Cambridge Assessment International Education

Specimen Paper Answers

Paper 2

Cambridge IGCSE®

Additional Mathematics 0606

Cambridge O Level

Additional Mathematics 4037

For examination from 2020

Version 1

Cambridge Pathway

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Contents

Introduction

The main aim of this booklet is to exemplify standards for those teaching Cambridge IGCSE Additional Mathematics 0606 and to show examples of very good answers.

This booklet contains answers to Specimen Paper 2 (2020), which has been marked by a Cambridge examiner. Each answer is accompanied by a brief commentary explaining its strengths and weaknesses. These examiner comments indicate where and why marks were awarded and how answers could be improved

The Specimen Papers and mark schemes are available to download from the School Support Hub www.cambridgeinternational.org/support.

2020 Specimen Paper 2
2020 Specimen Paper 2 Mark Scheme

Past exam resources and other teacher support materials are also available on the School Support Hub.

The mark scheme should be read alongside the examiner comments.

Types of mark

M	Method marks, awarded for a valid method applied to the problem.
A	Accuracy mark, given for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B	Mark for a correct result or statement independent of Method marks.

Assessment overview

All candidates take **two** papers.

All candidates take:		**and:**	
Paper 1	2 hours 50%	**Paper 2**	2 hours 50%
80 marks		80 marks	
Candidates answer all questions		Candidates answer all questions	
Scientific calculators are required		Scientific calculators are required	
Assessing grades A* – E		Assessing grades A* – E	
Externally assessed		Externally assessed	

Assessment objectives

The assessment objectives (AOs) are:

AO1 Demonstrate knowledge and understanding of mathematical techniques

Candidates should be able to:

- recall and use mathematical manipulative techniques
- interpret and use mathematical data, symbols and terminology
- comprehend numerical, algebraic and spatial concepts and relationships.

AO2 Apply mathematical techniques

Candidates should be able to:

- recognise the appropriate mathematical procedure for a given situation
- formulate problems into mathematical terms and select and apply appropriate techniques.

Weighting for assessment objectives

The approximate weightings allocated to each of the assessment objectives (AOs) are summarised below.

Assessment objectives as a percentage of the qualification

Assessment objective	**Weighting in IGCSE %**
AO1 Demonstrate knowledge and understanding of mathematical techniques	50
AO2 Apply mathematical techniques	50

Assessment objectives as a percentage of each component

Assessment objective	**Weighting in components %**	
	Paper 1	**Paper 2**
AO1 Demonstrate knowledge and understanding of mathematical techniques	50	50
AO2 Apply mathematical techniques	50	50

Question 1

Specimen answer

1 Solve $xy = 3$,

$$x^4y^5 = 486.$$ [3]

From $xy = 3$, $x = \frac{3}{y}$. Substitute into $x^4y^5 = 486$ to get $\left(\frac{3}{y}\right)^4 y^5 = 486$

$\frac{81}{y^4} \times y^5 = 486$, $81y = 486$ $y = 6$, $x = \frac{3}{y} = \frac{3}{6} = \frac{1}{2}$

Examiner comment

Question 1

The candidate has written a concise and correct solution to this problem. All the steps are clearly presented and the method is complete, as is required in the rubric on the front of the examination paper. The method mark is earned at the point where the candidate states $81y = 486$ and, as this is correct, the accuracy marks are earned for the correct evaluation of each unknown.

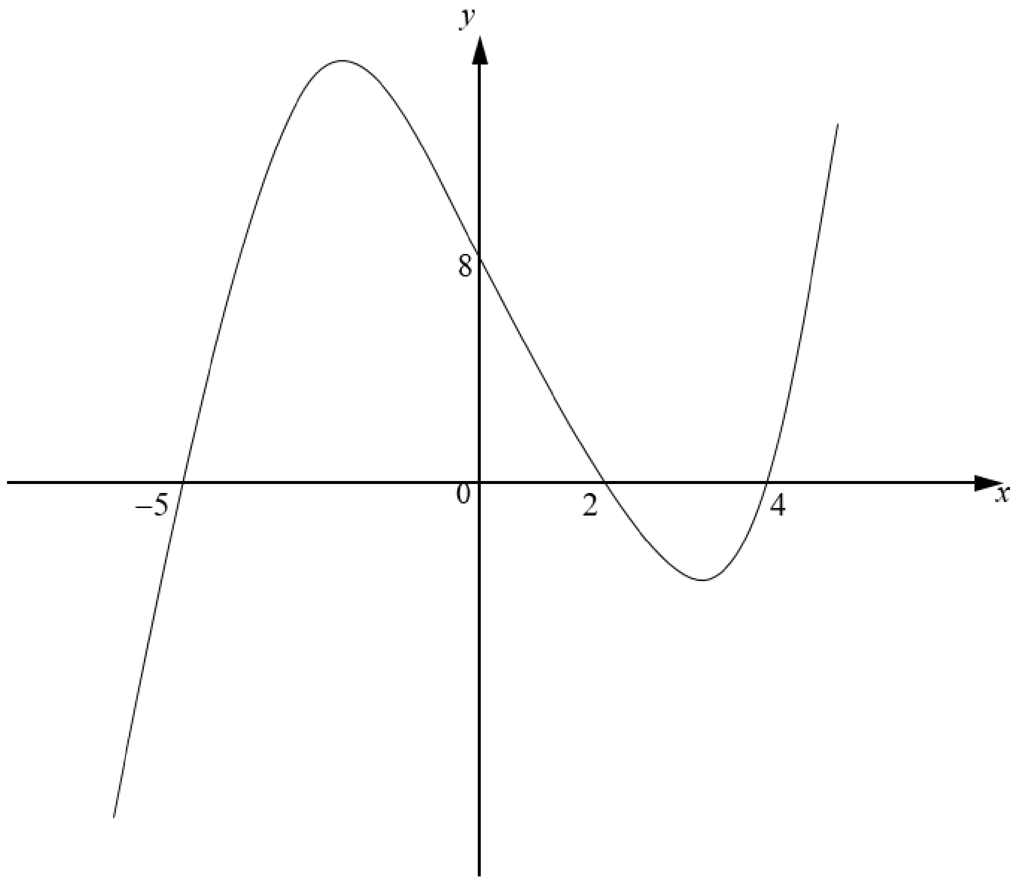
It is to be expected that candidates who score the highest grades in this paper will achieve full marks for a standard question assessing a simple technique, such as this.

