MATHEMATICS D

Paper 4024/11
Paper 1

Key messages

To do well on this paper, candidates should have covered the whole syllabus and learned the necessary formulae. They should clearly show all the necessary working for each question and be able to perform calculations accurately. It is important to read the questions on the paper carefully, to highlight the key points and the form in which the answer is required, and to give the result to a suitable degree of accuracy.

General comments

The paper contained questions accessible to all candidates. Areas of the syllabus that require further attention include estimation and rounding to a given number of significant figures, symmetry and geometry including angles of elevation and vectors.

It is clear from errors made on the paper how important it is to practise constantly basic non-calculator work, in particular multiplication tables and positioning of the decimal point in calculations.

Presentation of the work was usually good with most candidates showing sufficient clear working. Incorrect answers are best crossed out and replaced rather than written over. There is no need to delete working if it is not to be replaced, as the original attempt may gain some credit. A check should be made to ensure that the answer is sensible and that the answer in the working has been correctly transcribed to the answer space.

It is important that candidates note the instruction on the cover page that 'the omission of essential working will result in loss of marks' as opportunities to score some marks for steps within the working will be lost otherwise.

It is important that candidates write all numbers clearly. Too often numbers are written hurriedly and are not formed correctly.

A number of candidates were unable to attempt a significant number of the questions on the paper which could have been due to insufficient preparation or entry to an examination which was too advanced for their skills.

Comments on specific questions

Question 1

(a) This part was answered well with answers occasionally incorrectly written as $1\frac{-7}{24}$ or $-1\frac{7}{24}$.

(b) This part was also answered well with the common error being the incorrect placement of the decimal point.

Answers: **(a)** $\frac{17}{24}$ **(b)** 0.52

Question 2

(a) This part was answered well. Misunderstanding of indices included giving 9^0 as 0, finding 9^{2-0} and trying to divide the indices to give $9^0 = 1$.

(b) The correct answer was frequently seen. The final value should not contain indices. The index, rather than 9, was sometimes inverted and common incorrect answers included 3, 81, $\sqrt{9}$, $\frac{1}{9^2}$ and $\frac{1}{81}$.

Answers: **(a)** 80 **(b)** $\frac{1}{3}$

Question 3

(a) This part was answered well.

(b) This part was answered well.

Answers: **(a)** 24 **(b)** 120

Question 4

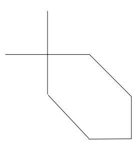
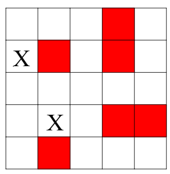
Some candidates were able to convert each of the two given numbers to 1 significant figure, although many gave the numerator as 100 or 10, or the denominator as 2 or 0. Others were unaware that 1000.00 was not a value given to one significant figure. The correct division of 1000/0.02 was then sometimes calculated accurately but a few answers of 50000 were then divided by 100 (as 1000 and 0.02 were both multiplied by 100 to convert them into whole numbers in order to carry out the division).

Answer: 50 000

Question 5

(a) This part was rarely correct, with many hexagons drawn. A few answers were just missing the line running vertically above AB.

(b) This part was more successful but still proved challenging for some.

Answers: **(a)**  **(b)** 

Question 6

Candidates who realised that the total distance needed to be found first and thus found $(10 \times 1\frac{1}{2} + 7)/2$ were nearly always successful. There were a number of misconceptions with many candidates finding a numerator of (10 + 7), (10/1.5 + 7), (10 + 7/0.5) or $(10 \times 1.5 + 7/0.5)$.

Answer: 11

Question 7

(a) There were many correct answers with a common error being to not line up the decimal places in 9.6 + 7, thus ending with an answer of 10.3.

(b) The correct answer was often seen. Candidates should try to use the correct horizontal fraction line, rather than '/', or put brackets round the numerator giving (x – 7)/3 as the final answer. The final answer should be written in terms of *x*, not in terms of *y*. Wrong answers included $\frac{x+7}{3}$ and $\frac{7-x}{3}$. Some candidates did not understand the meaning of $f^{-1}(x)$ and tried to give a numerical answer.

Answers: **(a)** 16.6 **(b)** $\frac{x-7}{3}$

Question 8

The common difficulty with proportionality questions is failure to read the question carefully enough and thus choose the wrong formula. Here there were attempts to use $y = kx$ rather than $y = kx^2$. Those recognising the correct proportionality were often able to find *k* correctly although there were occasionally problems with the division of fractions, sometimes from thinking $(½)^2$ was 1. Some quoted the correct formula but omitted the square in their calculation giving *k* as $^2/_5$ but then recovered to give a final answer of 40. Some candidates failed to form any formula or equation.

Answer: 80

Question 9

(a) There were frequent correct answers. On some scripts, the correct answer was given in the working but either '=4' or just '4' were seen on the answer line. The answer line gave '*x*' as the start of the answer so '> 4' was expected for a candidate's answer.

(b) The question asks for integer values but it was fairly common to see only one of the two expected answers given on the answer line. Common errors were to leave the answer as $-4 < n \leqslant -2$, to include decimal answers, or to give an answer of '–7, –6, –5'. It was clear that the word 'integer' was not well understood.

Answers: **(a)** $x > 4$ **(b)** –3 and –2

Question 10

Some candidates lost marks through not knowing the difference between mean, mode and median.

(a) This part was answered well.

(b) The common error was to not order the numbers before finding the one in the middle, thus giving an answer of 5.

(c) The numbers given were such that cancelling out the negative numbers with the positive numbers made the addition on the numerator easier. A few answers were not simplified from 0/7.

Answers: **(a)** –2 **(b)** –1, **(c)** 0

Question 11

(a) This part was generally answered well**.** Common wrong powers were 4, 5 and –5.

(b) To be successful, the powers of one of the numbers should be changed so that it has the same power as the other, for example $55 \times 10^6 - 2.1 \times 10^6$. Many candidates wrote out the numbers in full but often failed to line up the digits correctly, which lead to an answer of 3.4 × power of 10. A number of candidates tried to subtract the numbers and then add, or subtract, the powers.

Answers: **(a)** 1.2×10^{-4} **(b)** 5.29×10^7

Question 12

Although there were many correct answers, candidates experienced a number of issues with signs in this question, due to the less familiar form of the two equations. Those attempting to rearrange the first equation sometimes wrote $x = 12 - 2y$, although those rearranging correctly nearly always reached the correct answers. For the elimination method, which was used more often, the common method was to double the first equation and then add but there were sometimes a number of sign errors in their chosen method with common wrong answers of $x = 2$, $y = 7$ or $x = 14$ (or –14), $y = 13$. Candidates who checked their answers in both equations, thus realising there must be an error, were sometimes able to correct their work.

Answer: $x = -2$, $y = 5$

Question 13

(a) This part was generally answered correctly. Some measured incorrectly on their protractors, with 68° rather than 72° for the angle at the top left of the diagram. Some measured the angle from the wrong line and there were a few who did not attempt the question, possibly due to lack of a protractor.

(b) There were many correct fractions found but these were not always reduced to the simplest form, as asked for in the question.

(c) This part was frequently correct but there were a number of incorrect attempts using 120 with 168 or with the answer to part **(b)**.

Answers: **(a)** Correct line **(b)** $\frac{7}{15}$ **(c)** 240

Question 14

(a) Candidates were generally able to read the correct value from the table.

(b) There were some correct answers in the range but many incorrect answers came from rounding 32.25 to 32.2 or 32.3 and then reading from the table.

(c) Candidates often read the appropriate value from the table but the majority then had problems deciding on the position of the decimal point in the answer. Common wrong answers were 31.8 and 318, along with 31.08. Finding 50^2 as 2500 would have given a rough estimate to help find the size of their answer. A long multiplication was not required. A number of candidates did not attempt this part.

Answers: **(a)** 0.106 **(b)** 5.678 to 5.68(0) **(c)** 3180

Question 15

(a) The common wrong answer was $11 - 6t$. A few candidates tried to solve an equation.

(b) There were two common approaches to the question – simplify each component of the numerator or write out the numerator in full. Errors in simplifying each component included giving 2^3 as 6 and $(x^2)^3$ as x^5. Occasionally the final answer included $^8/_6$ rather than $^4/_3$. In a question of this type, it is better not to try to convert the numerical part to a mixed number. At times, y was placed on the numerator but not as y^{-1}.

Answers: **(a)** $5 - 6t$ **(b)** $\frac{4x^2}{3y}$

Question 16

(a) The correct answer was fairly common although there were problems with the signs and some answers gave the gradient.

