebra or Autograph to draw some graphs. Print these out and challenge learners to match up graphs with their equations and solutions of the equation f(x)=0. Alternatively, learners could attempt problems that require them to solve a quadratic that requires rearrangement. Don Steward also some problems that require simplifying before they can be solved: http://donsteward.blogspot.co.uk/2011/09/two-root.html

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 5 **Past papers:** Jun 13 Paper 12 Q 24b, 24c Jun 13 Paper 21 Q 9 Nov 13 Paper 12 Q 17 Nov 13 Paper 21 Q 5 Jun 14 Paper 11 Q 20b Jun 14 Paper 21 Q 1c Jun 14 Paper 22 Q 2c, 2d Nov 14 Paper 11 Q 5 Nov 14 Paper 12 Q 18 Nov 14 Paper 21 Q 7c Jun 15 Paper 11 Q 17 Jun 15 Paper 12 Q 18 Jun 15 Paper 22 Q 9a Nov 15 Paper 12 Q 16a

Syllabus ref	Learning objectives	Suggested teaching activities
21 Graphical representa-tion of inequalities	Represent linear inequalities graphically. (Linear Programming problems are not included.)	Show how a number line may be used to represent in 1D the solution of an inequality such as $x > 6$ or $0 \leqslant x < 3$, using an unfilled circle to represent a boundary which is not included and a filled-in circle to represent a boundary which is included. Start representing inequalities in 2D by drawing a line such as $x = 3$. Ask your learners to identify sets of coordinates where $x > 3$ and plot these. Identify the line as the boundary between the regions $x > 3$ and $x < 3$. Do similarly for lines such as $y = 6$, $y = 4x - 1$, $x + 2y = 6$. In particular, identify for the learners the possible methods for determining which region is which, in the case of sloping lines – for instance, seeing which inequality is satisfied by the origin or another point not on the line. Give the learners practice in drawing the graphs of inequalities and in identifying the inequalities

Syllabus ref	Learning objectives	Suggested teaching activities
		represented on a given graph (for which they may have to determine the equation of a straight line, studied in Unit 3). Work on inequalities. The graphical approach is covered in section 16.1: http://www.cimt.org.uk/projects/mepres/allgcse/bkc16.pdf *Maths is Fun* has work on graphing inequalities: www.mathsisfun.com/algebra/graphing-linear-inequalities.html **Extension activities:** Ask learners to identify co-ordinates (x, y) that satisfy a set of inequalities, where x and y are integers.

Past papers and specimen papers

Specimen paper:
2018 Specimen Paper 1 Q 12

Past papers:
Jun 13 Paper 12 Q 5
Nov 13 Paper 11 Q 17
Nov 13 Paper 12 Q 15
Jun 14 Paper 12 Q 15
Nov 14 Paper 12 Q 23
Jun 15 Paper 11 Q 21
Jun 15 Paper 12 Q 11
Nov 15 Paper 12 Q 18
Nov 15 Paper 21 Q 5c

Syllabus ref	Learning objectives	Suggested teaching activities
41 Categorical, numerical and grouped data	Construct and use cumulative frequency diagrams. Estimate and interpret the median, percentiles, quartiles and interquartile range for cumulative frequency diagrams.	Start with a frequency table for a grouped distribution (learners' own data from previous statistics work could be used here) and take the opportunity to revise how to find the mean of such a distribution, using the mid-points of the intervals, and how to draw a frequency polygon to represent the distribution. Then combine the frequencies for the first two classes and ask what information is given by this, so helping learners to see that cumulative frequency gives information about values less than the upper bound of each class, and so the diagram must be plotted using upper bounds. Draw cumulative frequency diagrams and read information from them, showing how the median can be obtained directly and how to interpret the varying steepness of the graph. Introduce the concepts of percentiles, quartiles and interquartile range. Give learners practice in obtaining and using these to compare distributions. Work on cumulative frequency at section 9.5: http://www.cimt.org.uk/projects/mepres/allgcse/bkb9.pdf Mathematics teacher K Pitchford describes an unusual (but effective) method for teaching quartiles: http://mathssandpit.co.uk/blog/?p=1145 Olympic weights – a handling data rich task from mathematics teacher Adam Briggs that involves cumulative frequency and also box and whisker diagrams: www.ncetm.org.uk/public/files/411886/Olympic+Weights+-+A+Handling+Data+Rich+Task.doc **(F)** **Extension activities:** Ask learners to consider how cumulative frequency data can be used to estimate probabilities. Why would this only be an estimate? How reliable would the estimate be?

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 8 **Past papers:** Jun 13 Paper 11 Q 18 Jun 13 Paper 12 Q 21 Nov 13 Paper 12 Q 14 Nov 13 Paper 21 Q 4c Nov 14 Paper 11 Q 22 Nov 14 Paper 22 Q 11b Jun 15 Paper 12 Q 20 Jun 15 Paper 21 Q 10c Nov 15 Paper 12 Q 20

Syllabus ref	Learning objectives	Suggested teaching activities
7 Standard form	Use the standard form $A \times 10^n$ where n is a positive or negative integer and $1 \leqslant A < 10$.	Show learners the statement 7000 = 7 x 10^3 and ask them if this is true. How do they know? Can they give another example like this one? Look at diameter of Sun: approximately 1 400 000 km. How can we rewrite this? 1.4 x 10^6. Explain that this method used to deal with very large or very small numbers. Astronomy uses large distances and data using stars is a good source of large numbers. Use this opportunity to define standard form and discuss the correct notation. The scale of the universe is an interactive app that shows a variety of objects, varying from astronomical bodies to sub-atomic particles. Click on images to reveal more information, including measurements in standard form: http://htwins.net/scale2/ Alternatively, you could use *Estimating Length Using Standard Form*, a resource from the UK Department for Education's Standards Unit, to introduce the topic. This asks learners to estimate lengths of a variety of objects using standard form: www.nationalstemcentre.org.uk/elibrary/resource/1961/estimating-length-using-standard-form-n4 **(F)** Practise conversion to and from standard form. **(I)** Show the learners how to use their calculators to calculate with standard form and how to use the laws of

Syllabus ref	Learning objectives	Suggested teaching activities
		indices when possible to simplify standard form calculations without a calculator. Learners should practise solving a variety of problems involving numbers written in standard form. Section 1.7 is about standard form: http://www.cimt.org.uk/projects/mepres/allgcse/bka1.pdf **Extension activities:** Learners research sizes of very large or very small objects and produce some comparisons, for example, how many trips to the moon would be required in order to cover the same distance as travelling from the Earth to one of the outer planets.
Past papers and specimen papers		
Specimen paper: 2018 Specimen Paper 1 Q 4 **Past papers:** Jun 13 Paper 11 Q 17 Jun 13 Paper 12 Q 18 Nov 13 Paper 11 Q 6 Nov 13 Paper 12 Q 6 Jun 14 Paper 12 Q 5 Nov 14 Paper 11 Q 19 Nov 14 Paper 12 Q 12 Jun 15 Paper 11 Q 5 Jun 15 Paper 12 Q 9 Nov 15 Paper 11 Q 16 Nov 15 Paper 12 Q 10		

Syllabus ref	Learning objectives	Suggested teaching activities
28 Geometrical terms **30 Similarity**	Use and interpret the geometrical terms similarity and congruence. Solve problems and give	One starting point is to tell learners you have a sketch of a triangle, ask them to identify the minimum amount of information they would need to be given in order to draw the triangle themselves. Use this to lead into a discussion about the requirements for congruence in triangles. Move on to ensure that learners can write a set of logical statements, including an appropriate congruence

Syllabus ref	Learning objectives	Suggested teaching activities
and congruence	simple explanations involving similarity and congruence. Calculate lengths of similar figures.	condition, so that they can prove two triangles are congruent. Give learners practice in setting out explanations showing congruence. Work on congruence: https://mathsmadeeasy.co.uk/gcse-maths-revision/congruent-shapes-gcse-revision-and-worksheets/ An activity from William Emeny which combines constructions with congruence conditions to create a tangram puzzle: www.greatmathsteachingideas.com/2010/11/04/tangram-constructions-fun/ **(F)** Use the work on enlargements in Unit 5 to find unknown sides in similar triangles, via using the scale factor of enlargement. Derive the ratios for similar triangles, relating them to the scale factor and showing how other ratios can be obtained. Give learners practice in finding missing lengths in similar shapes. Work on similarity: https://www.tes.com/teaching-resource/similar-shapes-worksheet-6192956 Please note – at this stage learners only need to compare lengths in similar shapes. Volumes and areas will be considered in Unit 8. **Extension activities:** Learners generate some congruence conditions for other shapes, for example parallelograms.
Past papers and specimen papers		