Mark awarded = 3 out of 3

Question 2

Specimen answers

2 **(a)** On the axes below, sketch the graph of $y = \frac{1}{5}(x-2)(x-4)(x+5)$, showing the coordinates of the points where the graph meets the coordinate axes.



Roots at $-5, 2, 4$ [2]

y intercept at $\frac{1}{5}(-2)(-4)(5) = 8$

(b) Explain why your sketch in part **(a)** can be used to solve $(x-2)(x-4)(x+5) \leqslant 0$. [1]

The roots of $\frac{1}{5}(x-2)(x-4)(x+5)$ and $(x-2)(x-4)(x+5)$ are the same.

(c) Hence solve $(x-2)(x-4)(x+5) \leqslant 0$. [1]

$2 \leqslant x \leqslant 4$

Examiner comment

Question 2(a)

Below the graph, the candidate has indicated the positions of the roots and the *y*-intercept. Although it is not necessary to write this information separately, this is a good idea as the candidate is able to reference the information and reduce the possibility of an error.

Even though the question asks for the 'coordinates of points where the graph meets the coordinate axes', the correct marking of the points –5, 2, 4 on the *x*-axis and the point 8 on the *y*-axis is sufficient for the mark. The shape is reasonable and sufficient for the mark, as the question asks for a sketch, not an accurate graph.

Mark awarded = 2 out of 2

Question 2(b)

The candidate has understood that the basic principle that the solution of the inequality depends upon the roots of the graph. As the roots of the graph in part (a) would be the same as the roots of the graph of $y=(x-2)(x-4)(x+5)$ the inequality can be solved using the graph. The explanation given is good enough for the mark to be awarded.

Mark awarded = 1 out of 1

Question 2(c)

The candidate has interpreted the section of the graph in the fourth quadrant correctly and the inequality stated is correct for that section. As the graph is of a cubic function, there is a section of the graph in the third quadrant that also satisfies the inequality. This section is where $x \leqslant -5$. Without this inequality, the answer is incomplete and so does not score.

Mark awarded = 0 out of 1

Total mark awarded = 3 out of 4

Question 3

Specimen answers

3 Functions g and h are such that

$$\mathrm{g}(x) = 2 + 4\ln x \quad \text{for } x > 0,$$
$$\mathrm{h}(x) = x^2 + 4 \quad \text{for } x > 0.$$

(a) Find g^{-1}, stating its domain and its range. [4]

$$\text{Let } y = \mathrm{g}(x) \text{ then } y = 2 + 4\ln x$$
$$x = 2 + 4\ln y$$
$$\frac{x-2}{4} = \ln y$$
$$y = \mathrm{e}^{\frac{x-2}{4}}$$
$$\mathrm{g}^{-1}(x) = \mathrm{e}^{\frac{x-2}{4}}$$

Domain: all real values of x

Range: $x > 0$

(b) Solve $\mathrm{gh}(x) = 10$. [3]

$$\mathrm{g}(\mathrm{h}(x)) = \mathrm{g}(x^2 + 4)$$
$$10 = 2 + 4\ln(x^2 + 4)$$
$$8 = 4\ln(x^2 + 4)$$
$$\ln(x^2 + 4) = 2$$
$$x^2 + 4 = \mathrm{e}^2$$
$$x^2 = \mathrm{e}^2 - 4$$
$$x = \pm\sqrt{\mathrm{e}^2 - 4}$$

For gh, $x > 0$ so $x = \sqrt{\mathrm{e}^2 - 4}$

(c) Solve $\mathrm{g}'(x) = \mathrm{h}'(x)$. [3]

$$\mathrm{g}'(x) = \frac{4}{x} \qquad \mathrm{h}'(x) = 2x$$

$$\frac{4}{x} = 2x \qquad x^2 = 2 \qquad x = \pm\sqrt{2}$$

Examiner comment

Question 3(a)

The candidate has stated a full and correct method for finding the inverse function. The method mark is earned for the third step in their working on the left hand side and the accuracy mark is earned for the fourth step. The candidate has chosen to swop the variables first and then make y the subject. This is a slightly better approach than making x the subject and then swopping the variables as it avoids the possibility of leaving the answer in terms of y, which would not be allowed.

The domain has been stated in an acceptable form and earns the mark.

The range is not acceptable, as incorrect notation has been used. Writing $y > 0$ would have been acceptable for the mark. It is important that candidates take care to use the correct notation when answering questions. This is especially true for questions assessing functions, where errors of this type are commonly made.

Mark awarded = 3 out of 4

Question 3(b)

A detailed and well-presented solution has been given to this part of the question. All the steps have been clearly stated and the logic of the answer is clear. The candidate has correctly considered the possibility of the positive and negative square roots. They have justified discarding the negative root in this case, as it would be outside the domain for gh. The candidate has left their answer in exact form. As this is not a question in context, this form is acceptable and earns full marks.

Mark awarded = 3 out of 3

Question 3(c)

Many candidates misinterpret or misread the 'dashed' notation and equate the inverse functions. Candidates need to take care to read the notation carefully as inverse function notation and dashed derivatives look similar when a question is read at speed. This candidate does not make such an error and correctly interprets the use of the 'dashed' notation for the first derivative. The two derivatives have been equated correctly and solutions found. It is important that candidates understand that the domains of both functions still need to be considered. This should have resulted in the candidate discarding the negative square root once again in this case. The candidate has not done this and therefore the accuracy mark cannot be given.