(b) This question was generally not done well. Many candidates did not know the correct formula for finding distance with $\sqrt{[(0 + 10)^2 - (7 + (-1))^2]}$ being a common wrong starting point. Those who quoted the correct formula sometimes had issues with signs, commonly 7 – 1 rather than 7 – (–1). Those reaching $\sqrt{164}$, often felt they had made an error, expecting their answer to be an exact root, and so started again with an incorrect formula (as the word 'integer' was not always understood). Others attempted to find the square root of 164, which was not what the question required. A significant number of candidates did not attempt the question.

Answers: **(a)** (5, 3) **(b)** 164

Question 17

(a) Given that the number of plants being measured was 90, some candidates were unaware that the end of the cumulative frequency graph was at the point (6, 90). A few diagrams showed the curve descending from (4, 77) to the *t* axis, others finished at the point (7, 100) and some showed a random curve which had no link with the table. There were a number of no responses.

(b)(i) The common wrong answer was 2.95, as candidates read off the graph at the point where the cumulative frequency was 50, instead of 45.

(ii) The common wrong answer was 66 and a few candidates tried to find the number of leaves with length greater than 3.5 cm.

Answers: **(a)** Curve from (4,77) to (6, 90) via (5, 87) **(b)(i)** 2.8 **(c)** 67 or 68

Question 18

(a) The common wrong answers were 13 and 15.

(b) Those who recognised that the circle was split into 10 sectors, each the size of the one shown, were successful. The most common wrong answer was 45, which came from measuring the angle shown on the diagram. A number of candidates did not attempt this part.

(c) There was little understanding of what is an angle of elevation. A few candidates reached 18°, for the base angle of the triangle with vertices at O, the position of seat 2 and the bottom of the wheel, but did not carry on to find the angle of elevation. Some tried to use the seat number in their calculations. A significant number of candidates made no attempt to answer this part.

Answers: **(a)** 14 **(b)** 36 **(c)** 72

Question 19

(a) This was generally answered well. The common wrong answers were $5a(5a - 5)$ and $5a(a - 1)$.

(b) This part was not answered as well as part **(a)** with the common wrong answer being $(3b - 4)^2$.

(c) Most candidates were able to correctly factorise one pair of terms with many going on to give the correct answer. A few miscopied *t* as 1 from their working to the answer line.

Answers: **(a)** $5a(5a - 1)$ **(b)** $(3b - 4)(3b + 4)$ **(c)** $(2x + 3)(2y + t)$

Question 20

(a) The position of D was nearly always correct but some diagrams did not show the construction arcs required.

Both parts of **(b)** asked for the construction inside the quadrilateral but it was rare to see the loci restricted to ABCD.

(b)(i) Some candidates were able to draw the correct angle bisector .

(ii) Many candidates were able to draw the correct perpendicular bisector of BC.

(c) Those whose constructions in **(b)** were correct were often able to correctly identify the positions of P and Q at the ends of the required line. Occasionally the labelling was not clear enough to identify which part of the angle bisector was the intended answer. Some drew a new line, labelling it PQ, often from C to the point of intersection of their answers to **(b)**.

Answers: **(a)** Quadrilateral with visible arcs **(b)(i)** Bisector of angle ABC **(ii)** Perpendicular bisector of BC **(c)** PQ labelled

Question 21

Few candidates answered this question well.

(a) The common wrong answer was (23, 6) and many answers bore no connection to the equations of the three given lines.

(b) Some candidates were able to give both inequalities correctly but $y > 6$ alone was a common wrong answer. Many inequalities involved 23. Answers should be strict inequalities.

(c) Finding the coordinates of A, B and C gives the range of values for points within the triangle. The value of h has to lie between 18 and 23 so the greatest value inside the triangle is 22. Similarly k must lie between 6 and $7\frac{2}{3}$, so the greatest value is 7. Few answers were correct, although some gave one correct value, and only a few showed any working. Occasionally the value for h was smaller than that for k.

Answers: **(a)** (18, 6) **(b)** $y > 6$ and $y < \frac{x}{3}$ **(c)** $h = 22$ and $k = 7$

Question 22

Few candidates answered this question well.

(a) The correct answer was sometimes seen with some giving an acceptable unsimplified answer. Some candidates gave the acceleration of the train rather than the retardation. There were many answers simply quoting a formula, which was not always the correct formula.

(b) Those appreciating that the area under the graph represented the distance and using either the formula for the area of a trapezium or the areas of the three separate sections generally reached the stage of equating the two distances, although the given distance was often not converted to metres. Some answers simply used 1.2 km divided by a time (usually 80 seconds).

Answers: **(a)** $\frac{v}{10}$ **(b)** 20

Question 23

(a) There were many correct answers seen.

(b)(i) Those giving the number of elements generally gave the correct value. Others gave a list of the four values of P ∪ Q.

(ii) There was some confusion in this part with some candidates giving the numbers in both P and Q, rather than listing the values of x/y.

Answers: **(a)** Correct diagram **(b)(i)** 4 **(ii)** –1, 1, $\frac{1}{2}$, –4, 4, 2

Question 24

A significant number of candidates did not attempt part **(b)**.

(a) This part was generally well done, but some answers were spoilt by giving a final answer of 8*ab*. Others subtracted, rather than added, the vectors.

(b) (i) The best approach to find *k* is to use ratios. It was rare to see the correct answer. Common wrong answers were the numbers 2, or 4, or answers that contained vectors.

(ii)(a) Some candidates noticed that, by putting X on BC such that XE is parallel to BF, they were able to deduce, from part **(b)(i)**, that $\overrightarrow{AX} = 9\mathbf{a}$ and $\overrightarrow{XE} = 3\mathbf{b}$. The properties of parallelogram CDEX were then used to find $\overrightarrow{CD}$ and $\overrightarrow{DE}$. Few candidates demonstrated a sound understanding of the properties of vectors.

(ii)(b) Many gave a vector answer which was parallel to the correct answer but it was commonly incorrectly given as 6**a**, –6**a** or 3**a**.

Answers: **(a)** 6**a** + 2**b** **(b)(i)** 3 **(ii)(a)** 3**b** **(ii)(b)** –3**a**

Question 25

(a) This was well answered by many candidates.

(b) (i) The common wrong answer was *N* + 2 whilst many answers were only numbers.

(ii) Few candidates recognised that the sequence involved square numbers.

(c) There were a number of correct answers, often found by inspection of the sequences rather than using the formulae found in the two earlier parts. Some thought *N*, rather than 2*N* + 1, was 25.

Answers: **(a)** 11, 36 **(b)(i)** 2*N* + 1 **(ii)** $(N + 1)^2$ **(c)** 169

Question 26

(a) Candidates generally found at least two elements correctly. There was some confusion over signs, and other incorrect answers were found by multiplying the two matrices.

(b) The common error was to square the individual elements of **A.**

(c) Some realised they needed to find $\mathbf{B}^{-1}$ but not many were able to deduce that *k* was ½.

Answers: **(a)** $\begin{pmatrix} -6 & -6 \\ 3 & 3 \end{pmatrix}$ **(b)** $\begin{pmatrix} -2 & -6 \\ 3 & 7 \end{pmatrix}$ **(c)** $\frac{1}{2}$

MATHEMATICS D

Paper 4024/12
Paper 1

Key messages

In order to do well in this paper, candidates need to have covered the whole syllabus and learned the necessary formulae and facts. They should be able to recognise, and carry out correctly, the appropriate mathematical procedures for a given situation. They should perform calculations accurately and clearly show all necessary working in the appropriate place.

General comments

There were questions on the paper accessible to all candidates. There were many scripts where candidates had made good attempts at all questions. In a few cases, responses suggest that some candidates had found most of the paper challenging. This may indicate that the demand of the paper was not well matched to the candidates' current performance; alternative offerings for such candidates may prove more appropriate.

Topic areas that proved to be challenging this session involved calculations with time (**Question 5(b)**), calculating the median (**Question 8(b)**), working with bounds (**Questions 9(b)** and **25(b)**), practical problems involving volume and capacity **18(b)** and vector problems (**Question 27(b)(ii)**). More specifically, many candidates found approximation and the use of appropriate degrees of accuracy challenging. Candidates are advised to reinforce their understanding of the integer class of number. Some candidates were very competent at performing standard techniques, and yet were unable to recognise the appropriate mathematical procedure required in a novel context.

In general, candidates would benefit from improving their skills in manipulating fractions and in basic arithmetic, particularly when negative numbers are involved. Most candidates showed competence when calling upon multiplication facts.