Syllabus ref	Learning objectives	Suggested teaching activities
Specimen papers: 2018 Specimen Paper 1 Q 13 2018 Specimen Paper 2 Q 8a **Past papers:** Jun 13 Paper 21 Q 2c Nov 13 Paper 21 Q 7ai Jun 14 Paper 21 Q 8 Nov 14 Paper 12 Q 13 Jun 15 Paper 12 Q 10 Nov 15 Paper 12 Q 13		

Syllabus ref	Learning objectives	Suggested teaching activities
36 Trigono-metry	Apply the sine, cosine and tangent ratios for acute angles to the calculation of a side or an angle of a right-angled triangle (angles will be quoted in, and answers required in, degrees and decimals of a degree to one decimal place). Solve trigonometrical problems in two dimensions including those involving angles of elevation and depression and bearings.	Ask learners to draw a right-angled triangle with an angle of 40° and adjacent side of 2, 4 or 5 cm and to measure the opposite side and compare the results (revising their knowledge of enlargement from Unit 5 and reviewing the idea of similarity). Apply their ideas to finding the opposite side when the adjacent is 1 cm and defining this length in cm as tan 40° and show the length they have drawn is consistent with this value on the calculator. Use enlargement to show this is opposite ÷ adjacent for all their triangles drawn. Repeat the process with one or two other angles and then show how tangent ratios can be used to find unknown sides and then angles in right-angled triangles. Similarly define cosine and sine ratios and use all three ratios. Include solving problems where the learners need to interpret the situation, sketching a diagram to show the right-angled triangle needed. Include problems requiring the use of Pythagoras' theorem as well as trigonometry (studied in Unit 3). Give practice in solving problems. **Extension activities:** Learners try some simple problems in 3 dimensions, for example, finding angles in a pyramid or triangular prism. **Learning resources:** Interactive pages on basic trigonometry: www.catcode.com/trig/index.html

Syllabus ref	Learning objectives	Suggested teaching activities
		Maths Is Fun also has an introduction to trigonometry, although this goes beyond the requirements for this unit: www.mathsisfun.com/algebra/trigonometry.html A chapter about trigonometry and sections 4.4 to 4.7 are suitable for this unit, together with work on Pythagoras, theorem from section 4.3: http://www.cimt.org.uk/projects/mepres/allgcse/bka4.pdf

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 2 Q 4a **Past papers:** Jun 13 Paper 11 Q 8b Jun 13 Paper 21 Q 6a Nov 13 Paper 21 Q 6 Nov 14 Paper 21 Q 8b Nov 14 Paper 22 Q 4b Jun 15 Paper 21 Q 2, 5

Unit 7: Indices, graphing inequalities, probability, sequences, circles

Recommended prior knowledge

- Work with indices, squares and square roots (see Units 1 and 5)
- Solve quadratic equations by factorising (see Units 5 and 6)
- Manipulate algebra (see Units 1 – 3)
- Work with square roots (see Unit 1)
- Find simple probabilities (see Unit 4)
- Calculate with fractions and decimals (see Unit 2)
- Find areas and circumferences of circles (see Unit 4)
- Find volumes of prisms (see Unit 5)
- Use symmetry properties of circles (see Unit 4)

Context

Work on solving quadratic equations studied in Unit 6 is extended to include equations which do not factorise. Work on indices is extended to include fractional indices. Elementary ideas of probability were studied in Unit 4; this unit moves on to considering the probabilities of combined events. The topic of sequences involves both number and algebra, as number patterns are explored and generalised. The work on linear sequences could be studied earlier, after Unit 3. The remaining work in the syllabus about circles is covered in this unit.

Outline

Revision of solving quadratic equations which factorise leads on to the new methods required to solve those which do not factorise. The laws of indices are applied to find the meaning of fractional indices and these are then used to solve a variety of problems. Possibility spaces and tree diagrams provide the basis for the introduction of probabilities of combined events. Linear number sequences are studied first, progressing to explore other sequences. From knowing how to find the circumference and area of whole circles, studied in Unit 4, learners move on to calculating areas and arc lengths for sectors, where the angle is given in degrees. Angle properties involving symmetry were also met in Unit 4, and learners now meet the remaining angle properties of the circle and use the full range of these angle properties to solve problems.

Syllabus ref	Learning objectives	Suggested teaching activities
19 Indices	Use and interpret fractional indices.	Remind learners of the work done on other indices in Unit 5. Use the laws of indices to discuss the meaning of an index such as ½ by considering $a^{1/2} \times a^{1/2} = a^1$ etc. Move on to consider fractions with numerators other than one by applying the result $(x^m)^n = x^{mn}$, for example writing $8^{2/3}$ as $(8^{1/3})^2$. Ask learners how they would extend this process to evaluate $8^{-2/3}$. Practise evaluating numerical expressions containing roots and indices. Ensure that learners understand the

Syllabus ref	Learning objectives	Suggested teaching activities
		difference between being asked to simplify and being asked to evaluate. Progress to simplifying algebraic expressions containing roots and indices. Learners also need to solve simple equations involving indices, for example $64^b=4$ **Extension activities:** Ask learners to use some results they are given to evaluate a square or cube root without a calculator. For example, if told that ask them to evaluate to 2 decimal places. (They should already know that which will allow them to tackle the problem by writing 0.006 as 6/1000). Alternatively, learners could apply what they have learned about indices to simplify expressions written using square or cube roots. **Further resources:** Work on indices, including fractional indices in section 1.6: http://www.cimt.org.uk/projects/mepres/allgcse/bka1.pdf *Using Indices*, a resource from the UK Department for Education's Standards Unit, introduces fractional indices: www.nationalstemcentre.org.uk/elibrary/resource/1969/using-indices-n12 **(F)** Some of the activities in *Manipulating Surds*, a resource from the UK Department for Education's Standards Unit, are suitable for this level: www.nationalstemcentre.org.uk/elibrary/resource/1968/... **(F)**

Past papers and specimen papers

Specimen paper:
2018 Specimen Paper 1 Q 18

Past papers:
Jun 13 Paper 11 Q 20
Jun 13 Paper 12 Q 16
Nov 13 Paper 11 Q 8
Nov 13 Paper 12 Q 12
Jun 14 Paper 11 Q 13
Jun 14 Paper 12 Q 19
Jun 15 Paper 11 Q 12
Nov 15 Paper 11 Q 14

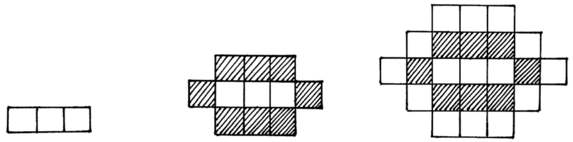
Syllabus ref	Learning objectives	Suggested teaching activities
20 Solutions of equations and inequalities	Solve quadratic equations either by use of the formula or by completing the square.	Remind learners how to solve quadratic equations which factorise, by giving them a few to solve, including as the last one an equation which does not factorise. Use this to discuss the methods available for solving such an equation, reminding them of the graphical methods used in Unit 5 to obtain approximate solutions. Then introduce the methods of using the formula or completing the square, as methods which give the exact solution of a quadratic equation. Learners need to practise all of the methods for solving quadratics and consider when to use each one. **Extension activities:** Learners compare exact solutions of equations from algebraic methods with the solutions obtained from graphical methods and comment on the difference. **Further resources:** Work on solving quadratics in sections 10.12 and 10.14: http://www.cimt.org.uk/projects/mepres/allgcse/bkb10.pdf Mathematics teacher William Emeny has a card-sorting task that summarises all methods for solving quadratics and asks learners to consider when each method should be used: www.greatmathsteachingideas.com/2012/02/16/quadratic-equations-the-main-ideas-a-card-sort-to-support-conceptual-understanding/ **(F)**

Past papers and specimen papers
Past papers: Jun 13 Paper 12 Q 24d Jun 13 Paper 21 Q 7 Jun 13 Paper 22 Q 8 Nov 13 Paper 22 Q 11b Jun 14 Paper 11 Q 20 Jun 14 Paper 21 Q 11 Jun 14 Paper 22 Q 10 Nov 14 Paper 22 Q 8d Jun 15 Paper 22 Q 9c Nov 15 Paper 22 Q 6