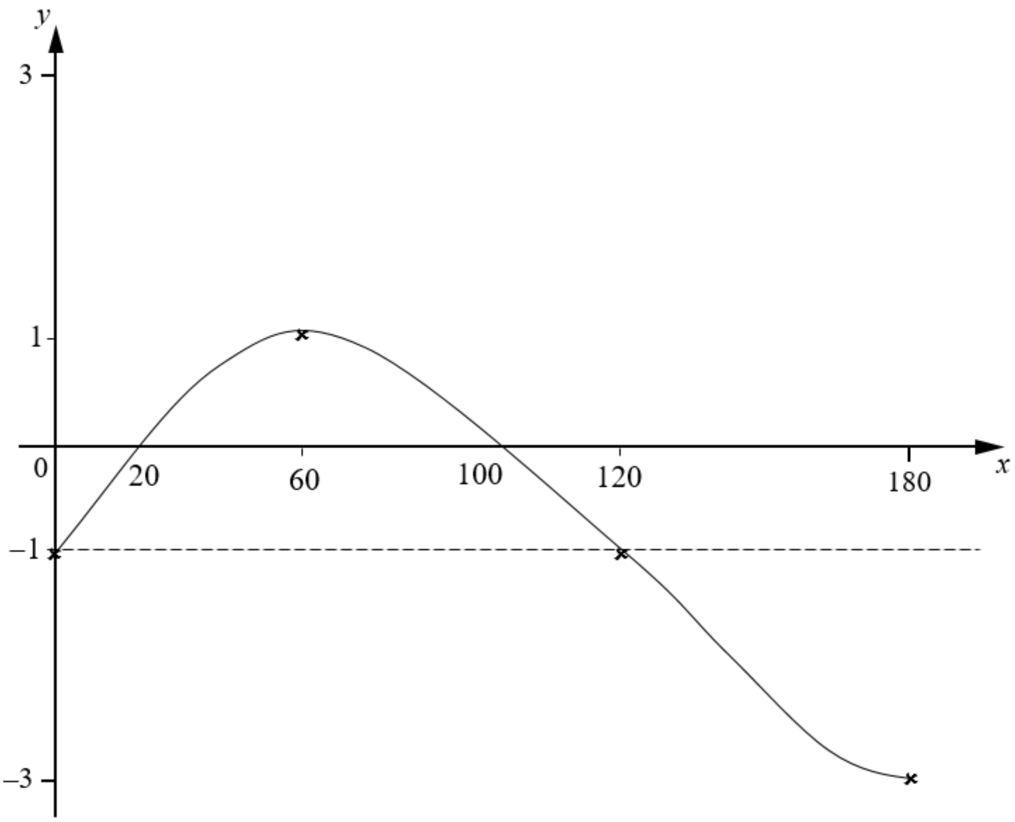
Mark awarded = 2 out of 3

Total mark awarded = 8 out of 10

Question 4

Specimen answer

4 On the axes below, sketch the graph of $y = 2\sin\frac{3}{2}x - 1$ for $0° \leqslant x \leqslant 180°$, showing the coordinates of the points where the graph meets the coordinate axes. [4]



Period = $\frac{360}{3/2} = 240°$; intercepts at 0, 120, 240

Amplitude = 2; max (60, 1) min (180,–3)

Midline is $y = -1$

$2\sin\frac{3}{2}x = 1$ $\sin\frac{3}{2}x = \frac{1}{2}$ $\frac{3}{2}x = 30$ $x = \frac{60}{3} = 20$

By symmetry roots at 20°, 100°.

Examiner comment

Question 4

The candidate has made good use of the space beneath the graph to do some working out. Whilst this is not essential, it is useful. The candidate's logic is clear to see in their step-by-step approach and is as follows.

Firstly, they find the period of $y = \sin\frac{3}{2}x$ and then state its roots (0, 0), (120, 0), (240, 0). They are able to determine that the maximum point has an *x*-coordinate of 60 and the minimum point an *x*-coordinate of 180. Next, the amplitude is written down. This allows the candidate to successfully state the coordinates of the turning points in the required domain. The graph can be drawn at this stage, with the key points being marked. Once this has been done, the candidate finds the roots of $y = 2\sin\frac{3}{2}x - 1$ that are between 0 and 180 and marks these on the sketch to complete the question. The shape of the curve is reasonable and good enough for full marks to be awarded.

Mark awarded = 4 out of 4

Question 5

Specimen answers

5 **(a)** A 6-character password is to be chosen from the following 9 characters.

letters	A	B	E	F
numbers	5	8	9	
symbols	*	$		

Each character may be used only once in any password.

Find the number of different 6-character passwords that may be chosen if

(i) there are no restrictions, [1]

6 from 9 $\quad {}^{9}P_{6} = 60480$

order matters

(ii) the password consists of 2 letters, 2 numbers and 2 symbols in that order, [2]

order still matters

LL AND NN AND SS
2 from 4 AND 2 from 3 AND 2 from 2

$$ {}^{4}P_{2} \times {}^{3}P_{2} \times {}^{2}P_{2} = 12 \times 6 \times 2 $$
$$ = 144 $$

(iii) the password must start and finish with a symbol. [2]

order still matters

S __ __ __ __ S

1 from 2 AND 4 from 7 AND 1 symbol remaining

$$ {}^{2}P_{1} \times {}^{7}P_{4} \times 1 = 2 \times 840 $$
$$ = 1680 $$

(b) An examination consists of a section A, containing 10 short questions, and a section B containing 5 long questions. Candidates are required to answer 6 questions from section A and 3 questions from section B.