Presentation of work was usually good. Candidates are reminded to write in dark blue or black pen. Except for diagrams and graphs, candidates should not write in pencil. Some candidates did not appear to bring geometrical instruments into the examination. Candidates should bear in mind that it is to their advantage to make sure they provide sufficient working and that this working is set out neatly and legibly. Working makes it possible for marks, where they are available, to be awarded for correct methods and intermediate results.

Candidates should be encouraged to pay careful attention to the wording, the numbers given, the units used and the units required in the answer. Candidates should only write the final answer to each question in the answer space. Care must be taken to ensure that answers obtained in the working are accurately transferred to the answer space. When an answer is to be changed, candidates are advised to cross out their original attempt and show their new solution clearly, rather than to write over previous work.

Comments on specific questions

Question 1

(a) A large proportion of candidates answered this fraction calculation correctly. The most common method errors were from calculations such as

$$\frac{42-15}{35} = \frac{27}{35}\ ;\ \frac{6-3}{7-5} = \frac{3}{2}\ ;\ \frac{6}{35} - \frac{3}{35} = \frac{3}{35}$$

(b) Many responses indicated a poor appreciation of place value. Many candidates tried to simplify the given fraction by cancelling, but poor arithmetic, combined with misplaced decimal points, led to incorrect answers such as $\frac{90}{0.45} = \frac{900}{45} = \frac{1}{20}$.

A significant number of candidates left their answer in an unsimplified form such as $\frac{9000}{45}$; $\frac{10}{0.05}$.

Answer: **(a)** $\frac{9}{35}$ **(b)** 200

Question 2

(a) Attempts at this part showed that many candidates would benefit from improving their experience of classifying statistical data. Only a minority used tally marks, others listed the masses in each interval. A few candidates gave probabilities instead of frequencies.

(b) From the total frequency of 20, given in the question, candidates were expected to multiply their minimum frequency by $\frac{360}{20}$. Some attempted to do this, but errors in evaluation were common.

Many candidates did not understand what was required, and gave answers such as 4, from $\frac{20}{5}$; 72°, from $\frac{360^o}{5}$; 7 < *m* < 9; calculation of a sector area.

Answer: **(a)** 7, 8, 5 all three **(b)** 90°

Question 3

Most candidates dealt competently with this question.

A few did not read the question carefully, and assumed that *y* is directly proportional to *x*, or inversely proportional to x^2.

A common error was to evaluate $\frac{5}{10}$ as 2, or as $\frac{1}{5}$. Some, from $\frac{1}{6} = \frac{k}{30}$, obtained $k = 180$, or $k = \frac{1}{5}$.

A small minority converted $\frac{1}{6}$ to a decimal, moving on to obtain approximate answers.

Answer: $\frac{1}{2}$

Question 4

(a) The majority of candidates knew to substitute $\frac{1}{2}$ for *x*, and obtained $\frac{1}{2} \div 4$, or $\frac{0.5}{4}$, or $\frac{\frac{1}{2}}{4}$.

Subsequent operations showed weaknesses in some candidates' ability to handle basic calculations. Typical wrong answers were 8, 2, $\frac{1}{2}$, 1.25.

(b) Many candidates made a sound attempt with this question, though some did not seem to understand the meaning of $f^{-1}(x)$.

Some candidates looked to find $f^{-1}(\frac{1}{2})$ or similar, resulting in a numerical value only.

Occasionally, an answer was given that was not expressed in terms of *x*.

Answer: **(a)** $\frac{1}{8}$ **(b)** $4x$

Question 5

Answers to this question showed that many candidates seem to be unfamiliar with a timetable as given in the question, and only a minority gave the correct answers.

(a) Most candidates did not realise that the journey time is the difference between the time at A and the time at E. Common wrong answers were 30; 150; 98.

(b) Many candidates ignored the appointment time and gave answers such as 09 03; 17 12; 17 33 or 2 30.

In cases where the appointment time was considered, only in a small number of responses was there evidence of a realisation that the journey time was 32 minutes and so the latest bus from B would be the one which departed at 14 33, arriving at D at 15 05. A few, having calculated the journey time of 32 minutes, said that she needed to catch a bus at 2.58 p.m.

Answer: **(a)** 68 **(b)** 14 33 or 2.33 p.m.

Question 6

(a) Responses to this part were generally correct. Occasional wrong answers were 3.82; 3.86; 3.8.4 or 3.804

(b) Responses to this part were generally correct. Common wrong answers were 9, from one-half of 18; 10, from five-ninths of 18 or $\frac{1}{9}$, from $\frac{2}{18}$.

Answer: **(a)** 3.84 **(b)** 4

Question 7

A good knowledge of angle properties linked to isosceles triangles and parallel lines was demonstrated by candidates in this question.

(a) Responses to this part were generally correct. Some gave 110°, seemingly as angle *CBE*, not angle *CBF*. Others gave 43°, from 180° – 102° – 35°.

(b) This part was answered successfully by the majority of candidates. Common wrong answers were 67°, from 102° – 35° or 102°

Answer: **(a)** 78° **(b)** 70°

Question 8

(a) Usually answered correctly, with common wrong answers 9; 5 or 2.

(b) Responses showed that many candidates should revisit the methods used to find the median from a frequency table. With a frequency of 30, the median is halfway between the 15th and 16th ordered value. Frequent wrong answers were 1; 5.5, from $\frac{7+4}{2}$; 6, from $\frac{30}{5}$; $1\frac{2}{3}$ (the mean value) or 10, from $(0\times9+1\times6+2\times7+3\times4+4\times2+5\times2)/5$.

Answer: **(a)** 0 **(b)** 1.5

Question 9

Responses indicated that some candidates should develop their understanding of bounds.

(a) This part was answered well, many candidates appreciating that the lower bound is 0.5 cm^2 less than 8 cm^2. The usual wrong answer was 7.95 cm^2.

(b) Candidates found this part more challenging, with few realising that the lower bound for the length is obtained by dividing the *lower* bound for the area by the *upper* bound of the width. Some candidates appeared not to have read the question carefully and took the length to be 2 cm, and gave the answer 1.5, from 2 – 0.5.The most common wrong answers were 3.5, from 8/2 = 4 then 4 – 0.5 or 5, from 7.5/1.5.

Answer: **(a)** 7.5 **(b)** 3

Question 10

Candidates who realised that to obtain an answer correct to 1 significant figure, a suitable approximation is to correct the given numbers to 2 significant figures, usually obtained $\frac{40\times\sqrt{36}}{3000}$ and hence $\frac{40\times6}{3000}$. Attempts to evaluate this expression varied. Some candidates did not attempt to give their answer correct to 1 significant figure as instructed in the question; others gave an answer correct to 1 decimal place. Many obtained $\frac{2}{25}$, but either gave this as their answer, or evaluated it as 12.5; or, having obtained 0.08 went on to give the answer 0.1.

Some candidates ignored, or failed to see the instruction to use, suitable approximations to calculate an estimate. Others incorrectly used 3 or 30 instead of 3000; or 40 or 4 instead of 36.

Answer: 0.08

Question 11

The combination of calculating means using years and months was challenging for many candidates.

Only a minority realised that the correct method was first of all to calculate the total age of the four people, by multiplying 14 years 3 months by 3, and adding 15 years and 3 months. The mean of the four people is then obtained by dividing the total age by 4. Sound attempts at doing this obtained a total age of 58 years or 696 months. However, not all attempts to divide these numbers by 4 were successful. A common error was to give 14.5 years as 14 years 5 months.

Some candidates attempted to evaluate 3×14.3+15.3, treating months as decimals of a year. Many candidates added 14 years 3 months to 15 years 3 months and divided the result by 2; or by 4. A few started by multiplying 14 years 3 months by 4.

Answer: 14 years 6 months

Question 12

Many candidates would benefit from improving their knowledge of the laws of indices.

(a) The correct answer was obtained by candidates who realised that $a^{2x} = (a^x)^2 = 5^2$.

Wrong answers included $\frac{5}{2}$; 5 ; 10 ; $10x$; 10^x ; 25^x ; 5^{10} or a^{10} .

(b) The correct answer was obtained by candidates who realised that $a^{-x} = \frac{1}{a^x}$.

Wrong answers were many and very varied.

Answer: **(a)** 25 **(b)** $\frac{1}{5}$

Question 13

Some candidates showed that they would benefit from obtaining a deeper understanding of histograms and, in particular, that frequency is represented by the area of a rectangle and not its height.

(a) The usual wrong answer was 20.

(b) Most drew the first rectangle: base 40 to 50, height 3.