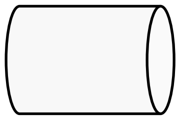
Syllabus ref	Learning objectives	Suggested teaching activities
40 Probability	Calculate the probability of simple combined events using possibility diagrams and tree diagrams where appropriate. (In possibility diagrams outcomes will be represented by points on a grid, and in tree diagrams outcomes will be written at the end of branches and probabilities by the side of the branches.)	Build on the work of Unit 4 and start with a simple event such as the score on a spinner; ask for the probability of the total score being a certain number when the spinner is spun twice. Discuss with the learners strategies such as listing outcomes and show them how to represent the outcomes on a possibility diagram. To introduce tree diagrams, use an example such as choosing balls at random from a bag, when there are different numbers of balls of different colours. First do examples where the first ball is replaced and a second ball chosen. Later, to introduce conditional probability, do an example where the first ball is not replaced and so the probabilities on the second branches are changed. Use reasoning such as finding a fraction of a fraction to help learners see when to multiply and when to add combined probabilities – this can be accompanied by listing outcomes using suffix notation where appropriate (such as R_1, R_2, R_3 where there are three red balls) to obtain a list of equally likely outcomes. At first, give learners examples which require them to complete drawn tree diagrams and calculate probabilities. Then progress to learners drawing their own tree diagrams to represent situations. Finally, include examples where a tree diagram is not required (although learners may wish to draw it) but where learners can solve the problem using logic and addition / multiplication of probabilities as appropriate. **(F)** **Extension activities:** Learners solve problems where the final probability of an outcome is given they need to work backwards from this to work out information such as how many counters of each colour were in a bag. **Further resources:** Suitable work on probability in section 5.3 onwards: https://www.cimt.org.uk/projects/mepres/allgcse/bka5.pdf *Developing an Exam Question: Probability*, a resource from the UK Department for Education's Standards Unit, involves learners in using and analysing past examination questions as they review key ideas about probability: www.nationalstemcentre.org.uk/elibrary/resource/2057/developing-an-exam-question-probability-s7

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 2 Q 2e **Past papers:** Jun 13 Paper 21 Q 4a Jun 13 Paper 22 Q 10a Nov 13 Paper 11 Q 20 Nov 13 Paper 12 Q 21 Jun 14 Paper 11 Q 24 Jun 14 Paper 12 Q 21 Jun 14 Paper 22 Q 4 Nov 14 Paper 12 Q 21 Nov 14 Paper 22 Q 3 Jun 15 Paper 11 Q 25 Jun 15 Paper 21 Q 10a Jun 15 Paper 22 Q 11a Nov 15 Paper 11 Q 7 Nov 15 Paper 12 Q 23

Syllabus ref	Learning objectives	Suggested teaching activities
22 Sequences	Continue a given number sequence; recognise patterns in sequences and relationships between different sequences (includes linear sequences, quadratic and cubic sequences, exponential sequences and simple combinations of these). Generalise sequences as simple algebraic statements (including expressions for the *n*th term).	Start with some simple linear sequences, expressed in numbers or in patterns using matchsticks, etc. Ask learners to explain the patterns and how to find the next term, the 10th term, etc. leading on to finding the *n*th term. Make comparisons with gradient and the equation for a straight line. Lead on to other basic number patterns such as squares, cubes, powers of 2 and triangle numbers before other quadratic sequences. Consider examples where one sequence can be generated from another, such as starting with the square number sequence, then showing learners a sequence that is $2n^2$ and asking them to look for connections. Look at second differences for quadratic sequences, etc. and how these or simultaneous equations can be used to find the *n*th term. Alternatively, learners can approach quadratics by using the general form of the *n*th term, $an^2 + bn + c$, substituting n=1, n=2, etc. and building up patterns of differences: 1st term: 2nd term: 3rd term: $a + b + c$ $4a + 2b + c$ $9a + 3b + c$ etc. 1st differences: $3a + b$ $5a + b$ $7a + b$ etc. 2nd differences: $2a$ $2a$ etc. Learners can compare these with the results for a particular sequence in order to find values for *a*, *b* and *c* and consequently the *n*th term. For an activity using sequences, learners could explore the number of games, rounds, etc. needed for any number of competitors for a knockout tournament such as the Wimbledon tennis tournament. Another practical context comes from engineering. Engineers use v-blocks for a variety of purposes, including counting large numbers of work pieces quickly. If an engineer's v-block is available, learners can use this to generate a sequence of triangular numbers in context, perhaps using pencils. A (slightly blurry but usable) image showing a v-block holding drills is available here: www.flickr.com/photos/morethanmaths/2263004958 This could also be done using diagrams and provides a practical context for a sequence of numbers. Alternatively, learners could work on an investigation such as 'surrounded' where learners surround a shape with layers of squares, as shown below:

Syllabus ref	Learning objectives	Suggested teaching activities
		 starting shape 1st layer 2nd layer This can generate a variety of sequences of different types, by considering numbers of squares in each layer, or total number of squares at each stage, or even extending the investigation into 3D, by surrounding a cuboid with layers of cubes. **Extension activities:** Learners can use the *n*th term to solve problems. For example, if they are told that the first and second terms of a sequence with the *n*th term ax^2+b/x are 7 and 14, they should be able to form and solve equations in order to find the values of *a* and *b*. Alternatively, learners could investigate or research sequences such as the Fibonacci sequence. **Further resources:** A chapter about number patterns: http://www.cimt.org.uk/projects/mepres/allgcse/bkb12.pdf The Nrich site has many puzzles about sequences, such as this one about triangle numbers: http://nrich.maths.org/2274/ *Analysing Sequences*, a resource from the UK Department for Education's Standards Unit, is intended for learners working at A Level, but may be adapted to suit learners taking a lower level qualification: www.nationalstemcentre.org.uk/elibrary/resource/1970/analysing-sequences-n13

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 17 **Past papers:** Jun 13 Paper 21 Q 3 Jun 13 Paper 22 Q 7 Nov 13 Paper 11 Q 24 Nov 13 Paper 12 Q 23 Jun 14 Paper 11 Q 12 Jun 14 Paper 22 Q 6 Nov 14 Paper 11 Q 18 Nov 14 Paper 12 Q 11 Jun 15 Paper 11 Q 26 Jun 15 Paper 12 Q 26 Nov 15 Paper 11 Q 18 Nov 15 Paper 12 Q 24

Syllabus ref	Learning objectives	Suggested teaching activities
35 Mensuration	Solve problems involving arc length and sector area as fractions of the circumference and area of a circle.	Ask learners to find the area of a whole circle and then a fraction of it, progressing quickly on to find the area of a sector of any angle; similarly for calculating arc length. Learners should be able to give answers in terms of pi. Where tasks are in context, answers should be to a suitable degree of accuracy. Include also some problems using fractions of cylinders, such as finding the volume of a piece of cheese whose cross-section is a sector. **(F)** **Extension activities:** Investigating practical problems can provide a significant challenge, for example, designing a dipstick for a cylindrical tank:  Dan Meyer's cheese cutting problem is a rich and challenging task.: http://threeacts.mrmeyer.com/luckycow/

Syllabus ref	Learning objectives	Suggested teaching activities
		Learners should be able to apply their ideas to making nets of cones or frustums. Asking learners to make a conical party hat of a given height that will fit the circumference of their head is a fun but challenging activity that also involves some trigonometry in 3D (this is introduced formally in the next unit). **(F)**

Past papers and specimen papers

Past papers:
Jun 13 Paper 11 Q 19
Jun 13 Paper 12 Q 17
Nov 13 Paper 21 Q 10
Nov 13 Paper 22 Q 8
Jun 14 Paper 11 Q 7
Nov 14 Paper 21 Q 10b
Nov 14 Paper 22 Q 5c
Jun 15 Paper 21 Q 8
Jun 15 Paper 22 Q 4b
Nov 15 Paper 22 Q 8

