Cambridge IGCSE

Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

7566243408

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/52**

Paper 5 (Core) **May/June 2019**

1 hour

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 24.

Answer **all** the questions.

SQUARE ROOTS WITHIN SQUARE ROOTS

This investigation looks at sequences of terms with square roots.

You can form a sequence by using square roots within square roots.

$\sqrt{6}$, $\sqrt{6+\sqrt{6}}$, $\sqrt{6+\sqrt{6+\sqrt{6}}}$, $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}$, …

You can calculate the first four terms of this sequence as follows.

$\sqrt{6} = 2.4494\ldots$

$\sqrt{6+\sqrt{6}} = \sqrt{6+2.4494\ldots} = \sqrt{8.4494\ldots} = 2.9068\ldots$

$\sqrt{6+\sqrt{6+\sqrt{6}}} = \sqrt{6+2.9068\ldots} = \sqrt{8.9068\ldots} = 2.9844\ldots$

$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}} = \sqrt{6+2.9844\ldots} = \sqrt{8.9844\ldots} = 2.9974\ldots$

1 **(a)** Complete each part of the calculation below to work out the next term of the sequence, writing each decimal as far as the 4th decimal place.

$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}} = \sqrt{6+\ldots\ldots\ldots\ldots} = \sqrt{\ldots\ldots\ldots\ldots\ldots} = \ldots\ldots\ldots\ldots\ldots$

(b) As the sequence continues, the terms get closer and closer to an integer.
This integer is the *integer limit* of the sequence.

Write down the integer limit of this sequence.

.....

2 Here is a similar sequence of square roots.

$$\sqrt{30},\quad \sqrt{30+\sqrt{30}},\quad \sqrt{30+\sqrt{30+\sqrt{30}}},\quad \sqrt{30+\sqrt{30+\sqrt{30+\sqrt{30}}}},\quad \ldots$$

(a) Calculate the first three terms, writing each decimal as far as the 4th decimal place.

..... , ... , ...

(b) Write down the integer limit of this sequence.

.....

3 **(a)** Complete this table for sequences similar to those in **question 1(a)** and **question 2**.

1^{st} term	Integer limit
$\sqrt{2}$	
$\sqrt{6}$	
$\sqrt{12}$	4
$\sqrt{20}$	
$\sqrt{30}$	
$\sqrt{42}$	7

(b) **(i)** Use **part (a)** to find the first term of the sequence that has an integer limit of 8.

.....

(ii) Calculate the 2nd term of the sequence in **part (i)**, writing the decimal as far as the 4th decimal place.

.....

4 The general sequence is $\sqrt{k}$, $\sqrt{k+\sqrt{k}}$, $\sqrt{k+\sqrt{k+\sqrt{k}}}$, $\sqrt{k+\sqrt{k+\sqrt{k+\sqrt{k}}}}$, …
The integer limit of the sequence is the integer N.

For such sequences, $k = N(N-a)$, where a is a constant.

(a) Use the last row of the table in **question 3(a)** to find the value of a.

.....

(b) Use $k = N(N-a)$ to show that

$$\sqrt{90},\quad \sqrt{90+\sqrt{90}},\quad \sqrt{90+\sqrt{90+\sqrt{90}}},\quad \sqrt{90+\sqrt{90+\sqrt{90+\sqrt{90}}}},\quad \ldots$$

has an integer limit of $N = 10$.

(c) Find the first three terms, **in square root form**, of the sequence that has an integer limit of $N = 26$.

..... , ... , ...

5 Here is the general form of another sequence of square roots with integer limit N.

$$\sqrt{k},\ \sqrt{k+2\sqrt{k}},\ \sqrt{k+2\sqrt{k+2\sqrt{k}}},\ \sqrt{k+2\sqrt{k+2\sqrt{k+2\sqrt{k}}}},\ \ldots$$

For such sequences, $k = N(N-a)$, where a is a constant.

When $k = 24$ the integer limit of the sequence is $N = 6$.

(a) Find the value of the constant a.

.....

(b) **(i)** Find the value of k when the integer limit is $N = 5$.

.....

(ii) Write down the 3rd term of the sequence in **part (i)** in square root form.

.....

(iii) Calculate the 3rd term, writing the decimal as far as the 4th decimal place.

.....

6 Here is the general form of another sequence of square roots with integer limit N.

$$\sqrt{k},\quad \sqrt{k+5\sqrt{k}},\quad \sqrt{k+5\sqrt{k+5\sqrt{k}}},\quad \sqrt{k+5\sqrt{k+5\sqrt{k+5\sqrt{k}}}},\quad \ldots$$

For this sequence, $k = N(N-a)$, where a is a constant.

The sequence $\sqrt{14},\quad \sqrt{14+5\sqrt{14}},\quad \sqrt{14+5\sqrt{14+5\sqrt{14}}},\quad \sqrt{14+5\sqrt{14+5\sqrt{14+5\sqrt{14}}}},\quad \ldots$
has an integer limit of $N = 7$.

Find the value of the constant a.

.....

Question 7 is printed on the next page.

7 Here is the general sequence with integer limit N.

$$\sqrt{k},\ \sqrt{k+x\sqrt{k}},\ \sqrt{k+x\sqrt{k+x\sqrt{k}}},\ \sqrt{k+x\sqrt{k+x\sqrt{k+x\sqrt{k}}}},\ \ldots$$

For all such sequences, $k = N(N-a)$, where a is a constant that depends on the value of x.

(a) (i) Use **question 4**, **question 5** and **question 6** to complete the table.

	x	a
Question 4	1	
Question 5		
Question 6	5	

(ii) Write down an expression for k in terms of N and x.

.....

(b) Find the integer limit, N, of this sequence.

$$\sqrt{7},\ \sqrt{7+6\sqrt{7}},\ \sqrt{7+6\sqrt{7+6\sqrt{7}}},\ \sqrt{7+6\sqrt{7+6\sqrt{7+6\sqrt{7}}}},\ \ldots$$

.....