Though many drew the second rectangle correctly, a few had a base 50 to 90, or omitted to draw the ordinate at $t = 80$. The usual error was to have a height of 3.

Answer: **(a)** 40 **(b)** rectangles: base 40 to 50, height 3 ; base 50 to 80, height 1

Question 14

Many candidates found this question to be very difficult. Few gave answers which were integers.

Attempts to solve the inequality were often correct, getting as far as a statement such as $-5x < 12$. Too often, though, this led to $x < -2.4$. Those who wrote $x > -2.4$, seldom went on to give the correct answers.

Some candidates seemed to interpret 'the two solutions' requested in the question as having to solve two inequalities, and either attempted to solve the given equation in two different ways, or else provided one of their own in addition to the given one.

Answer: –2 and –1

Question 15

(a) Many correct responses were seen for this part. Some candidates gave answers that were angles; or stated 'clockwise' or 'anticlockwise'; or stated the value 3.

(b) This part was answered correctly by few candidates. Some responses gave just two angles, others gave three angles which did not sum to 180°. The answer 70° was mentioned more frequently than 72°.

Answer: **(a)** 5 **(b)** 72, 70, 38 all three

Question 16

Generally answered well by those who know what is meant by standard form. Some candidates do not yet show an understanding of the notation associated with standard form, $A \times 10^n$, where $1 \leqslant A < 10$ and n is an integer.

(a) Some misinterpreted 'standard form' and gave an answer such as 360000000. Others made errors with the index, so powers of 6, 7, and –8 (sometimes with 36) were regularly seen.

(b) (i) Some found 45×10^{-7}, but either gave this as their final answer, or gave the final answer with a power of –8. Others gave answers involving addition of 5 and 9, or with a power of 25.

(ii) Few correct responses were seen for this part. Incorrect answers included 3×10^{-4} ; 3×10^{-14} or 3×10^{-16}.

Answer: **(a)** 3.6×10^8 **(b)(i)** 4.5×10^{-6} **(ii)** $(\pm)3 \times 10^{-8}$

Question 17

(a) Generally answered correctly. A few tried to find $\frac{70}{110} \times 100$.

(b) Generally answered correctly. Common wrong answers were 25%, from $\frac{15000-12000}{12000} \times 100$; 30%, from $\frac{15000-12000}{100}$; 80% from $\frac{12000}{15000} \times 100$. Some candidates used 15 000–12 000=13 000.

Answer: **(a)** 77 **(b)** 20

Question 18

(a) The most common errors were 302, by including a non-existent top; 330, from finding the volume of the inside; 118, from adding just three areas; using 5×11 four times; using lengths of edges.

(b) The few correct answers seen were obtained by those who realised the cubes would not fill the entire volume inside the box, and that the box holds just two layers of 15 cubes each. The most popular answer was 41, from dividing the volume of the box by the volume of one cube. Some looked to divide the surface area obtained in part **(a)** by 8, or by 4, or by 2.

Answer: **(a)** 236 **(b)** 30

Question 19

Answers to this question showed that many candidates would benefit from developing their understanding of probability and the manner in which probabilities of combined events are calculated. Many answers for probabilities had values greater than 1.

(a) Some correct answers were seen. Incorrect answers arose from failure to obtain 0.3 from 1 – 0.7; putting fractions over 9 for the second arrow – treating the problem as a 'non-replacement' one; using 0, 0.6, 0.7 instead of 0.3. At times, probabilities were not written on the branches, even when the correct values were used in part **(b)(ii)**.

(b) (i) Many realised that it was necessary to use 0.7. Some obtained the correct answer, but answers such as 0.7×0.7=0.14; 0.7×0.7=4.9; 0.7×2=1.4 and 0.7+0.7=0.14 were common.

(ii) This part proved challenging for many candidates. Many responses only considered one of the two possibilities and gave the answer 0.21. Other incorrect answers were 0.7, from the first branch for hit once; 1 – 0.49, for at least once, instead of exactly once. Too often a wrong answer was given without any working being shown.

Answer: **(a)** Probabilities 0.7 and 0.3 on the correct branches **(b)(i)** 0.49, **(ii)** 0.42

Question 20

(a) Most candidates seemed to know how to find the gradient though some were let down by insecure directed number skills. The common wrong answer was 2.

(b) Many candidates attempted to find the equation of the line correctly, though not all built upon their answer to part **(a)**. Some gave either $y = –2x + c$, or $–2x + 4$ as their final answer.

(c) Many candidates misinterpreted this question as finding the midpoint of the two given points, so obtained the wrong answer (–1.5, 7). In situations such as this candidates are advised to draw a simple sketch. Some candidates, attempting the question correctly, were let down by insecure directed number skills.

Answer: **(a)** –2 **(b)** $y = –2x + 4$ **(c)** (3, –2)

Question 21

(a) There were many good attempts at this part gaining a completely correct answer. Some incorrect elements arose from arithmetic errors. A few candidates attempted to multiply the two matrices.

(b) Most candidates showed that they were familiar with an inverse matrix. Some calculated the determinant as 7; or 17; or –17; or omitted it altogether. Others candidates should revisit how to find the adjoint matrix.

Answer: **(a)** $\begin{pmatrix} 7 & 9 \\ -15 & -16 \end{pmatrix}$ **(b)** $-\frac{1}{7}\begin{pmatrix} -4 & -1 \\ 5 & 3 \end{pmatrix}$

Question 22

(a) This part was answered well by many candidates. The usual mistake was to treat the expression as the difference of two squares. There were some careless slips such as $3a(3a–2a)$

(b) This part was answered well by many candidates. The common wrong answers were $(2–5t)^2$; $(4–5t)(4+5t)$.

(c) Most candidates dealt with this question competently, though some found the negative signs challenging.

Answer: **(a)** $3a(3a–2)$ **(b)** $(2–5t)(2+5t)$ **(c)** $(x+3d)(2c–y)$

Question 23

This question was sometimes omitted, perhaps because of a lack of geometrical instruments, or perhaps because many candidates do not have sound understanding of constructions and loci. Occasionally there was confusion between a perpendicular bisector of a side and the bisector of an angle.

(a) Those who attempted this part usually measured the angle correctly. The common wrong answer was 82°.

(b) This part was done well by many candidates. Better accuracy was achieved by those who joined the points of intersection of two intersecting arcs than those who drew the common tangent to two touching arcs.

(c) Most candidates realised that the circle centre *C*, radius 5 cm, was required, though some of these did not draw the complete circle.

(d) Many candidates positioned *P* and *Q* at the two points of intersection of the constructions in the previous two parts and measured the length of *PQ* to the required accuracy. Others positioned *P* and *Q* at the wrong places.

Answer: **(a)** 97 to 99 inclusive **(d)** 4.3 to 4.9 cm dependent on correct constructions drawn.

Question 24

Attempts at this question showed that some candidates need to gain a better understanding of the properties of a speed-time graph, and to appreciate that:

- the 'D-S-T triangle' is not a valid method when there is an acceleration or a deceleration;
- an acceleration is represented by a part of the graph that has a positive gradient;
- a deceleration is represented by a part of the graph that has a negative gradient;
- The distance travelled during a given interval is obtained by finding the area under the graph for this interval.

(a) Those who obtained the correct answer usually used $\frac{30+12}{2}$, or its equivalent. Wrong answers included 6, from $\frac{30}{5}$, or from $\frac{12\times 5}{10}$; 24, from $\frac{12\times 10}{5}$ or 42, from $\frac{1}{2}(12+30)(10)\div 5$; 18, from $30-12$.

(b) Those candidates who realised that the acceleration is the gradient of the line joining (40, 12) to (60, 30) usually obtained the correct answer. A few calculated the *average* acceleration over the whole interval. Others calculated the deceleration.

(c) Answered well by candidates who recognised which area to calculate.

Answer: **(a)** 21 **(b)** $\frac{18}{20}$ **(c)** 420

Question 25

(a) There were many correct, or partially correct, responses to this part. Common errors included an incorrect inequality symbol; incorrectly converting $7x + 5y = 35$ to the form $y = \ldots$; using $x < 5$ and $y < 4$.

(b) Candidates who realised that it was necessary to find the gradient of *OA* (= 3.5) usually chose the integer 3 as their answer. A few candidates got as far as finding the coordinates of the point *A*, (10/7, 5), but more candidates gave $k = 5/4$ (the gradient of *OB*) as their answer. Other wrong answers were 5/7 or 7/5, coming from the line $7x + 5y = 35$.

Answer: **(a)** $7x + 5y > 35$ and $x < 4$ and $y < 5$ **(b)** 3

Question 26

(a) Usually correct, though sometimes with arithmetic slips.