Syllabus ref	Learning objectives	Suggested teaching activities
32 Angles	Calculate unknown angles and give simple explanations using the following geometrical properties: (f) angle in a semi-circle (h) angle at the centre of a circle is twice the angle at the circumference (i) angles in the same segment are equal (j) angles in opposite segments are supplementary.	If a dynamic geometry computer package such as GeoGebra, Geometer's Sketchpad or Cabri-géomètre is available, this may be used to demonstrate these angle properties, such as varying the angle at the centre and showing that the angle at the circumference is always half its value. If it is not available, learners can draw and measure angles to see the relationships. Proofs are not required of these angle properties, but it is worthwhile to show learners how they can be obtained, using the properties of isosceles triangles and the exterior angle to a triangle, etc. Give learners practice at calculating unknown angles in circle diagrams (using also the work done in Unit 4) and in giving reasons for how they obtained their answers. **(F)** Emphasise that learners are expected to be able to state reasons such as "the angle in a semicircle is 90 degrees" and that these need to be stated using correct mathematical language and naming angles in an unambiguous way. A useful activity to practise this is to give learners a diagram and ask them to write a description of it: for example, angle APB=AQB, etc. then pass the description to another learner who has to reproduce the diagram. If the original description was not sufficiently clear or accurate, the learner drawing the diagram should challenge the one who wrote the description to improve it. **(F)** **Extension activities:** Questions that have a high level of difficulty (such as those requiring several steps in order to reach a solution) can provide a suitable level of challenge. Alternatively, learners could be given a diagram and asked to deduce the minimum information that they would need to provide in order for someone else to be able to deduce a given length or angle. **(F)** **Further resources:** Work on angles and circles in sections 3.8 and 3.9: http://www.cimt.org.uk/projects/mepres/allgcse/bka3.pdf A set of GeoGebra applets for demonstrating circle theorems. These could be used by the teacher or by learners: www.tes.co.uk/teaching-resource/Demonstrations-of-Circle-theorems-6117839/ 'Discovering circle theorems', an activity from the National Centre for Excellence in Teaching Mathematics (NCETM): www.ncetm.org.uk/public/files/280228/NCETM_Mathematics_Department_Workshops_Circle_Theorems_Resource_Sheet_HT1.CIR.2.pdf
Past papers and specimen papers		

Syllabus ref	Learning objectives	Suggested teaching activities
Specimen paper: 2018 Specimen Paper 1 Q 25 **Past papers:** Jun 13 Paper 11 Q 14 Jun 13 Paper 12 Q 23 Nov 13 Paper 11 Q 22 Nov 13 Paper 12 Q 22 Jun 14 Paper 21 Q 4 Jun 14 Paper 22 Q 11 Nov 14 Paper 12 Q 20 Nov 14 Paper 21 Q 10a Nov 14 Paper 22 Q 2 Jun 15 Paper 11 Q 19 Jun 15 Paper 21 Q 3b Jun 15 Paper 22 Q 4a Nov 15 Paper 11 Q 24 Nov 15 Paper 12 Q 17 Nov 15 Paper 22 Q 3a		

Unit 8: More complex graphs and formulae, accuracy, cones, pyramids and spheres

Recommended prior knowledge

- Constructing and interpreting graphs (see Units 3 and 5)
- Rounding (see Unit 3)
- Areas of circles and sectors (see Units 4 and 7)
- Similarity in 2 dimensional shapes (see Unit 6)
- Trigonometry in right-angled triangles (see Unit 6)

Context

Graphical work from Unit 5 is extended to include cubic, reciprocal and exponential functions and applying rates of change graphically. Earlier work on linear graphs is developed to consider parallel and perpendicular lines. Earlier work on changing the subject of simple formulae is extended to include more complex formulae. Earlier work on rounding is extended to discuss the limits of accuracy, with the use of upper and lower bounds to problems. This section could be studied earlier in the course if wished. Three-dimensional work on symmetry, surface area and volume is extended to include cones and other pyramids and spheres. Trigonometry in right-angled triangles and similarity are also applied in three dimensions.

Outline

Learners draw graphs in each of the forms cubic, reciprocal and exponential, to gain familiarity with the properties of each of these types of curve. The graphs are used to solve equations, including, where necessary, drawing an appropriate line on the graph. Graphs are also used to estimate the gradient of a curve by drawing a tangent and in application to distance-time and speed-time graphs. The equations of lines that are parallel or perpendicular to another line are explored and the gradient properties are used to draw these. Finally, algebraic manipulation skills are extended by learning how to transform more complex formulae. When discussing the limits of accuracy, first the upper and lower bounds for data given to specified accuracy are identified. Then these bounds are used in calculations to find the upper and lower bounds of solutions to problems. Work on sectors of circles, studied in Unit 7, is applied to forming a cone and showing how the surface area formula is obtained. Symmetry properties and problems involving surface area and volume are studied for cones, then other pyramids and spheres. Similarity is applied to finding areas and volumes of similar shapes and solids. Trigonometry in right-angled triangles, used to solve two-dimensional problems in Unit 6, is now applied in three-dimensions.

Syllabus ref	Learning objectives	Suggested teaching activities
25 Graphs of functions	Construct tables of values and draw graphs for functions of the form $y = ax^n$ where $n = -2, -1, 0, 1, 2, 3$ and simple sums of not more than three of these and for functions of the form $y = ka^x$ where a is a positive integer. Interpret graphs of reciprocal and exponential functions. Solve equations approximately by graphical methods; estimate gradients of curves by drawing tangents.	Use ICT if available, or let learners work in groups to produce graphs of form $y = ax^n$ where $n = -2, -1, 0, 1, 2, 3$ and simple sums of these to compare and learn to recognise the shapes of these. For exponential graphs, you may wish to make links with exponential sequences studied in Unit 7, and to interpret exponential graphs in terms of growth. Discuss the gradient of a chord of a curve and define the gradient of a curve as the gradient of the tangent to the curve at that point. Practice drawing tangents 'by eye' and using them to estimate the gradient of a curve (see also the next section of this unit). **(I)** Draw a graph such as a quadratic or cubic and use it to solve equations, including cases where a straight line such as $y = 4x - 1$ needs to be drawn to solve the appropriate equation. Many graphing programs accept inputs using function notation. This could be used to introduce learners to the idea of function notation during graphical work in preparation for further work in Unit 10. **Extension activities:** Learners match a set of graphs to their equations by considering the shape of the graph and properties such as *y*-intercepts. **Further resources:** Work on plotting graphs of curves at section 13.4, identifying common functions at section 13.11 and using graphs to solve equations at section 13.12: https://www.cimt.org.uk/projects/mepres/allgcse/bkc13.pdf *Interpreting Functions, Graphs and Tables*, a resource from the UK Department for Education's Standards Unit, has materials for developing learners understanding of the links between algebraic, tabular and graphical representations of a variety of functions: www.nationalstemcentre.org.uk/elibrary/resource/2006/interpreting-functions-graphs-and-tables-a7 Online graphical calculator: www.desmos.com/ Computer programs such as GeoGebra, Autograph or Omnigraph are useful for using graphs to solve equations, as well as for drawing graphs.
27 Coordinate geometry	Determine the equation of a straight line parallel to a given line, e.g. find the equation of a	Ask learners to plot sets of graphs on a graphics calculator or in a graphing package with the same gradient but different intercepts / constants. Ask learners to deduce that parallel lines have the same gradient and to say how this relates to the equations used.

Syllabus ref	Learning objectives	Suggested teaching activities
	line parallel to $y = 4x - 1$ that passes through (0, -3). Find the gradient of parallel and perpendicular lines, e.g. find the gradient of a line perpendicular to $y = 3x + 1$; find the equation of a line perpendicular to one passing through the coordinates (1, 3) and (–2, –9).	Give learners a pairs of lines that intersect and ask them to suggest two other lines that would enclose a parallelogram. Collect in results from learners to get a number of different possible answers, but show that all the results contain two sets of parallel lines. Ask learners to plot a number of rhombuses and kites and to find the equations of the diagonals – these will need to be extended to cross the *y* axis. Ask them to deduce what happens to the equations of lines that are perpendicular to one another to establish the rule. The Nrich task Parallel Lines is a useful interactive tool: http://nrich.maths.org/5609 Give practice in finding equations of parallel lines, including examples that pass through particular points. Move on to consider perpendicular lines. Learners could draw these on paper or using a graphing package such as GeoGebra. They should then find the gradients and equations of their lines before deducing the connection between the gradients of perpendicular lines. Alternatively, the Nrich task Perpendicular Lines: http://nrich.maths.org/5610 could be used to support your learners' investigation of perpendicular lines. *Connecting Perpendicular Lines*, a resource from the UK Department for Education's Standards Unit, asks learners to use graphical calculators or graphing software to investigate perpendicular lines: www.nationalstemcentre.org.uk/elibrary/resource/2008/connecting-perpendicular-lines-a10 **(I)** Ensure that learners are able to find an equation of a line that is parallel to another line or perpendicular to it that goes through a particular point, by deducing the gradient and substituting the point to find the constant. **(F)** **Extension activities:** Learners attempt to use graphing software such as GeoGebra, or the online graphical calculator at Desmos.com, to build up a picture from line segments. If this is not possible, a similar task could be attempted using pencil and paper methods. These tasks from Nrich are also useful: Enclosing Squares: http://nrich.maths.org/763 and Painting Between the Lines: http://nrich.maths.org/7031 **Learning resources** Work on straight line graphs: https://www.cimt.org.uk/projects/mepres/book8/bk8_14.pdf