Find the number of different selections of questions that can be made if

(i) there are no further restrictions, [2]

	A		**B**
order does not matter	6 from 10	and	3 from 5

$$^{10}C_6 \times {}^5C_3 = 210 \times 10$$
$$= 2100$$

(ii) candidates must answer the first 2 questions in section A and the first question in section B. [2]

A		**B**
Q1 Q2 __ __ __ __	AND	Q1 __ __
4 from 8 remaining	AND	2 from 4 remaining

$$^8C_4 \times {}^4C_2 = 70 \times 6$$
$$= 420$$

Examiner comment

Question 5(a)

Questions on this topic need some interpretation. When this is done correctly, candidates score well, as in this case.

(i) The candidate has understood that, for a password, order is important. This means that the required answer is a permutation not a combination. The evaluation of the correct permutation has been carried out using the nPr facility on the calculator. This is perfectly acceptable for the mark. **1/1**

(ii) As the question is still about the number of passwords possible, the candidate has stated that 'order still matters'. This is useful and has helped the candidate to carry on, correctly, using permutations and the product rule for counting. The candidate has stated what they are attempting to do and this has helped them form a correct 3-term product. This has been correctly evaluated and both marks awarded. **2/2**

(iii) This part of the question is similar to the previous part of the question in that permutations are still required, along with the product rule for counting. The candidate has considered that the password can start with one of the two symbols and must end with the other. The 4 characters in between must be chosen from the remaining 4 letters and 3 numbers, hence '4 from 7'. Full marks are awarded for this solution. **2/2**

Mark awarded = 5 out of 5

Question 5(b)
(i) In this part of the question, it does not matter in what order the questions in section A and section B are answered. This means that combinations need to be used in the solution. For section A, 6 questions from 10 must be answered when order does not matter. For section B, 3 questions from 5 must be answered when order does not matter. This candidate has clearly understood and indicated this. The candidate is given the method mark for the full method of applying combinations and the product rule for counting. The accuracy mark is earned when this is correctly evaluated and the correct numerical answer stated. **2/2**

(ii) This candidate has again written their interpretation of the question down before they have attempted to apply the mathematics. They have been very successful in using this approach. Question 1 and question 2 of section A must be answered. This does not mean that order is important as they could be answered at any point. Therefore, combinations are still required in this part. The candidate has understood and indicated that, for section A there are 4 questions to select from the remaining 8 and for section B there are 2 questions to select from the remaining 4. The point at which they write the product of the correct combinations earns them M1 and the correct final answer is given A1.
An excellent answer to this whole question. **2/2**

Mark awarded = 4 out of 4

Total mark awarded = 9 out of 9

Question 6

Specimen answers

6 A particle P travels in a straight line such that, t s after passing through a fixed point O, its velocity v m s^{-1} is given by $v = \left(e^{\frac{t^2}{8}} - 4\right)^3$.

(a) Find the speed of P at O. [1]

$$t = 0 \qquad v = (e^0 - 4)^3 = (1-4)^3 = (-3)^3 = -27$$

(b) Find the value of t for which P is instantaneously at rest. [2]

$$\left(e^{\frac{t^2}{8}} - 4\right)^3 = 0$$
$$e^{\frac{t^2}{8}} - 4 = 0$$
$$e^{\frac{t^2}{8}} = 4$$

$$\ln 4 = \frac{t^2}{8}$$
$$t = \sqrt{8\ln 4}$$
$$t = 3.3302...$$

(c) Find the acceleration of P when $t = 1$. [4]

$$a = \frac{dv}{dt}$$
$$a = 3\left(e^{\frac{t^2}{8}} - 4\right)^2 \left(\frac{2t}{8}e^{\frac{t^2}{8}}\right)$$
$$a = 3\left(e^{\frac{t^2}{8}} - 4\right)^2 \left(\frac{t}{4}e^{\frac{t^2}{8}}\right)$$

When $t = 1$

$$a = 3\left(e^{\frac{1}{8}} - 4\right)^2 \left(\frac{1}{4}e^{\frac{1}{8}}\right)$$
$$a = 3(1.1 - 4)^2 \left(\frac{1}{4} \times 1.1\right)$$
$$a = 6.93825$$
$$a = 6.94 \quad \text{(to 3 sf)}$$

Examiner comment

Question 6(a)

The candidate understands that it is necessary to find the velocity when $t = 0$. The candidate found this correctly as $-27\ \text{ms}^{-1}$. However, they did not understand, or perhaps forgot, the difference between velocity and speed. To have earned the mark, the candidate needed to state the magnitude of the velocity. The correct answer to this part was, therefore, $27\ \text{ms}^{-1}$.

Mark awarded = 0 out of 1

Question 6(b)

The candidate has given a clear and correct answer to this part. In the second line of working, the candidate correctly takes the cube root of both sides of the equation and correctly attempts to solve it. The method mark is awarded for the first line on the right hand side, $\ln 4 = \frac{t^2}{8}$. The final answer is given correct to more than 3 significant figures and the mark scheme does not penalise this. This solution earns full marks.

Note that this candidate applies a correct method to arrive at the equation $e^{\frac{t^2}{8}} = 4$. Incorrect removal of the cube root using $e^{\frac{3t^2}{8}} - 4^3 = 0$ also gives the answer 3.33. When candidates use this method, which is clearly not correct, no marks are given, even though the numerical value 3.33 is obtained. It is very important, therefore, that candidates make sure they show all their working to show that they are not using their calculators to solve equations. This instruction is given in the rubric on the front of the examination paper.

Mark awarded = 2 out of 2

Question 6(c)

The correct solution depends upon understanding that $a = \frac{dv}{dt}$. This candidate has appreciated this and correctly applied the chain rule twice to obtain the correct expression for the acceleration in terms of t. This has earned them the first two marks. They then substitute $t = 1$ into their expression and earn the third mark.

The candidate then makes a premature approximation error. They use $e^{\frac{1}{8}} = 1.1$ in their calculation. This value is only accurate to two significant figures. This has resulted in their final answer being inaccurate and the final mark cannot be awarded. Candidates need to make sure that any working values used are to correct to more than three significant figures to be confident that their final answer is sufficiently accurate when rounded.