(b)(i) Many candidates identified the difference of 3, though instead of using this as $3n$ gave the answer $n + 3$. Others gave a numerical answer such as 19 or 64.

(ii) There were fewer correct answers than in the previous part, as many still gave a linear expression in *n*. Some realised that square numbers were involved and gave n^2, $(n + 1)^2$, $n(n + 2)$. Others gave a number or did not attempt this part.

(c) Some candidates realised that the answer could be obtained from their responses to part **(b)(i)** and part **(b)(ii)**. However, some of these subtracted the wrong part from the other, or else simplified $(n + 2)^2 - 3n + 4$. Most candidates either did not attempt this part, or gave a numerical answer, or attempted to find the *n*th term of the black beads sequence.

Answer: **(a)** 49, 19, 30, **(b)(i)** $3n + 4$, **(ii)** $(n + 2)^2$, **(c)** $n^2 + n$

Question 27

(a) Many responses suggested that candidates would benefit from familiarising themselves with the modulus notation of a vector. An answer such as $\begin{pmatrix}-1\\4\end{pmatrix}$, from adding the two vectors, was very common.

Those who understood the modulus notation usually started correctly, though errors in working, such as $\sqrt{(-3)^2+4^2}=\sqrt{7}$; $\sqrt{25}+\sqrt{4}=\sqrt{29}$; $\sqrt{4}$ = 4 occurred frequently.

(b) (i) Most candidates did not attempt this part, or else left their answer as the sum of two vectors instead of giving a single vector. Those who gave a single vector usually gave the correct top row, though the bottom row was sometimes $k + 4$ or $-4k$.

(ii) Many candidates did not attempt this part. Some candidates did not recognise that vector *OM* is a multiple of vector *OT* and worked from $2k - 3 = 24$ to get $k = 13.5$. Others realised that vector *OM* is a multiple of vector *OT* but used the same letter k as the multiplying factor, instead of a different letter, and obtained $k = 4$. A few candidates correctly noticed that the gradients of *OM* and *OT* are equal and solved the equation $\frac{16}{24}=\frac{4}{2k-3}$.

Answer: **(a)** 7 **(b)(i)** $\begin{pmatrix}-3+2k\\4\end{pmatrix}$ **(ii)** $4\frac{1}{2}$

MATHEMATICS D

Paper 4024/21
Paper 2

Key messages

- To succeed in this paper, candidates need to have studied the full content of the syllabus and remember the necessary formulas and apply them appropriately.
- Candidates should use a suitable degree of accuracy in their working. Final answers should be rounded correct to three significant figures where appropriate.
- Where candidates are required to show a result, they must set their work out logically, showing all stages of working leading to the required result.

General Comments

The question paper consisted of a range of question types ranging from routine tasks to more complex problem-solving questions. Candidates of all abilities were able to demonstrate their understanding of a range of mathematical skills. Scripts covering the whole mark range were seen and candidates appeared to have sufficient time to complete the paper.

In general, candidates showed good understanding of standard questions involving arithmetic, algebraic manipulation and trigonometry. Candidates found it harder to access questions where the process required needed to be identified, for example the use of basic trigonometry in **Question 11**.

In questions requiring candidates to show a result, key steps were sometimes omitted. They should show that they are taking a square root, rather than leaving this to be implied, for example.

Candidates need to take care when using mathematical formulas. They were usually able to quote the sine and cosine rules correctly, but errors were sometimes seen in the quadratic formula. The formulas for surface area and volume were provided in **Question 8**, but many candidates did not realise that the question involved a hemisphere rather than a sphere.

In some places, candidates gave an inaccurate final answer due to inappropriate rounding of intermediate results. Typically, candidates should show the value to at least one more significant figure than given in the question. Values of $\frac{22}{7}$ or 3.14 were sometimes used for π which also led to inaccurate final answers. It is important that candidates retain sufficient figures in their working and only round their final answer to three significant figures. Final answers rounded to two significant figures are not acceptable.

Comments on Specific Questions

Section A

Question 1

(a)(i) Most candidates answered this part correctly. Some candidates rounded the answer to $504 which was not appropriate as $503.50 is an exact amount. Some gave the answer as the pay increase rather than the amount earned after the increase and a few also multiplied by either 7 or 24. A common incorrect method was to work out $12.50×38 and add the result to $12.50×0.06.

(ii) Most candidates answered this part correctly. Some gave an answer of 88% which was the percentage of his original earnings rather than the percentage reduction. A common error was to divide by 462, the new amount, rather than 525, the original amount.

(iii) Candidates who identified that $2472 was 103% of the original pay and carried out a reverse percentage calculation reached the correct answer. Many candidates interpreted the question incorrectly and calculated 97% of $2472 leading to the incorrect answer of $2397.84.

(b) Many candidates were able to identify which exchange rate to use in each part of their calculation and apply it correctly to reach the correct result. Some candidates multiplied by both exchange rates rather than dividing to convert from pounds to dollars. Some candidates did not subtract the £260 before converting from pounds to dollars and others simply converted the £260 to dollars.

Answers: **(a)(i)** 503.50 **(ii)** 12 **(iii)** 2400 **(b)** 192

Question 2

(a) Candidates who identified the correct midpoints were usually able to calculate the products correctly and give the correct value for the mean. The most common errors were to use incorrect midpoints, such as 3 in place of 2.5 or the upper bounds of the groups, or to use the group widths rather than midpoints. Most candidates divided their sum by 150, however some simply calculated the sum of frequencies divided by either 50 or 150.

(b) Most candidates were able to draw a histogram with correct width bars with a linear scale added to the frequency density axis. When frequency densities were calculated they were usually correct and bars were drawn accurately. Many candidates used the frequencies as the heights rather than calculating frequency densities. A small number of candidates left gaps between the bars. A small number of frequency polygons were seen.

(c) Few candidates understood that they could estimate the number of calls longer than 25 minutes by adding half of the frequency of the $20 < t \leqslant 30$ group to the frequency of the $30 < t \leqslant 50$ group. Most candidates calculated the percentage of calls over 20 minutes.

Answers: **(a)** 14.35 **(b)** Correct histogram **(c)** 19.3

Question 3

(a) Candidates found it challenging to find the bearing of *A* from *C* and answers of 140 or 050 were common. A small number of candidates appeared to measure the bearing which was not appropriate as the diagram was not drawn to scale.

(b) Most candidates identified that they needed to use the cosine rule to find *BC* and were able to apply it correctly to reach the required result. Some missed key steps from their answer, such as showing the square root symbol in their working or showing a more accurate value than 59.4, such as 59.37. A small number of candidates used 220° in place of 140° in the cosine rule. Some candidates attempted to use the sine rule or Pythagoras theorem.

(c) Candidates who identified that the sine rule could be used to find angle *B* usually reached the correct value of 24.3° for this angle and often went on to give the bearing correctly. Some successfully used the cosine rule to find angle *B*. Candidates who started by calculating angle *C* usually made no further progress to find the correct bearing. A small number of candidates thought that this could be answered without the use of trigonometry and attempted to subtract various values from 360° or 180°.

Answers: **(a)** 040 **(b)** 59.4 correctly derived **(c)** 204.3

Question 4

(a) Many candidates gave the correct probability. A small number gave an answer of either $\frac{4}{9}$ or $\frac{1}{9}$.

(b)(i) Many candidates understood that they needed to square their previous answer to find the probability of two odd numbers. A small number added their two fractions or treated this as a situation without replacement.

(ii) The most common answer in this part was $\frac{20}{81}$, the probability of odd followed by even. Few candidates remembered to consider the other case of even followed by odd. Again in this part, some candidates added their two fractions or treated this as a situation without replacement.

(c) Candidates who realised that the problem had now changed to one where the balls were not replaced often considered both cases of odd followed by odd and also even followed by even and reached the correct result. Some were confused about the number of balls in the bag and used denominators of 8 and 7, even though the question had stated that there were 9 balls in the bag. In some cases, candidates added probabilities when they should have been multiplied, and then multiplied the results of these additions. Candidates who reached a probability greater than 1 did not identify that their answer must have been incorrect.

Answers: **(a)** $\frac{5}{9}$ **(b)(i)** $\frac{25}{81}$ **(ii)** $\frac{40}{81}$ **(c)** $\frac{4}{9}$

Question 5

(a) Most candidates were able to eliminate the fractions and expand the brackets correctly as the first step of solution of the equation. Some made sign errors in their rearrangement to a quadratic equation, but then many showed a correct method to solve their quadratic. A few candidates did not rearrange their quadratic to equal zero, but instead attempted to solve $y^2 - y = 6$ for example.