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 2 Q 3, 6 **Past papers:** Jun 13 Paper 11 Q 23 Jun 13 Paper 21 Q 10 Jun 13 Paper 22 Q 11 Nov 13 Paper 21 Q 9 Nov 13 Paper 22 Q 9 Jun 14 Paper 21 Q 7 Nov 14 Paper 11 Q 20 Jun 15 Paper 21 Q 9 Nov 15 Paper 12 Q 22 Nov 15 Paper 22 Q 9

Syllabus ref	Learning objectives	Suggested teaching activities
24 Graphs in practical situations	Apply the idea of rate of change to easy kinematics involving distance-time and speed-time graphs, acceleration and retardation. Calculate distance travelled as area under a linear speed-time graph.	Remind learners of their work using travel graphs in Unit 4. Extend the work on speed to include curved distance-time graphs, drawing tangents to the curve to estimate the speed. Introduce a straight-line velocity-time graph and discuss the meaning of its gradient, acceleration. Show that distance travelled is the area under a horizontal speed-time graph and then extend this to the area under any straight line speed-time graph (for instance by considering the trapezium as the average of the two rectangles using initial and final speeds). Give practice in solving problems. **(I)** Work on velocity-time graphs: http://online.cctt.org/physicslab/content/Phy1/lessonnotes/constantvelocity/lessonvelocitygraphs.asp **Extension activities:** Ask learners to use data from a cycle or motor race to draw graphs and calculate appropriate rates of change. This could produce some very complex graphs, so it may be best to ask

Syllabus ref	Learning objectives	Suggested teaching activities
		learners to work in groups and share the task between them.

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 26 **Past papers:** Jun 13 Paper 11 Q 13 Jun 13 Paper 12 Q 22 Nov 13 Paper 11 Q 23 Nov 13 Paper 12 Q 25 Jun 14 Paper 11 Q 18 Jun 14 Paper 12 Q 20 Nov 14 Paper 12 Q 22 Jun 15 Paper 22 Q 2 Nov 15 Paper 11 Q 20 Nov 15 Paper 12 Q 25

Syllabus ref	Learning objectives	Suggested teaching activities
17 Algebraic representa-tion and formulae	Transform more complicated formulae.	Revise earlier work from Unit 5 on transforming simple formulae and extend this to transform formulae involving powers and roots or where the new subject appears more than once. Where possible, use formulae that learners have met elsewhere in their studies, for instance in Physics. **Extension activities:** Many questions involving distance-time graphs generate formulae that involve fractions (these are covered more fully in Unit 10). Learners could be encouraged to try to form and solve equations involving two vehicles travelling a given distance where one takes a certain amount of time longer than the other. (There are past examination questions of this type listed in the algebraic fractions topic in Unit 10.) **Further resources:** Work on transforming formulae: https://www.cimt.org.uk/projects/mepres/allgcse/bka2.pdf

Past papers and specimen papers
Past papers: Jun 14 Paper 11 Q 9 Nov 14 Paper 22 Q 8a Nov 15 Paper 11 Q 15

Syllabus ref	Learning objectives	Suggested teaching activities
10 Limits of accuracy	Give appropriate upper and lower bounds for data given to a specified accuracy (e.g. measured lengths). Obtain appropriate upper and lower bounds to solutions of simple problems (e.g. the calculation of the perimeter or area of a triangle) given data to a specified accuracy.	Discuss a problem such as 'Will a piece of furniture 550 mm wide fit in a space 550 mm wide?' Include the need for specified accuracy of the data (in this example to the nearest mm or the nearest cm) and the definitions of upper and lower bounds. Ensure that learners are clear that whilst bounds are linked to ideas of rounding, they are boundary values, not values that could be rounded to a given value, hence bounds should not be written using recurring decimals. Extend examples such as this to include more than one piece of furniture fitting in another space, discussing the maximum and minimum total widths and the difference between their widths, again defining upper and lower bounds. Progress to problems needing multiplication or division, such as area or speed. **Extension activities:** Learners attempt problems with multiple steps, selecting the appropriate bound at each stage. **Further resources:** Examples of bounds on slides 14.5 and 14.6: http://www.cimt.org.uk/projects/mepres/book9/y9s14os.pdf (A useful hint – to display a PDF file full screen, like a PowerPoint presentation, hold down the Control key and press L.) Don Steward has a set of images and problems related to bounds: http://donsteward.blogspot.co.uk/2013/12/highest-and-lowest-bounds.html **(F)**

Past papers and specimen papers

Syllabus ref	Learning objectives	Suggested teaching activities
Specimen paper: 2018 Specimen Paper 2 Q 4b **Past papers:** Jun 13 Paper 21 Q 6b Jun 13 Paper 22 Q 5 Nov 13 Paper 11 Q 5 Nov 13 Paper 12 Q 11 Jun 14 Paper 12 Q 14 Jun 14 Paper 21 Q 6b Nov 14 Paper 12 Q 9 Nov 14 Paper 21 Q 4b Nov 15 Paper 11 Q 11 Nov 15 Paper 12 Q 9		

Syllabus ref	Learning objectives	Suggested teaching activities
31 **Symmetry** **28** **Geometrical terms** **35** **Mensuration**	Recognise symmetry properties of the pyramid (including cone). Use and interpret vocabulary of simple solid figures: pyramid, cone, sphere. Solve problems involving the surface area and volume of a sphere, pyramid and cone (formulae will be given for the surface area and volume of the sphere, pyramid and cone).	Ask learners to construct hollow cones from a sector of a circle. Have the same radii for the circles but ask them to choose different sector angles. Ask them to investigate how the base radius of the cone is related to their choice and how the surface area of the cone changes. Derive the formula $A = \pi rl$ from their results. Construct a square-based and a rectangular-based pyramid with vertex above the centre of the base. Use these and the cone models to discuss the symmetries of pyramids. Demonstrate the general formula for the volume of a pyramid by showing how three square-based pyramids with the vertex above a corner of the base fit together to make a cube. (Discuss also the symmetry properties of such a pyramid.) Give the learners the general formula for any pyramid's volume and apply this to cones and other pyramids. Use the formulae for the volume and surface area of a sphere and solve problems involving any of the above solids. **(F)** **Extension activities:** Working with compound shapes, for example finding the volume of a toy made from a cone joined to a hemisphere, can provide a high level of challenge.

Syllabus ref	Learning objectives	Suggested teaching activities
		Further resources: Work on symmetry properties is at section 3.6: http://www.cimt.org.uk/projects/mepres/allgcse/bka3.pdf **Extension activity:** Don Steward has a challenging task that involves finding volumes of a set of containers: http://donsteward.blogspot.co.uk/2012/04/tubs.html
Past papers and specimen papers		
Past papers: Nov 13 Paper 11 Q 19 Nov 13 Paper 12 Q 16 Jun 14 Paper 11 Q 23 Jun 14 Paper 22 Q 9 Nov 14 Paper 12 Q 14, 15 Jun 15 Paper 12 Q 24 Jun 15 Paper 21 Q 4 Nov 15 Paper 21 Q 8a Nov 15 Paper 22 Q 4		

Syllabus ref	Learning objectives	Suggested teaching activities
30 Similarity and congruence	Use the relationship between areas of similar triangles, with corresponding results for similar figures, and extension to volumes of similar solids.	Find in terms of π the surface area and volume of spheres of radius 2 cm and 5 cm and compare the results. Lead on to substituting *a* and *ka* for *r* in the sphere formulae and hence show that an enlargement of scale factor *k* produces an area enlargement of scale factor k^2 and volume scale factor of k^3. Alternatively, this could be approached by starting from a problem in context, for example, looking at how much air is required to fill a small soccer ball, then asking learners to calculate the requirements for a larger version. Showing a demonstration or a video clip will address any misconceptions that doubling the radius would double the amount of air required. Generalise these results for other solids and for the areas of similar shapes in two dimensions. Go on to use these facts in solving problems, incorporating revision of the work on solids.