Note that this candidate has made no errors when differentiating $e^{\frac{t^2}{8}}$ even though they have done this mentally. Applying the chain rule to differentiate $e^{f(x)}$ as $f'(x)e^{f(x)}$ can be a common error in understanding when answering questions of this complexity. Candidates often do not realise that they also need to apply the chain rule again to this term to complete the differentiation. It is common for candidates to differentiate $e^{f(x)}$ as $e^{f(x)}$ in such cases. These candidates may improve by using a written approach to differentiate expressions of this complexity.

Mark awarded = 3 out of 4

Total mark awarded = 5 out of 7

Question 7

Specimen answers

7 Variables x and y are such that when $\lg y$ is plotted against x^2, a straight line graph passing through the points (1, 0.73) and (4, 0.10) is obtained.

(a) Given that $y = Ab^{x^2}$, find the value of each of the constants A and b. [4]

$$\lg y = \lg A + x^2 \lg b$$
$$0.73 = \lg A + \lg b \quad ①$$
$$0.10 = \lg A + 4\lg b \quad ②$$
$$②-① \quad -0.63 = 3\lg b$$
$$\lg b = -0.21$$
$$b = 10^{-0.21} = 0.0165... = 0.617$$

$$\lg A = 0.73 + 0.21$$
$$\lg A = 0.94$$
$$A = 10^{0.94}$$
$$A = 8.7096...$$
$$A = 8.71$$

(b) Find the value of y when $x = 1.5$. [2]

$$y = (8.71)(0.617)^{1.5^2} = 2.9387...$$
$$y = 2.94$$

(c) Find the positive value of x when $y = 2$. [2]

$$2 = (8.71)(0.617)^{x^2}$$
$$\frac{2}{8.71} = 0.617^{x^2}$$
$$x^2 = \log_{0.617}\left(\frac{2}{8.71}\right)$$
$$x^2 = 3.0$$
$$x = \sqrt{3} = 1.73$$

Examiner comment

Question 7(a)

The method of forming and solving a pair of simultaneous equations is correct although it is not the simplest approach. From the statement $\lg y = \lg A + x^2 \lg b$, it is clear that the value of $\lg b$ is the gradient and so it may be simpler and quicker to state $\lg b = \frac{0.73 - 0.10}{1 - 4} = -0.21$. The statement $\lg b = -0.21$ earns the first mark. The second step is dependent on candidates knowing the correct base. The notations 'lg' indicates base 10 and so the candidate earns the next mark at the point where they state $b = 10^{-0.21}$. A common error in such questions is to use base e rather than base 10 for this step. The third mark is given for $\lg A = 0.94$ and the final mark for $A = 10^{0.94}$. Note that, in this part of the question, the candidate has maintained the accuracy of their values and their final answers are correct to the required accuracy.

Mark awarded = 4 out of 4

Question 7(b)

The method used here is correct and the first line earns M1. The candidate has used the rounded values they obtained in part (a) for *A* and *b* in this part. This means that their final answer is not accurate to three significant figures. However, the mark scheme allows that answers which round to 2.9 are acceptable here. Since 2.94 rounds to 2.9, the final mark has been earned and can be given. This candidate could have avoided this inaccuracy by using the exact values $A = 10^{0.94}$ and $b = 10^{-0.21}$ or by using more accurate decimals, such as *A* = 8.7096 and *b* = 0.6165.

Accepting values which round to 2.9 or 3.0 is a valid thing to do here as questions of this type are often based upon experimental data or the reading of graphs, which are both subject to error. When this is the case, greater accuracy than is reasonable should not be claimed for derived values.

Mark awarded = 2 out of 2

Question 7(c)

Again, in this part, the method used is fully correct. The only issue is the use of the rounded decimals in the first line of working. This first line is sufficient for the method mark. The candidate correctly applies the definition of a logarithm to the equation they have in their second line of working leading to the equation $x^2 = 3.0$. The value 3.0 is another rounded value. This candidate benefits from the answer $\sqrt{3}$ being acceptable.

Premature approximation errors almost always result in inaccurate, unacceptable answers and should be avoided.

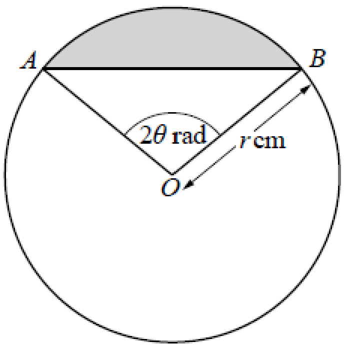
Mark awarded = 2 out of 2

Total mark awarded = 8 out of 8

Question 8

Specimen answers

8



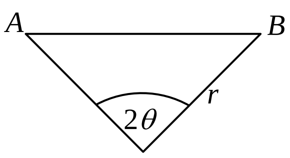
The diagram shows a circle, centre O, radius r cm. The points A and B lie on the circle such that angle $AOB = 2\theta$ radians.

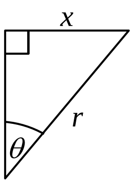
(a) Given that the perimeter of the shaded region is 20 cm, show that $r = \dfrac{10}{\theta + \sin\theta}$. [3]

Perimeter shaded region = arc AB + chord AB

arc $AB = r\text{'}\theta\text{'} = r(2\theta) = 2r\theta$

chord AB →





$$\sin\theta = \frac{x}{r}$$
$$x = r\sin\theta$$
$$2x = 2r\sin\theta$$

$$\therefore \quad 20 = 2r\theta + 2r\sin\theta$$
$$r = \frac{10}{\theta + \sin\theta} \quad ✓$$

(b) Given that r and θ can vary, find the value of $\frac{\mathrm{d}r}{\mathrm{d}\theta}$ when $\theta = \frac{\pi}{6}$. [4]

$$r = \frac{10}{\theta + \sin\theta} = 10(\theta + \sin\theta)^{-1}$$

$$\frac{\mathrm{d}r}{\mathrm{d}\theta} = -10(\theta + \sin\theta)^{-2}(1 + \cos\theta)$$