(b) In this part, many candidates correctly eliminated the fraction and reached $2p - pt = 4t + 1$. The next step should have been to isolate the t terms, but many candidates were not able to do this successfully. Candidates who had reached a formula with two terms in t were often able to factorise and rearrange to make t the subject of their formula. In this part, sign errors were common when rearranging and some candidates did not include t = in their final answer.

(c) Many candidates answered this part correctly. It was common to see the denominator factorised correctly but candidates were less successful in factorising the numerator. Some candidates attempted to cancel terms incorrectly without using any factorisation.

Answers: **(a)** $y = -3$, $y = 2$ **(b)** $t = \frac{2p-1}{4+p}$ **(c)** $\frac{3x-2}{x+4}$

Question 6

(a) (i) Candidates who identified that angle *ABC* was the angle in a semicircle reached the correct answer in this part. It was common to see an answer of 22° resulting from the incorrect assumption that angle *BFC* was a right angle.

(ii) In this part, even if the angle was stated correctly, few candidates were able to give a correct reason. Candidates should be able to state the reason ‘angles in the same segment are equal’ as is stated in the syllabus. It was common to see vague reasons such as ‘same chord’ or ‘angles in segment’ or incorrect reasons such as ‘isosceles triangle’ or ‘alternate angles’.

(iii) Some candidates were able to apply the fact that angles in opposite segments are supplementary to find this angle correctly.

(iv) Candidates who had the correct value for angle *CDE* often found this angle correctly. In other cases it was unusual to see candidates using the parallel lines to find angle *DCF* correctly so they were seldom able to find the correct value for angle *BCD*.

(b) Many candidates identified that the sum of the angles in a pentagon is 540°, set up and solved an equation and found the size of the largest angle correctly. Only a small proportion of candidates simply found the value of x. Where 540° was not used, it was common to see candidates setting up an equation using either 360° or 180° as the angle sum.

(c) In a calculation involving bounds, candidates should find the bounds of the given quantities and use these in their calculation. In this case, as the lower bound was required and the calculation was a subtraction, they needed to use the upper bounds in their calculation. Few candidates were able to do this and it was common to see the given angles subtracted from 360° with a correction of 0.5° made to the result which is an incorrect method. Some candidates did find bounds before the subtraction, but usually found the lower bounds so their final answer was the upper bound of the value of *y*. A small number of candidates thought that 360° should be adjusted to 359.5° which is incorrect as 360° is an exact value.

Answers: **(a)(i)** 38° **(ii)** 38°, angles in same segment are equal **(iii)** 112° **(iv)** 106° **(b)** 156° **(c)** 105.5°

Section B

Question 7

(a) (i) Many correct answers were seen in this part. Candidates were usually able to show the correct method to find the gradient of the line, but some made errors in the calculation and gradients of 2 or $-\frac{1}{2}$ were sometimes obtained.

(ii) It was common for candidates to identify that the gradients of the two lines were the same, but they were not always able to use the given coordinates to work out the *y*-intercept of line M. Some used the value 3, the *y*-coordinate given in the question. A number of candidates gave the gradient of line M as $\frac{1}{2}$, a result of using perpendicular lines.

(b) (i) Almost all candidates were able to calculate the missing value of *y*.

(ii) Most candidates used the given scale to draw axes and plotted the points accurately. Some good, smooth curves were seen, but many candidates joined from $x = 1.5$ to $x = 2$ with a line rather than a curve.

(iii) Most candidates were able to draw a tangent at the correct point, although they made errors in their calculation of the gradient. It was common to see a positive gradient rather than a negative gradient. Some candidates calculated (change in *x*) ÷ (change in *y*). Some tangents touched the curve at $x = 1$, but were not accurately drawn leading to an inaccurate value for the gradient.

(iv) Candidates who identified that they needed to read from their curve at $y = 2$ gave correct solutions. Some read values at $y = 5$ and others attempted to solve the given equation algebraically.

Answers: **(a)(i)** $y = –2x + 5$ **(ii)** $y = –2x – 1$ **(b)(i)** 3.5 **(ii)** correct curve **(iii)** –2.4 to –1.6 **(iv)** 0.6 to 0.8, 4.2 to 4.4

Question 8

(a) Most candidates used Pythagoras theorem correctly to reach the required result. Most reached the stage $\sqrt{180}$ but many did not show the value 13.42 or 13.416 in their working. A small number of candidates attempted to answer this part using the formulas given in the question.

(b) Most candidates identified the correct formulas required in this part and calculated the surface area of the cone correctly. It was common to find the surface area of a sphere rather than a hemisphere and some also added the area of a circle from the base of the cone. A small number of candidates used the height, 12 cm, of the cone rather than the slant height, 13.4 cm. In calculations involving π, candidates should use their calculator value or 3.142 rather than 3.14 or $\frac{22}{7}$.

(c) Most candidates used the correct formulas in this part and calculated the volume of the cone correctly. Again, the formula for the volume of a sphere rather than a hemisphere was often used. Some candidates used the slant height, 13.4 cm, or the height of the solid, 18 cm, in place of the height of the cone, 12 cm.

(d) (i) Many candidates attempted to use the given ratios to find the dimensions of solid *B* and then calculate the surface area rather than use the area factor with their answer to part **(b)**. This strategy rarely led to the correct answer. Candidates who did use a ratio method usually multiplied by the scale factor, 3, rather than the area factor, 9. Some used the given ratio incorrectly and used a scale factor of $\frac{6}{2+6+1}$.

(ii) Similar errors were seen in this part, with some candidates multiplying by the scale factor, $\frac{1}{2}$, rather than the volume factor, $\frac{1}{8}$. It was also common to see the given ratio used incorrectly to result in a scale factor of $\frac{1}{2+6+1}$.

Answers: **(a)** 13.4 correctly derived **(b)** 479 **(c)** 905 **(d)(i)** 4310 **(ii)** 113

Question 9

(a) Many candidates used ratio correctly to calculate the time taken.

(b) (i) Candidates who understood that the time could be found by dividing the capacity by the rate gave the correct answer.

(ii) Candidates found forming the required equation challenging. Many of those who found two correct expressions for time subtracted the time using the small pump from the time using the large pump rather than the large from the small as required. Some correct elimination of fractions and rearrangement was seen, though few candidates showed a complete correct solution in this part.

(iii) Many candidates were able to solve the quadratic equation correctly, though the solutions were not always given correct to two decimal places as required. Some errors were seen usually resulting from an incorrect quadratic formula or from sign errors in evaluating the discriminant.

(iv) Some candidates substituted the positive solution into the correct expression for the time for the large pump. Many candidates were unable to then convert this time in minutes correctly into minutes and seconds, with 36.45 minutes often given as 36 minutes 45 seconds.

Answers: **(a)** 7 **(b)(i)** $\frac{2500}{x}$ **(ii)** $3x^2 + 60x - 10\,000 = 0$ correctly derived **(iii)** 48.59 and –68.59;
(iv) 36 minutes 27 seconds

Question 10

(a)(i) Many candidates were able to rotate triangle *B* through 180°, although some used an incorrect centre resulting in a translation of the correct answer. Some reflections of triangle *A* were seen.

(ii) Candidates were more successful at transforming triangle *A* using the given matrix than using directions as given in **(a)(i)**. Few correct matrix calculations were seen, with many candidates identifying that the given matrix would give an enlargement with scale factor 3 centred on the origin so no calculation was required.

(iii) Some candidates were able to identify the correct matrix for the inverse transformation. It was common to see answers of $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ where candidates knew the form of the matrix but were not able to state the correct values.

(iv) Many candidates identified that the transformation was an enlargement, but few were able to give a complete description. It was common to see a scale factor of –3 rather than $-\frac{1}{3}$ and the centre, if given, was often (0, 0).

(b) (i) Some candidates were able to find the vector correctly. Common errors were to add the two given vectors or to calculate $\begin{pmatrix} 8 \\ -2 \end{pmatrix} + 2\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ rather than $\begin{pmatrix} -8 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 6 \\ 3 \end{pmatrix}$.

(ii) Some candidates identified that $\overrightarrow{ST} = \frac{1}{4}\overrightarrow{SR}$ and reached a correct answer in this part. It was common for candidates to use the ratio incorrectly and assume that $\overrightarrow{ST} = \frac{1}{3}\overrightarrow{SR}$.