Syllabus ref	Learning objectives	Suggested teaching activities
		Extension activities: Films often use giant insects or giant or miniaturised people. Learners could work out the weight of an insect or person that has been transformed in this way. Newspaper stories about world record attempts involving giant food items could provide the starting point for an interesting task. Some pieces of artwork involve giant sized versions of everyday items. These could also form the basis of interesting questions for learners to pose and then work on. **Further resources:** Mathematics teacher Nathan Kraft has created some videos involving inflating soccer balls. These a presented in a way that should allow learners to pose their own questions and request further information for themselves: http://mrkraft.wikispaces.com/Soccer+Ball+Inflation One example of an interesting image of a large piece of artwork which could generate interesting mathematical questions is at: www.flickr.com/photos/58722587@N00/5081885214

Past papers and specimen papers
Past papers: Jun 13 Paper 11 Q 16 Jun 13 Paper 12 Q 11 Nov 13 Paper 11 Q 11 Nov 14 Paper 11 Q 17 Nov 14 Paper 12 Q 8 Jun 15 Paper 11 Q 13b

Syllabus ref	Learning objectives	Suggested teaching activities
36 Trigono-metry	Solve simple trigonometrical problems in three dimensions. (Calculations of the angle between two planes or of the angle between a straight line and plane	Use a flagpole supported by wires or similar situation to introduce a problem in three dimensions needing the use of trigonometry or Pythagoras' theorem, revising the work done in Unit 6 on this topic as necessary. Show learners how to identify the right-angled triangle required and teach them to draw a sketch of this triangle, showing the right-angle its true size to assist in the solution.

Syllabus ref	Learning objectives	Suggested teaching activities
	will not be required.)	Learners often find it difficult to visualise the triangle required. One method is to use pieces of wire or spaghetti with small pieces of poster mounting putty or non-hardening modelling clay to make 3D models and highlight the triangles that are used in solving questions. Alternatively, cardboard models with sections cut away to reveal the section required could be used. **(I)** Give practice in solving problems. **(I/F)** **Extension activities:** Ask learners to design a net for a pyramid shaped gift box with a given height and base dimension. **(I)** Alternatively, they could consider the level of accuracy for a practical problem, e.g. finding the height of a tree, building or mountain. How accurate are the measurements? What are the upper and lower bounds in each case? What effect will this have on the final answer? Giving values for sin, cos or tan that are insufficiently accurate is a common error in examinations; the results of this activity should demonstrate that small errors in measurement can have a significant impact on the result. **Further resources:** Work on 3-D geometry: http://www.cimt.org.uk/projects/mepres/allgcse/bkc18.pdf

Past papers and specimen papers
Past paper: Nov 15 Paper 22 Q 5

Unit 9: Matrices, sets, Venn diagrams, further trigonometry and vectors

Recommended prior knowledge
- Pythagoras' theorem and trigonometry in right angled triangles in 2D and 3D (see Units 3, 6 and 8)
- Using a column vector to describe a translation (see Unit 5)

Context
This unit introduces matrices. The work in this unit is applied to transformations in Unit 10 and so this topic has been left to Unit 9 in the course, although matrices could be introduced earlier in the course if desired. The unit moves on to introduce set notation and Venn diagrams. Scatter graphs are introduced for paired data. Trigonometry is extended from right-angled triangles to include acute and obtuse angled triangles. Column vectors have been met previously in Unit 5 in work on translations; the concept of vectors is now extended to cover the principal basics of vector geometry.

Outline
Rectangular, not square, matrices are used initially to help learners appreciate how to add and subtract matrices and to find the product of two matrices of appropriate order. Once learners have gained sufficient competence in these skills, they move on to consider 2 × 2 matrices and their algebra, leading to finding the determinant of a matrix and the inverse of a non-singular matrix. Knowledge and understanding of set language and Venn diagrams is gradually built up, starting with simple examples of first one then two sets. This leads on to considering combinations of the sets and using Venn diagrams and sets to solve problems. Scatter graphs for paired data are constructed and the various types of correlation examined. Lines of best fit are drawn and used to estimate values. Initially, right-angled trigonometry is used to find an unknown length in an acute angled triangle by splitting it into two right-angled triangles. The sine and cosine rules are derived and used to find unknown lengths and angles, with the sine and cosine function definitions being extended to include obtuse angles. The area formula ½ *ab* sin *C* for the area of a triangle is obtained and used. Attention then moves to vectors, starting with column vectors used in translations. From these, addition of vectors and the magnitude of vectors is discussed before moving on to more general representations of vectors as line segments, and position vectors. Sums and differences of coplanar vectors are used in geometrical problems.

Syllabus ref	Learning objectives	Suggested teaching activities
38 Matrices	Display information in the form of a matrix of any order. Solve problems involving the calculation of the sum and product (where appropriate) of two matrices and interpret the results. Calculate the product of a scalar quantity and a matrix.	Start with an example such as a 3 × 4 matrix of numbers of 4 items in a shopping order on three separate weeks, multiplied by a 4 × 1 matrix of prices. Then extend the 4 × 1 matrix to a 4 × 2 matrix to include the new prices after an increase. Use this example to discuss the layout and principles of finding the product of two matrices. You could use combining with the shopping order of a neighbour to demonstrate addition of matrices, and similarly doubling an order to demonstrate multiplication of a matrix by a scalar quantity. Apply these principles to rectangular matrices of different shapes, including using them to solve problems, before focusing more on 2 × 2 matrices. Discuss their algebra compared with the four operations using numbers, for instance, that order of multiplication matters, with **AB** being different from **BA** in general. Give the learners some products which have the identity matrix as answer, then lead on to calculating the

Syllabus ref	Learning objectives	Suggested teaching activities
	Use the algebra of 2 × 2 matrices including the zero and identity 2 × 2 matrices. Calculate the determinant and inverse of a non-singular matrix. (**A**$^{-1}$ denotes the inverse of **A**.)	determinant and inverse of a non-singular matrix. Show learners the use of inverse matrices in solving simultaneous equations (they will also be used in Unit 10 to find the coordinates of points after an inverse transformation). **Extension activities:** Learners solve algebraic problems with matrices. For example, if given two matrices where one or more of the elements are unknown but the product of these matrices is known, learners can form and solve either simple linear equations (for cases with one unknown element in a matrix) or simultaneous equations (for cases with two or more unknowns). **Further resources:** An introduction to matrix algebra: www.sosmath.com/matrix/matrix0/matrix0.html *Maths is Fun* also has work on matrices: www.mathsisfun.com/algebra/matrix-introduction.html Work on matrices is included in this text (aimed at A Level learners, but the first section is suitable for learners studying at this level): https://www.cimt.org.uk/projects/mepres/alevel/fpure_ch9.pdf

Past papers and specimen papers

Specimen paper:
2018 Specimen Paper 1 Q 10

Past papers:
Jun 13 Paper 11 Q 24a
Jun 13 Paper 12 Q 13
Nov 13 Paper 22 Q 7
Jun 14 Paper 11 Q 25
Jun 14 Paper 12 Q 12
Nov 14 Paper 11 Q 11
Nov 14 Paper 12 Q 26
Jun 15 Paper 11 Q 10, 16
Jun 15 Paper 12 Q 21
Nov 15 Paper 12 Q 6, 26
Nov 15 Paper 21 Q 3

Syllabus ref	Learning objectives	Suggested teaching activities
2 Set language and notation	Use set language and set notation, and Venn diagrams, to describe sets and represent relationships between sets as follows: Definition of sets, e.g. $A = \{x : x \text{ is a natural number}\}$ $B = \{(x, y) : y = mx + c\}$ $C = \{x : a \leqslant x \leqslant b\}$ $D = \{a, b, c...\}$ Use Venn diagrams to solve problems. Notation: • Union of A and B: $A \cup B$ • Intersection of A and B: $A \cap B$ • Number of elements in set *A*: $n(A)$ • " .. is an element of ..." $\in$ • " .. is not an element of … " $\notin$ • Complement of set *A*: A' • The empty set: $\varnothing$ • Universal set: ξ • A is a subset of B: $A \subseteq B$ • A is a proper subset of B: $A \subset B$ • A is not a subset of B: $A \nsubseteq B$ • A is not a proper subset of B: $A \not\subset B$.	Start with a list of items such as some colours, which you write in set form, as in *D* in the learning outcomes. Then write a set for colours of the rainbow. Use the brackets notation and also ask questions about elements of sets to introduce this language and the symbols $\in$, $\notin$ and $n(A)$. Alternatively, you could make sets from the learners themselves, for example, listing a group of learners, then the ones with brown hair, or who like football. Draw a Venn diagram to show the two sets, together with the universal set of colours. Use an example such as $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8, 10\}$ to draw a Venn diagram with mutually exclusive sets. This can also be done with the learners forming a human Venn diagram, standing in the appropriate place depending on whether they like football or cycling or neither as an example. Use other examples to give the range of different types of Venn diagrams and of the notation of describing the elements of a set. Give the learners practice in the language and diagrams used. Then use the diagrams drawn already to discuss the meaning of different areas of the Venn diagram and introduce the rest of the required set language and notation. Give learners practice in this, including using Venn diagrams to solve problems to find the number of elements in a set. **Extension activities:** Some problems involving Venn diagrams have a high level of difficulty, for example cases where most statements involve combinations such as "10 people had breakfast but did not do any exercise". Learners should also use Venn diagrams to deduce probabilities. **Further resources:** *Maths is Fun* has an introduction to sets and Venn diagrams: www.mathsisfun.com/sets/venn-diagrams.html Please note – this page uses the symbol U to denote the universal set. This is a widely used alternative to ξ but is not required for this syllabus. Work on set notation in chapter 1: http://assets.cambridge.org/0521539021/sample/0521539021WS.pdf Large hoops can be marked with string or chalk to create human Venn diagrams.