When $\theta = \frac{\pi}{6}$

$$\frac{\mathrm{d}r}{\mathrm{d}\theta} = -10\left(\frac{\pi}{6} + \sin\frac{\pi}{6}\right)^{-2}\left(1 + \cos\frac{\pi}{6}\right)$$
$$= -70.46848...$$
$$= -70.5 \text{ (to 3 sf)}$$

Examiner comment

Question 8(a)

Here, the candidate has interpreted the question correctly and found expressions, in r and θ, for the arc length and chord length required to form the perimeter. The statement $2x = 2r\sin\theta$ earns the first mark and the second mark is awarded for the formation of the correct equation, $20 = 2r\theta + 2r\sin\theta$. The question requires the candidate to *show that* $r = \frac{10}{\theta + \sin\theta}$ and so the answer has been given. In order to earn the final mark, therefore, the candidate needs to demonstrate that they are able to manipulate the equation to the required form. At least one interim step between the initial equation and the final answer should be given in order to earn the last mark. For example,

$$20 = 2r\theta + 2r\sin\theta$$
$$20 = 2r(\theta + \sin\theta)$$
$$r = \frac{10}{\theta + \sin\theta}$$

would have been sufficient for 3 marks.

Mark awarded = 2 out of 3

Question 8(b)

It is easier to differentiate this expression using the chain rule than the quotient rule or product rule. This candidate has done this successfully. The expression for $\frac{\mathrm{d}r}{\mathrm{d}\theta}$ on the second line is correct and sufficient for M1 A2. The candidate has then correctly substituted $\theta = \frac{\pi}{6}$. The final answer is incorrect and the final mark cannot be awarded. The candidate has either not realised that, for any calculus, angles must be in radians or they have forgotten to check that their calculator is in radian mode. It is essential that candidates understand this point and make the necessary calculator check for any question that requires them to find, or work with, angles.

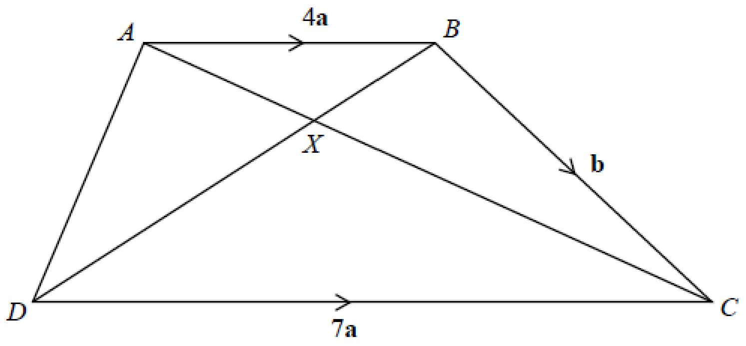
Mark awarded = 3 out of 4

Total mark awarded = 5 out of 7

Question 9

Specimen answers

9



In the diagram $\overrightarrow{AB} = 4\mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{DC} = 7\mathbf{a}$. The lines *AC* and *DB* intersect at the point *X*.
Find, in terms of **a** and **b**,

(a) $\overrightarrow{DB}$, [1]

$$\overrightarrow{DB} = \overrightarrow{DC} + \overrightarrow{CB} = \overrightarrow{DC} - \overrightarrow{BC} = 7\mathbf{a} - \mathbf{b}$$

(b) $\overrightarrow{DA}$. [1]

$$\overrightarrow{DA} = \overrightarrow{DB} + \overrightarrow{BA} = 7\mathbf{a} - \mathbf{b} - 4\mathbf{a} = 3\mathbf{a} - \mathbf{b}$$

Given that $\overrightarrow{AX} = \lambda\overrightarrow{AC}$ find, in terms of **a**, **b** and λ,

(c) $\overrightarrow{AX}$, [1]

$$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} = -\overrightarrow{DA} + \overrightarrow{DC} = \mathbf{b} - 3\mathbf{a} + 7\mathbf{a}$$
$$= 4\mathbf{a} + \mathbf{b}$$

$$\therefore \overrightarrow{AX} = \lambda(4\mathbf{a} + \mathbf{b})$$

(d) $\overrightarrow{DX}$. [2]

$$\overrightarrow{DX} = \overrightarrow{DA} + \overrightarrow{AX}$$

$$= 3\mathbf{a} - \mathbf{b} + \lambda(4\mathbf{a} + \mathbf{b})$$

$$= (4\lambda + 3)\,\mathbf{a} + (\lambda - 1)\mathbf{b}$$

Given that $\overrightarrow{DX} = \mu\overrightarrow{DB}$,

(e) find the value of λ and of μ. [4]

$\overrightarrow{DX} = (4\lambda + 3)\,\mathbf{a} + (\lambda - 1)\mathbf{b}$

$\overrightarrow{DB} = \overrightarrow{DC} + \overrightarrow{CB} = \overrightarrow{DC} - \overrightarrow{BC} = 7\mathbf{a} - \mathbf{b}$

$\therefore \quad (4\lambda + 3)\mathbf{a} + (\lambda - 1)\mathbf{b} = 7\mu\mathbf{a} - \mu\mathbf{b}$

Equating components:

$4\lambda + 3 = 7\mu$ ①

$\lambda - 1 = \mu$ ②

From ②, $\mu = \lambda - 1$

Substitute into ①

$$4\lambda + 3 = 7(\lambda - 1)$$
$$4\lambda + 3 = 7\lambda - 7$$
$$10 = 3\lambda$$
$$\lambda = \frac{10}{3}$$

When $\lambda = \frac{10}{3}$ $\quad \mu = \frac{10}{3} - 1 = \frac{7}{3}$

Examiner comment

Question 9(a)

The notation/direction has been interpreted correctly as the vector from *D* to *B*. The mark scheme indicates that the candidate's final answer must be checked for accuracy and this is correct so the mark is awarded.