Answers: **(a)(i)** Triangle B at (2, –3), (3, –3), (3, –5) **(ii)** Triangle C at (3, 3), (3, 9), (6, 3) **(iii)** $\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$; **(iv)** Enlargement, centre (3, –1.5), scale factor $-\frac{1}{3}$ **(b)(i)** $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 9 \\ 0 \end{pmatrix}$

Question 11

(a) The best answers in this part involved stating the three pairs of equal angles together with correct reasons involving vertically opposite angles and alternate angles. Many candidates were not aware that this was required to show that the triangles were similar. It was common to see vague statements involving the triangles sharing point *R* and the sides being parallel. Some candidates stated that the sides were in proportion with no evidence to justify this.

(b) (i) Candidates who identified the right-angled triangle *ADQ* were often able to use trigonometry correctly to calculate *AQ*. Some inaccurate answers were seen where candidates had rounded the value of cos 55 to 0.57 in their calculation rather than using the accurate value. Other candidates gave an answer of 8.7 when an answer to a minimum of three significant figures is required.

(ii) In this part, those candidates who were able to identify the right-angled triangle *ARB* were often able to use trigonometry correctly to calculate *AR*. Some used a longer method of finding *BR* and then using Pythagoras theorem to find *AR*. Similar problems to those in the previous part involving inaccurate values were seen in here.

(iii) Many candidates were able to calculate the area of the triangle correctly using $\frac{1}{2}ab\sin C$. In some cases 55° was used as the angle in place of 35°. Some candidates calculated $\frac{1}{2}$ base × height using 9 cm as the height of the triangle, but others used Pythagoras theorem to first calculate *AR* and reached the correct result.

(iv) Most candidates found this part challenging. Some used the fact that the triangles were similar to work out the scale factor of the enlargement of triangle *ARB* to triangle *PRQ*. This scale factor was not always squared to reach the area factor to calculate the total shaded area. Candidates who found lengths in triangle *PRQ* often did many stages of calculation to reach the lengths they needed to find the area. This was sometimes successful, but errors in calculations often led to an incorrect final result.

Answers: **(a)** Complete proof of similarity **(b)(i)** 8.72 **(ii)** 7.37 **(iii)** 19.0 **(iv)** 19.6

MATHEMATICS D

Paper 4024/22
Paper 2

Key messages

To succeed in this paper, candidates need to have completed full syllabus coverage and be able to apply what they know in appropriate contexts.

Candidates should show all working clearly and attempt to present their work in an orderly fashion.

Numerical working should in general maintain 3 significant figure accuracy.

General comments

Candidates at all levels were able to demonstrate what they knew and could do. In addition, those questions containing a number of method steps challenged candidates to choose appropriate techniques and to organise their work into a coherent strategy.

In a number of questions decisions about the operations, and techniques required, had to be made from the context of the question, for example the arithmetic in **Question 1** and the trigonometry in **Question 11**.

Whereas many candidates know how to handle particular calculations and rules when they are clearly signalled, there is often difficulty in choosing the correct step in a problem solving situation.

The main question covering mensuration was such that, on this occasion, all the formulae needed had to be remembered by candidates. This led to some confusion at various levels. As well as inaccurately remembered formulae, it was quite common to see, for example, a volume formula used when an area one was needed.

Good work was consistently seen in every curriculum area with the exception of the Geometry content of **Question 8**. Few responses to this question gained good marks. As well as calculations, statements of justification were required in order to obtain full marks. Candidates were generally let down by being unable to give the reasons why, for example, certain angles were equal.

There were many well presented scripts that showed relevant working clearly. Some candidates need to be reminded of the importance of showing all their working, and the need for care in forming letters and numbers that can be read easily by examiners, and in some cases, by the candidates themselves.

Comments on specific questions

Section A

Question 1

(a) The basic calculations required at the start of this question, the fraction or the percentage of the cash price and the totals of monthly payments, were generally correct. The expected next step was to decide which of these amounts needed to be combined to give the total costs of options A and B. The correct decisions here usually resulted in a successful outcome for the whole question, although when it came to the final step, instead of basing their answer on the difference between the costs of each option, some candidates stated the cheaper option as their final answer. Some of the less successful attempts at this question used subtraction in the wrong places, and there were one or two sightings of the simple interest formula.

(b) The steps needed in this question seemed to be evident to many candidates. Most realised that they required the number of litres the car would use in travelling 240 kilometres. The cost could then be worked out. There was some confusion at this point in using the rate $0.85 per litre. Division was sometimes seen. Full marks required the final answer to be correct to **two** decimal places.

(c) Many candidates still rely on rote learning for reverse percentages, rather than trying to build up steps from the description of the question. This often resulted in the calculation of 15% of $322 and sometimes 85% or even 115%. It was pleasing to see an algebraic approach at the core of some of the successful solutions.

Answers: **(a)** A by 240 **(b)** 10.61 **(c)** 42

Question 2

(a) (i) The idea of cumulative frequency was understood by most candidates. Some tables were completed using four numbers leading to 120, such as 40, 60, 80, 100 or 24, 48, 72, 96.

(ii) Plotting was usually accurate and drawing acceptable. Some block graphs were seen but not many.

(b) (i) A good response in this part.

(ii) Many graphs were used correctly to obtain the interquartile range. Candidates had to remember that one small square horizontally represented two minutes, not one. It was clear in some solutions that candidates knew the quartiles would be found using 30 and 90 on the vertical scale, but they then gave their answer as 60, or used 60 again to read off the value given for the median in the previous part.

(c) There were many good answers in this part. There were candidates who based their products in $\Sigma f(x)$ on interval width, and others who used the cumulative frequencies for f. Division by 5 was sometimes seen.

Answers: **(a)(i)** 12 40 85 107 **(ii)** Correct cumulative frequency curve **(b)(i)** 47 to 49 **(ii)** 28 to 32 **(c)** 49.3

Question 3

(a) There were many good first steps seen by candidates when solving this part. The instruction to express their answer as a column matrix was ignored by some candidates.

(b) (i) There was a wide variety of answers in this part.

(ii) Candidates who arrived at the expected matrix in the previous part usually understood that the elements represented amounts of money rather than numbers of shirts.

(c) (i) This matrix multiplication was often carried through correctly, but there was a tendency for candidates to add all the elements to end up with a matrix containing a single element. There were many candidates who were not helped by having the multiplication laid out for them in the question.

(ii) Many candidates understood the idea of percentage profit. There were correct answers from candidates who had had difficulty with the matrix representation in previous parts. They clearly had understood what had been going on throughout the question.

Answers: **(a)** $\begin{pmatrix} 5 \\ 6 \\ 8 \end{pmatrix}$ **(b)(i)** $\begin{pmatrix} 440 \\ 540 \end{pmatrix}$ **(ii)** The amount Anya makes for men's T-shirts and women's T-shirts,

(c)(i) $\begin{pmatrix} 290 & 630 & 537.50 \end{pmatrix}$ **(ii)** 48.7%

Question 4

(a) (i) This part was done well by many candidates. A number of responses placed triangle *B* out of position but nevertheless demonstrated an understanding the basic geometrical properties of translation. Quite a number changed the size and orientation of *A*.

(ii) Candidates found this part more difficult. It was sometimes omitted. The most common error was to use a scale factor of +2. Solutions often showed the construction lines used.

(b) (i) This part was sometimes omitted. Given that the matrix representing the transformation was given in the question, there were not many matrix multiplications seen as the strategy to find the coordinates of triangle *Q*.

(ii) Many attempts picked up the idea of a scale factor of 3 from the matrix given. They realised that there would have to be an invariant line, usually an axis, but often there was uncertainty about which axis to choose. The description '*y* is the invariant line' was not accepted.

Answers: **(a)(i)** Triangle *B* at, $(4,-1)$, $(4,-4)$, $(5,-4)$ **(ii)** Triangle *C* at $(1,4)$, $(3,4)$, $(3,-2)$
(b)(i) Triangle *Q* at $(3,1)$, $(9,1)$, $(6,3)$ **(ii)** Stretch factor 3, *y*-axis invariant or parallel to *x*-axis

Question 5

(a) The basic principles of combining algebraic fractions to obtain a single numerator and a single denominator were usually evident. This often led to a correct conclusion. In other cases, problems arose removing brackets and collecting terms. A common error for example was the appearance of -10. Collecting terms were often mixed up with various degrees of false cancelling.

(b) Candidates with assured basic skills in algebraic manipulation did well in this part. There were accurate solutions using both factorisation and the formula. Others were let down again by mistakes in removing brackets and collecting like terms. A common error seen was $2x(x+1)=2x^2+2$. A number of candidates did not see the need to combine both sides of the equation given in the question into a single equation to solve.