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 2 Q 5 **Past papers:** Jun 13 Paper 11 Q 24 Jun 13 Paper 12 Q 10 Nov 13 Paper 11 Q 14 Nov 13 Paper 22 Q 5 Jun 14 Paper 12 Q 11 Jun 14 Paper 21 Q 2 Nov 14 Paper 11 Q 10 Nov 14 Paper 12 Q 6 Jun 15 Paper 11 Q 18 Jun 15 Paper 12 Q 8 Nov 15 Paper 12 Q 15 Nov 15 Paper 21 Q 4

Syllabus ref	Learning objectives	Suggested teaching activities
42 Statistical diagrams	Construct and interpret scatter diagrams. Understand what is meant by positive, negative and zero correlation with reference to a scatter diagram. Draw a straight line of best fit by eye.	This can be linked to earlier work on slope and intercept. **(F)** Start with a set of paired data. This could come from body measurements from the Census at School, or students could take their own. Measuring height and arm span can produce a good correlation. Alternatively, data from another subject, e.g. an experiment in science, or GDP and life expectancy of countries from geography. Show how to plot points. Emphasise that each point represents a pair of data. Learners often want to treat this as a ‘dot to dot’ graph, which is not correct. Explain the patterns of points and explain the terms positive, negative and zero correlation. Discuss cases where you would expect to see a correlation, for example, ice cream sales against temperature should show negative correlation. *Maths Is Fun* has a variety of examples: www.mathsisfun.com/data/scatter-xy-plots.html.

Syllabus ref	Learning objectives	Suggested teaching activities
		Give practice in plotting scatter graphs and describing the correlation. Show how to draw a ruled line of best fit by eye. Emphasise that the line must be straight (not curved) and should only be drawn in cases where there is a correlation. Some graphing packages such as Autograph allow you to show residuals. These are not required for this syllabus, but displaying them and dragging the line around in order to minimise them can be very helpful in developing the learners' understanding about the placement of the line of best fit and in helping them to realise that lines of best fit do not necessarily pass through the origin. The main use of a line of best fit is to estimate missing values, for example, estimating the arm span of a person whose height is known. Ask learners to use their lines to estimate missing values. **Extension activities:** Learners find equations for their line of best fit. If using data from e.g. their science experiments this could be interpreted in terms of the two variables. **(F)** **Learning resources** Work on scatter graphs: http://www.cimt.org.uk/projects/mepres/book9/bk9i8/bk9_8i3.html
Past papers and specimen papers		
Specimen paper: 2018 Specimen Paper 2 Q 2		

Syllabus ref	Learning objectives	Suggested teaching activities
36 Trigono-metry	Extend sine and cosine functions to angles between 90° and 180°. Solve problems using the sine and cosine rules for any triangle and the formula ½ *ab* sin *C* for the area of a triangle.	Give the learners a right-angled triangle where they are required to find a distance using the sine function, and solve this problem. Then give them an acute-angled triangle where they need to find a length for which the sine rule would be appropriate. Ask them how they can solve this (if they do not suggest dividing it into appropriate right-angled triangles then give them the hint by drawing in the appropriate perpendicular height). When this has been solved together, give the same situation using letters rather than numbers and use the same method to obtain the sine rule. After practice in using the sine rule to obtain sides and angles in acute-angled triangles, introduce an obtuse-angled triangle requiring use of the obtuse angle. Show that this can be solved using the supplementary

Syllabus ref	Learning objectives	Suggested teaching activities
		angle, and then extend the sine and cosine functions to include obtuse angles. You could demonstrate that $\sin\theta = \sin(180 - \theta)$ and $\cos\theta = -\cos(180 - \theta)$ when θ is obtuse using calculator values or extend the definitions to generate the sine and cosine waves at least as far as 180°. The cosine rule and the formula for the area of a triangle may be developed similarly to the sine rule activity above. Give practice in using these, including situations where learners have to decide which (or both) of the sine and cosine rules they need to use. **Extension activities:** Learners can apply the sine and cosine rule to solving problems in three dimensions. **Further resources:** Trigonometry in non-right-angled triangles is covered in 4.8 to 4.9: http://www.cimt.org.uk/projects/mepres/allgcse/bka4.pdf

Past papers and specimen papers

Specimen paper:
2018 Specimen Paper 2 Q 8b, 10

Past papers:
Jun 13 Paper 11 Q 8
Jun 13 Paper 21 Q 11
Jun 13 Paper 22 Q 6, 12b
Nov 13 Paper 12 Q 26
Nov 13 Paper 21 Q 12
Nov 13 Paper 22 Q 10
Jun 14 Paper 21 Q 9
Jun 14 Paper 22 Q 5
Nov 14 Paper 21 Q 5
Jun 15 Paper 11 Q 13
Jun 15 Paper 21 Q 7
Jun 15 Paper 22 Q 5
Nov 15 Paper 21 Q 11
Nov 15 Paper 22 Q 7

Syllabus ref	Learning objectives	Suggested teaching activities
37 Vectors in two dimensions	Describe a translation by using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$ or $\overrightarrow{AB}$ or ***a***. Add and subtract vectors. Multiply a vector by a scalar. Calculate the magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$. (Vectors will be printed as $\overrightarrow{AB}$ or ***a*** and their magnitudes indicated by modulus signs, e.g. $\lvert\overrightarrow{AB}\rvert$ or |**a**|. In all their answers to questions candidates are expected to indicate **a** in some definite way, e.g. by an arrow or by underlining, thus $\overrightarrow{AB}$ or a.) Represent vectors by directed line segments. Use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors. Use position vectors.	Revise the work on translations by asking learners to find the image of a point after a translation and then after a further translation, asking them to give the vector for the combined transformation. Generalise to representing vectors by directed line segments, to adding and subtracting column vectors and to multiplying a column vector by a scalar. Use Pythagoras' theorem to find the magnitude of a column vector. Introduce the notation $\overrightarrow{AB}$ and **a** for describing vectors and modulus signs to indicate magnitude. Discuss the relationship between vectors **a** and *k***a**. Use position vectors and show the sum and difference of two vectors. Use vectors to solve problems and demonstrate some properties of plane figures, e.g. that the diagonals of a parallelogram bisect each other, or that the medians of a triangle intersect, dividing the medians in the ratio 2:1. **Extension activities:** There are many examples of vector problems with a high level of challenge, for example showing whether points defined by three position vectors are co-linear, or finding the ratio of the magnitudes of two related vectors. **Further resources:** Work on vectors: http://www.cimt.org.uk/projects/mepres/allgcse/bkc19.pdf Don Steward has a starter activity based on a tangram to develop use of vector notation: http://donsteward.blogspot.co.uk/2010/01/vectors.html

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 7 **Past papers:** Jun 13 Paper 21 Q 8b Jun 13 Paper 22 Q 9a Nov 13 Paper 11 Q 21 Nov 13 Paper 22 Q 12 Jun 14 Paper 21 Q 10a Jun 14 Paper 22 Q 8 Nov 14 Paper 11 Q 23 Nov 14 Paper 22 Q 7a Jun 15 Paper 12 Q 25 Jun 15 Paper 21 Q 11 Nov 15 Paper 21 Q 6 Nov 15 Paper 22 Q 11

Unit 10: The language of functions, algebraic fractions, histograms, transformations using matrices

Recommended prior knowledge

- Matrices (see Unit 9)
- Transforming simple formulae (see Unit 5)
- Constructing draw bar graphs (histograms of equal width where the vertical scale is the frequency) (see Unit 3)
- Basic transformations (see Unit 5)

Context

The algebraic skills required for the course are completed in two ways: by using functions and by working further with algebraic fractions. The statistics element of the course is completed by considering how to represent data grouped in unequal intervals as histograms. Work on transformations studied in Unit 5 is taken further, particularly in using the work on matrices studied in Unit 9.