Mark awarded = 1 out of 1

Question 9(b)

The notation/direction has been interpreted correctly as the vector from *D* to *A*. The candidate has used their answer from part (a) here, as expected, rather than starting again. It is likely that writing $\overrightarrow{DA} = \overrightarrow{DB} + \overrightarrow{BA}$ has helped them to see the link between the first two parts of the question. The mark scheme indicates that the candidate's final answer must be checked for accuracy and this is correct so the mark is awarded.

Mark awarded = 1 out of 1

Question 9(c)

Since it is given that $\overrightarrow{AX}$ is a scalar multiple of $\overrightarrow{AC}$, the candidate has found an expression for $\overrightarrow{AC}$, this time using their answer to part (b). To complete the question, they have then multiplied by λ. The candidate has earned the mark at this stage.

Note that, as the question does not require the answer to be given in any particular form, there is no need to multiply the brackets out. Any candidate who does so and makes an error will not be awarded the mark, as the mark scheme indicates that the candidate's final answer should be the one considered. Candidates need to have good manipulation skills when working with vector algebra.

Mark awarded = 1 out of 1

Question 9(d)

This part is also fully correct. Making the vector statement $\overrightarrow{DX} = \overrightarrow{DA} + \overrightarrow{AX}$ has most likely helped them to see that the earlier parts of the question are useful here. Although it is possible to 'start again', it simply creates unnecessary work. It also increases the possibility of making an error. The unsimplified form on the second line earns two marks as the mark scheme allows an unsimplified answer and this answer is correct. The final answer is useful for the next part of the question, so this candidate has saved themselves some work in that part.

Mark awarded = 2 out of 2

Question 9(e)

The candidate has used their expression for $\overrightarrow{DX}$ from part (d). They have forgotten that, in part (a), they found an expression for $\overrightarrow{DB}$ and have started again to find this vector. The first method mark is awarded for their third line of working. The method the candidate has attempted to use is a correct one. They have made a sign error in the second equation, which should be $\lambda - 1 = -\mu$. As they have earned the first mark and have attempted to solve their equations when they write $4\lambda + 3 = 7(\lambda - 1)$, they have also earned the second mark. Neither of the accuracy marks have been awarded.

This could have been prevented if the candidate had carried out a check to test their values. For example:

$(4\lambda + 3)\mathbf{a} + (\lambda - 1)\mathbf{b} = 7\mu\mathbf{a} - \mu\mathbf{b}$

Left hand side , when $\lambda = \frac{10}{3}$ $\quad \left(4 \times \frac{10}{3} + 3\right)\mathbf{a} + \left(\frac{10}{3} - 1\right)\mathbf{b} = \frac{49}{3}\mathbf{a} + \frac{7}{3}\mathbf{b}$

Right hand side, when $\mu = \frac{7}{3}$ $\quad \left(7 \times \frac{7}{3}\right)\mathbf{a} - \frac{7}{3}\mathbf{b} = \frac{49}{3}\mathbf{a} - \frac{7}{3}\mathbf{b}$

The two sides are not equal and so an error has been made. Sign errors or arithmetic errors are most common and should be looked for first of all in such cases.

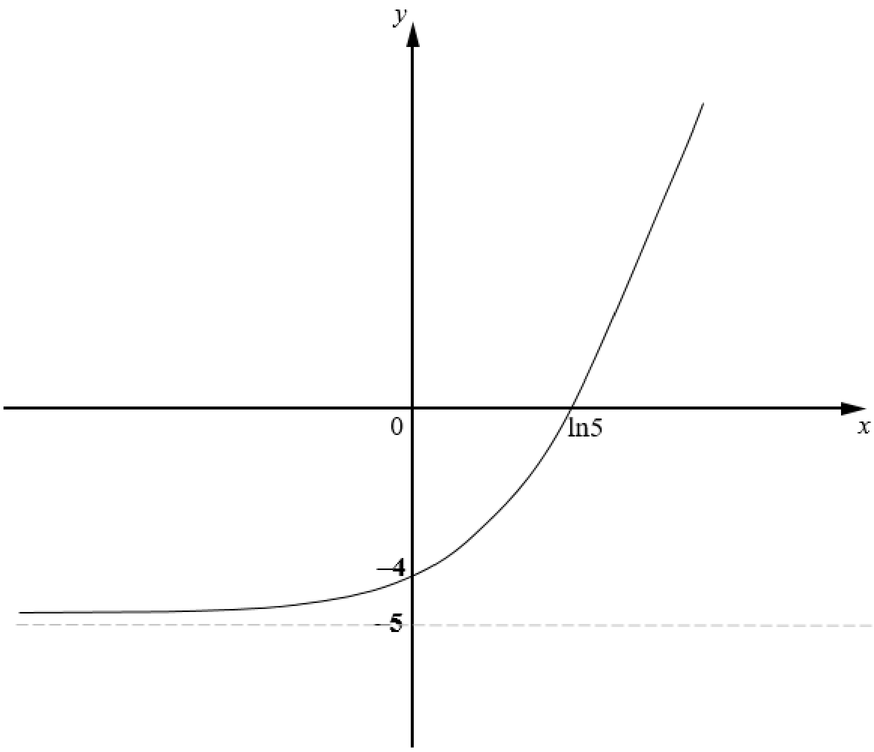
Mark awarded = 2 out of 4

Total mark awarded = 7 out of 9

Question 10

Specimen answers

10 **(a)** **(i)** Sketch the graph of $y = e^x - 5$ on the axes below, showing the exact coordinates of any points where the graph cuts the coordinate axes.



Translate e^x vertically

$\downarrow$

$\begin{pmatrix} 0 \\ -5 \end{pmatrix}$

$$e^x - 5 = 0$$
$$e^x = 5$$
$$\ln 5 = x$$

[3]

(ii) Find the range of values of k for which the equation $e^x - 5 = k$ has no solutions. [1]

No solutions when $k < -5$.