(c) (i) This part was generally well done.

(ii) Candidates still on track with the correct equations did well in this part. Generally the method of equalising coefficients was more reliable than substitution on account of the algebra needed to deal with fractions. Care was always needed however when subtracting two equations.

Answers: **(a)** $\frac{14-x}{(x-2)(x+1)}$ **(b)** –4 or 1.5 **(c)(i)** $3p+2n=4.80$, $5p+4n=9.00$ **(ii)** 0.60 1.50

Question 6

(a) (i) Most candidates realised that the set $A \cap B$ contained the element 15 only, but often gave this as their answer.

(ii) Generally well done, but care was needed not to omit 15. Many answers listed 10 and 20 only.

(iii) When understood, a common error was to give $\frac{7}{10}$ as the answer. When not fully understood, numbers could be given that were not probabilities. Sometime candidates thought that some kind of set was required.

(b) (i) A good number of correct answers were seen, with most candidates offering a Venn diagram as part of their working. Some of these diagrams were clearly annotated in algebraic terms. Some candidates understood the question but were unable to express it correctly in diagrammatic form.

(ii) This was a challenging part that was well understood by a minority of candidates. Many realised that a product of probabilities would be required here. The selection of appropriate ones generally proved elusive.

Answers: **(a)(i)** 1 **(ii)** 10, 12, 14, 15, 16, 18, 20 **(iii)** $\frac{7}{11}$ **(b)(i)** 8 **(ii)** $\frac{28}{45}$

Section B

Question 7

A popular choice of question.

(a) (i) There were many correctly completed tables. A common error was to give one entry as –4.5 and the other as +4.5.

(ii) The high standard of plotting and drawing seen over recent years was generally well maintained. Candidates scored well in this part of the question. Some made difficulties for themselves by opting to use different scales from those given in the question.

(iii) Correct tangents were frequently seen and very often followed up by forming the correct difference fraction for the gradient.

(iv)(a) Very few correct answers were seen in this part, but when *k* was found to be –2, candidates generally went on to a successful outcome in the next part. There were a few candidates who got very close to –2 by solving the equation $10 + 2x - x^2 = 0$ at this point. This was not accepted as this part of the question was the first step in the graphical method of solving this equation.

(iv)(b) Candidates who followed the instructions given in the question, and using their value of *k,* went on to gain marks.

(b) (i),**(ii)**,**(iii)**
The graphical context of this question involving indices proved challenging for many candidates. They seemed to have difficulty in relating the coordinates to the equation. When attempted, all three parts were usually correct. There was some isolated success with part **(i)**.

Answers: **(a)(i)** –4.5, –4.5 **(ii)** Correct smooth curve **(iii)** –2.4 to –1.6 dependent on tangent drawn **(iv)(a)** –2 **(iv)(b)** –2.4 to –2.3 and 4.3 to 4.4 **(b)(i)** 4 **(ii)** 3 **(iii)** 324

Question 8

This question was often omitted. It was clear from the responses to this question that candidates had difficulty in appreciating its implications.

In part **(a)** it was difficult for candidates to build up a good score since a mark for a 'correct' reason relied on the correct expression for the angle. Candidates sometimes stated, for example, that the angles were equal rather than the reason why they were equal.

Throughout this question, some candidates ignored the instruction to give the angles in terms of *y.* Answers as numbers of degrees were common, as were answers such as $G\hat{E}O = F\hat{E}O$.

(a) (i) Some correct expressions in terms of *y* were seen but few gave a satisfactory justification in terms of angle at the centre and angle at the circumference.

(ii) Some correct expressions were seen but few adequate reasons referring to radius and tangent were stated. Most candidates realised that a right angle was involved. There were some attempts to give alternative reasons such as using the angles of a triangle, but these were not developed into acceptable responses.

(iii) Some correct expressions were again seen, but often without full supporting reasons. Angle in a semicircle was rarely seen, but worthy references to a cyclic quadrilateral were sometimes written.

(b) This part was generally answered correctly.

(c) Better responses identified pairs of equal angles. It was expected that angles would be identified by the use of 3 letters, such as $D\hat{A}C$; 'angle *A*' was not accepted. Some of this work was spoiled by candidates giving numbers of degrees as justification for the statements made. There were also references to equal sides.

(d) Generally correct. Kite or rhombus were sometimes given.

(e) (i),(ii) Few responses to this final part of the question were correct. The common answers were 1 : 2 and 1 : 4. The correct answer to part **(ii)** was rarely seen.

Answers: **(a)(i)** $\frac{y}{2}$ angle at centre = twice angle at circumference **(ii)** $90 - y$ angle between radius and tangent = 90° **(iii)** $2y$ angle in a semicircle = 90° **(b)** *EFC* **(c)** Two pairs of equal angles given with conclusion **(d)** Trapezium **(e)(i)** 1 : 4 **(ii)** 1 : 8

Question 9

(a) A well answered question with 3 significant figure accuracy was usually achieved. Candidates should be aware that in calculations involving π, they should use their calculator value or 3.142. They should also be aware that the formula for the volume of a cylinder is never given as part of a question and therefore needs to be memorised.

(b) Completely correct solutions were not common, but most candidates picked up method marks for correct steps in the process. Successful candidates could see at once that the cross section consisted of a major sector and a triangle, and the area of the triangle could be calculated directly using the formula $\frac{1}{2}ab\sin C$. The problem became more difficult for candidates to organise when longer methods were adopted for the area of the triangle and when the area of the minor sector became part of the calculation. This led to confusion between addition and subtraction at the next stage. Again, the formulae required in this part must be memorised.

(c) Most candidates realised that the time required would be given by dividing the distance travelled by the speed. Correct solutions required the use of appropriate units in this calculation and the correct conversion of a number of minutes to minutes and seconds. Candidates working correctly in minutes often reached 2.33…, giving 2 minutes 33 seconds as their answer.

(d) The diagram here alerted many candidates to the use of the sine rule which they handled confidently to find $P\hat{Q}T$. There was often a successful conclusion, although in some cases this mark was lost because $P\hat{Q}T$ had lost its 3 significant figure accuracy.

Answers: **(a)** 7.54 **(b)** 53.7 **(c)** 2 minutes 20 seconds **(d)** 146.5°

Question 10

(a) By taking the expected route that the area given was $(x+4)(3x+4)$, some candidates quickly gained full marks for this part. Quite a number preferred to find the area of the frame first, using $(x+4)(3x+4)\ -3x^2$, adding the $3x^2$ back in to complete their equation. Many candidates however thought the total area was $(x+4)(3x+4)+3x^2$, making 'adjustments' to obtain the final equation. There seemed to be many unsuccessful attempts to work in reverse from the final equation.

(b) This part was generally well answered, with a good number of candidates able to factorise the given quadratic. This approach seemed more reliable than the use of the quadratic formula, where the negative signs in the discriminant often led to errors. This is another formula that candidates need to memorise.

(c) Few correct responses were seen here.

(d) There were very few correct answers seen. Many candidates here based all their working on the numbers 5 mm and 0.7 g, not realising that the 0.7 g related to volume. The common method adopted to incorporate the volume was to calculate the volume of the complete solid frame without the cut out for the picture.

Answers: **(a)** $3x^2 + 16x - 460 = 0$ correctly derived **(b)** 10 and $-\frac{46}{3}$ **(c)** 14 and 34 **(d)** 61.6

Question 11

(a) Candidates were able to use both trigonometry and Pythagoras' Theorem effectively to find *AY.* Full marks depended on showing how their value for *AY* rounded to 2.58. It was not sufficient to conclude their calculation with 2.58. Methods assuming that *AY was* 2.58 scored only the method mark for a correct expression involving Pythagoras' Theorem.

(b) Well answered with Pythagoras' Theorem again used effectively.

(c) Many candidates favoured the cosine rule in this part, sometimes finding the wrong angle. Others were able to split the triangle *APB* into two right angle triangles and find the relevant angle that way.

(d) (i) At this stage in the question it was clearly becoming more difficult to isolate the appropriate right angled triangle. It was unusual to see it drawn out to accompany the working in this part, although when this can be done, it is to be recommended. There was a good understanding of the meaning of angle of elevation and that the tangent ratio would be required. Care was always needed to use the correct dimensions in the final triangle used.

(ii) Again, better responses focused on the appropriate right angle triangle for which the data required was by now already known.

Answers: **(a)** Need to see 2.58 rounded from a correctly obtained 2.581 **(b)** 7.93 **(c)** 26.6° **(d)(i)** 11.2 **(ii)** 80.7°