Outline

Function notation is introduced formally, though it may have been used earlier, and skills of transforming formulae are applied to obtaining inverse functions. Earlier work on algebraic fractions is revised, and the rules of fractions applied to harder expressions and equations. In the statistics topic, the same data are compared using classes of equal width and classes of unequal width. This shows the need to use frequency density to represent the data in the latter case and learners are then given practice in using frequency density to draw and interpret histograms. The method of calculating an estimate of the mean is also revised and then applied where classes are of unequal width. Relevant matrix products are evaluated before going on to demonstrate their application to transformation geometry. Further work leads to obtaining the matrices for basic transformations and for combinations of transformations. The transformation from an object to its image is identified and described.

Syllabus ref	Learning objectives	Suggested teaching activities
26 Function notation	Use function notation, e.g. $f(x) = 3x - 5$, $f : x \rightarrow 3x - 5$ to describe simple functions. Find inverse functions $f^{-1}(x)$; use the notation	Many graphing programs accept inputs using function notation. This could be used to introduce learners to the idea of function notation during graphical work in earlier units. Develop the use of function notation by illustrating the efficiency of writing f(4) = 7 rather than 'when $x = 4$, $y = 7$'. Use the notation $f : 4 \rightarrow 7$ to illustrate functions viewed as mappings. Give practice in using both types of notation. To find the inverse of $f(x) = 3x - 5$, write as $y = 3x - 5$, then for the inverse function, write as $x = 3y - 5$ and rearrange to make y the subject.

Syllabus ref	Learning objectives	Suggested teaching activities
	$f^{-1}(x) = \dfrac{x+5}{3}$ and $f^{-1} : x \rightarrow = \dfrac{x+5}{3}$ to describe their inverses.	Demonstrate that the graph of $y = f^{-1}(x)$ is the reflection of the graph of $y = f(x)$ in the line $y = x$. Using graphing software, such as GeoGebra or Autograph, will make this very straightforward. Give learners practice in finding and using the inverses of functions. **Extension activities:** Learners investigate or research functions to find examples where the inverse function does not work for all values. **Further resources:** An short introduction to function notation: https://virtualnerd.com/algebra-1/relations-functions/functions/function-notation/f-of-x-definition *Maths Is Fun* has an introduction to inverse functions: www.mathsisfun.com/sets/function-inverse.html

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 1 Q 14 **Past papers:** Jun 13 Paper 21 Q 8a Jun 13 Paper 22 Q 9b Nov 13 Paper 11 Q 13 Nov 13 Paper 12 Q 3 Jun 14 Paper 12 Q 13 Nov 14 Paper 12 Q 4 Nov 14 Paper 21 Q 6 Jun 15 Paper 22 Q 7a Nov 15 Paper 11 Q 17 Nov 15 Paper 12 Q 4

Syllabus ref	Learning objectives	Suggested teaching activities
18 Algebraic manipula-tion	Manipulate algebraic fractions. Factorise and simplify rational expressions.	Revise the four rules of fractions with numbers (Unit 2) and algebraic fractions with numerical denominators (Unit 3). Apply the rules to other simple algebraic fractions, for instance simplifying those with common factors. Move on to look at examples where the numerator and/or denominator must be factorised before simplifying. Finally, apply these skills to solving quadratic equations resulting from the sum of two fractions with a linear function in the denominator. **Extension activities:** Questions that require learners to form and solve equations can be very challenging. There are a number of examples in the past examination questions listed for this topic. **(F)** **Further resources:** Work on manipulation: https://www.cimt.org.uk/projects/mepres/book9/bk9_11.pdf

Past papers and specimen papers
Specimen paper: 2018 Specimen Paper 2 Q 7, 10 **Past papers:** Jun 13 Paper 11 Q 25 Jun 13 Paper 12 Q 20 Nov 13 Paper 22 Q 11a Jun 14 Paper 12 Q 23 Jun 14 Paper 21 Q 1a, 1d Nov 14 Paper 21 Q 7b Jun 15 Paper 11 Q 11 Jun 15 Paper 21 Q 6 Jun 15 Paper 22 Q 9b Nov 15 Paper 11 Q 22 Nov 15 Paper 12 Q 16b Nov 15 Paper 21 Q 5b

Syllabus ref	Learning objectives	Suggested teaching activities
42 Statistical diagrams	Use frequency density to construct and read histograms with equal and unequal intervals.	Use data grouped in equal intervals and revise the work done in Unit 3 on calculating the mean and representing the data using a bar graph (histogram with equal intervals where the vertical scale is the frequency). Combine some of the groups for these data and calculate the mean again, thus showing that using grouped data only gives an estimate for the mean. Discuss how the newly grouped data should be represented fairly – comparing the new graph and the old, continuing to use frequency on the vertical axis, it is very clear that this gives a wrong representation. Show the learners that when area represents the frequency, the graphs are comparable. The Autograph program will draw histograms using frequency or frequency density so that an appropriate choice can be made. It also enables a demonstration of the effects when frequency is wrongly chosen, as is discussed here. Show the learners how to define frequency density clearly as frequency per cm or frequency per 10 cm, for instance, labelling their axis clearly to show the definition they have used, or else providing a key to show the frequency represented by an area such as 1 cm^2. Make sure the learners are aware that histograms are drawn for continuous data, not discrete data. Learners also need to be able to find frequencies from histograms, either by considering areas or by calculating frequencies from the frequency density and class width. Give practice in both drawing histograms and interpreting them. **(F)** **Extension activities:** Ask learners to consider how they could estimate the median value from a histogram. **Further resources:** https://www.cimt.org.uk/projects/mepres/allgcse/bkb8.pdf (section 8.7) Autograph is a powerful piece of software, but a licence fee is payable. A 30-day trial version can be downloaded from www.autograph-maths.com/download/

Past papers and specimen papers
Past papers: Jun 13 Paper 21 Q 10 Jun 13 Paper 22 Q 7 Nov 13 Paper 11 Q 12 Nov 13 Paper 12 Q 18 Nov 13 Paper 22 Q 14 Jun 14 Paper 22 Q 7b Nov 14 Paper 21 Q 4 Nov 14 Paper 22 Q 11a Jun 15 Paper 22 Q 11b Nov 15 Paper 22 Q 10

Syllabus ref	Learning objectives	Suggested teaching activities
38 Trans-formations	Use the following transformations of the plane: reflection (M), rotation (R), translation (T), enlargement (E) and their combinations. (If M(*a*) = *b* and R(*b*) = *c* the notation RM(*a*) = (*c*) will be used; invariants under these transformations may be assumed.) Identify and give precise descriptions of transformations connecting given figures. Describe transformations using coordinates and matrices. (Singular matrices are excluded.)	Ask learners to do a matrix multiplication such as $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 4 \\ 1 & 3 & 1 \end{pmatrix}$ then show them how this can be interpreted as transforming the triangle with coordinates (1, 1) (1, 3) and (4, 1) by reflecting it in the *x* axis. Invite the learners to explore with other triangles, etc. and other 2 × 2 matrices, giving them matrices which refer to the required transformations, until the learners can classify the basic matrices which represent certain transformations. Show learners how the matrix for a given transformation may be obtained by considering its effect on the square with vertices at (0, 0) (1, 0) (1, 1) and (0, 1) with the images of (1, 0) and (0, 1) forming the columns of the matrix. Show learners how the inverse of a matrix representing a transformation represents the inverse of the transformation. Use and describe combinations of transformations, using both coordinates and matrices. **Please note:** stretch and shear are **not** required for this syllabus, but did appear in previous syllabuses for this specification. Care will be therefore be needed when using textbooks or other resources that were

Syllabus ref	Learning objectives	Suggested teaching activities
		written for the previous versions of this syllabus. **Extension activities:** Matrix transformations are often used in computer animations. If any of your learners are enthusiastic coders or animators, they may be able to share their experiences of using transformations. Simple Wikipedia contains articles written in simple English. To access simple Wikipedia articles, change the *en* in the URL (web address) to *simple*, for example http://en.wikipedia.org/ would become http://simple.wikipedia.org/ . Not all articles have simplified versions and some of the ones that do are not very clear. Learners could be challenged to create or edit a simple Wikipedia article describing matrix transformations. This would also make a good end of course revision activity for any of the topics studied during the course. **Further resources:** Mathplanet has an introduction to transformations using matrices: www.mathplanet.com/education/geometry/transformations/transformation-using-matrices Work on transformations is included in (aimed at A Level learners): https://www.cimt.org.uk/projects/mepres/alevel/fpure_ch9.pdf (section 9.2)

Past papers and specimen papers
Past papers: Nov 13 Paper 12 Q 20 Jun 14 Paper 12 Q 10 Jun 15 Paper 21 Q 11b

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