Cambridge Assessment International Education

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1364711255

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/51**

Paper 5 Investigation (Core) **October/November 2021**

1 hour 10 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **8** pages. Any blank pages are indicated.

Answer **all** the questions.

ADDING SQUARE NUMBERS

This investigation looks at adding two or more square numbers to make another square number.
In this investigation all numbers are positive integers.

1 Complete the list of the first six square numbers.

$1^2 = 1$ $2^2 = ...$ $3^2 = 9$ $4^2 = ...$ $5^2 = ...$ $6^2 = 36$ [1]

2 **(a)** Work out

(i) 9^2,

..... [1]

(ii) 40^2.

..... [1]

(b) Show that $9^2 + 40^2 = 41^2$.

[2]

3

When $a^2 + b^2 = c^2$ then (a, b, c) is a *3-square set*.
a, b and c are positive integers.

Example
In **Question 2(b)**, $a = 9$, $b = 40$ and $c = 41$.
$9^2 + 40^2 = 41^2$, so (9, 40, 41) is a 3-square set.

When $a^2 + b^2 = c^2$ then $c = \sqrt{a^2 + b^2}$.

Use this formula and any patterns you notice to complete the table on the next page for 3-square sets.

a	b	c
3	4	5
5	12	13
7	24	25
9	40	41
11	60	
13	84	85
	112	113
	144	
19		181
21		221
25	312	313

[6]

4 When $a^2 + b^2 + c^2 = d^2$ then (a, b, c, d) is a *4-square set*.
It is possible to make a 4-square set using two rows in the table.

Example From the table row two $5^2 + 12^2 = 13^2$
row six $13^2 + 84^2 = 85^2$

Replace 13^2 in the second equation with $5^2 + 12^2$ from the first equation: $5^2 + 12^2 + 84^2 = 85^2$.

So (5, 12, 84, 85) is a 4-square set.

Use the same method with rows from the table to find two more 4-square sets.

(..... , ... , ... , ...) and (..... , ... , ... , ...) [3]

5 **(a)** Show that (6, 12, 12, 18) is a 4-square set.

[2]

(b) k is any positive integer greater than 1.
If (ka, kb, kc, kd) is a 4-square set, then $(ka)^2 + (kb)^2 + (kc)^2 = (kd)^2$.

Show that (a, b, c, d) must also be a 4-square set.

[2]

(c) The numbers in the 4-square set (6, 12, 12, 18) have common factors.

(i) Find a common factor of 6, 12, 12 and 18 that is greater than 1.

..... [1]

(ii) Use (6, 12, 12, 18) and **part (b)** to find a 4-square set where a, b, c and d do not have a common factor greater than 1.

(..... , ... , ... , ...) [2]

6 Here is another method for finding a 4-square set (a, b, c, d).

Choose two positive integers a and b with a less than b.

Then $c = \frac{a^2 + b^2 - 1}{2}$ and $d = \frac{a^2 + b^2 + 1}{2}$ make the 4-square set (a, b, c, d).

(a) Use this to find a 4-square set when

(i) $a = 2$ and $b = 3$,

(2, 3, ... , ...) [3]

(ii) $a = 2$ and $d = 43$.

(2, ... , ... , 43) [3]

(b) (i) Use your answers to **part (a)** and any patterns you notice to complete the table for 4-square sets that start with 2.

a	b	c	d
2	3		
2	5	14	15
2	7	26	27
2			43
2			

[3]

(ii) Write down an equation connecting c and d.

..... [1]

(c) When a and b are both even then $c = \frac{a^2 + b^2 - 1}{2}$ and $d = \frac{a^2 + b^2 + 1}{2}$ do not give a 4-square set.

Give an example to show this.

[2]

(d) When a and b are both odd there are no 4-square sets.

In a 4-square set, $d = 23$.

(i) Show that $a^2 + b^2 = 45$.

[1]

(ii) Find a 4-square set when $d = 23$.

(..... , ... , ... , 23) [2]

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