(b) Simplify $\log_a \sqrt{2} + \log_a 8 + \log_a \left(\frac{1}{2}\right)$ giving your answer in the form $p\log_a 2$, where p is a constant. [2]

$$\log_a 2^{\frac{1}{2}} + \log_a 2^3 + \log_a 2^{-1}$$
$$= \frac{1}{2}\log_a 2 + 3\log_a 2 - \log_a 2$$
$$= \frac{1}{2}\log_a 2 + 2\log_a 2$$
$$= \frac{5}{2}\log_a 2$$

(c) Solve the equation $\log_3 x - \log_9 4x = 1$. [4]

$$\log_3 x - \frac{\log_3 4x}{\log_3 9} = \log_3 3$$

$$\log_3 x - \frac{1}{2}\log_3 4x = \log_3 3$$

$$\log_3 x + \log_3 (4x)^{-\frac{1}{2}} = \log_3 3$$

$$\log_3 \left(x \times (4x)^{-\frac{1}{2}}\right) = \log_3 3$$

$$\frac{x}{2\sqrt{x}} = 3$$

$$\frac{\sqrt{x}}{2} = 3$$

$$\sqrt{x} = 6$$

$$x = \sqrt{6}$$

Examiner comment

Question 10(a)

(i) The candidate has drawn the asymptote at $y = -5$ and the graph clearly tends towards it in the third quadrant, with no doubling back or flicking up at the end. The shape of the graph in the first quadrant is also acceptable with no bending out or over at the end. The overall shape of the graph is, therefore, sufficiently good and earns the first B1. The remaining two marks are awarded for the correct root and *y*-intercept. The candidate has shown some working to support their values. The root is fully correct. Note that the root was required as an *exact* value and the candidate has followed this instruction. A rounded decimal for the root would not have been acceptable. They have made a sign slip in their calculation for the *y*-intercept. However, the correct value has been marked on the sketch and the sketch takes precedence over the working in this case, so full marks are allowed. Note also that coordinates are not required for full marks to be given. The marking of the values on the relevant axes is sufficient. **3/3**

(ii) The candidate has understood that the line $y = k$ and the curve $y = e^x - 5$ have no points in common when k is less than –5. However, the candidate has either used the wrong inequality sign or has omitted one solution. As $y = e^x - 5$ has a horizontal asymptote at $y = -5$, it is not possible for k to *equal* –5 either. The correct answer is $k \leqslant -5$. **0/1**

Mark awarded = 3 out of 4

Question 10(b)

The approach used by the candidate, converting all logs to powers of 2, is probably the simplest. It is possible to apply log laws first and then rewrite as a power of 2. This leads to $\log_a\left(\sqrt{2}\times 8\times \frac{1}{2}\right) = \log_a 4\sqrt{2}$ which is, perhaps, not as easy to manipulate into the required form.

The candidate, once again, presents a full, neat and logical, step-by-step, solution with all the key stages seen.

Mark awarded = 2 out of 2

Question 10(c)

The most concise solutions arise from converting all the logs to a consistent base. Using either base 3 or base 9 results in only one change of base and therefore the solution has fewer steps, although any consistent base is possible. At some point the constant 1 also needs to be converted to this consistent base. The candidate has chosen to write all terms as logs to the base 3 and does this correctly in the first line. This line earns the first mark and the third mark. The second mark is earned in the second line of working when $\log_3 9$ is written as 2. The final answer is incorrect as the candidate has square rooted rather than squaring. This slip may have been spotted if the candidate had checked their solution in the original equation using their calculator since $\log_3\sqrt{6} - \log_9 4\sqrt{6} \neq 1$. It is good practice to check the solutions to equations where possible.

Mark awarded = 3 out of 4

Total mark awarded = 8 out of 10

Question 11

Specimen answers

11 **(a)** **(i)** Show that $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - \sin\theta} = \sec^2\theta$. [3]

$$\operatorname{cosec} = \frac{1}{\sin}$$

$$\frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta} - \sin\theta} = \frac{\frac{1}{\sin\theta}}{\frac{1-\sin^2\theta}{\sin\theta}} = \frac{1}{\sin\theta} \times \frac{\sin\theta}{1-\sin^2\theta}$$

$$\text{LHS} \quad = \frac{1}{\cos^2\theta}$$

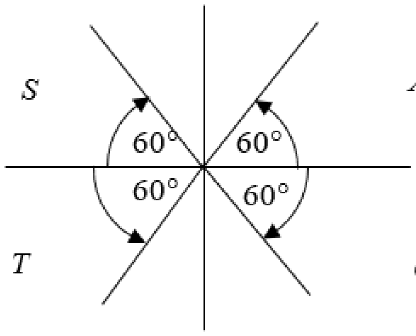
$$= \sec^2\theta$$

(ii) Hence solve $\frac{2\operatorname{cosec}\phi}{\operatorname{cosec}\phi - \sin\phi} = 8$ for $0^\circ < \phi < 360^\circ$. [3]

$2\sec^2\phi = 8 \quad \sec^2\phi = 4$

$\sec\phi = 2$ or -2 $\qquad \cos\phi = \frac{1}{2}$ or $-\frac{1}{2}$

$\frac{1}{\cos\phi} = 2$ or -2 $\qquad \phi = 60, 120, 240, 300$



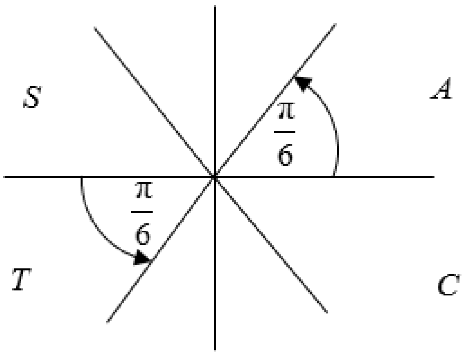
(b) Solve $\sqrt{3}\tan\left(x + \frac{\pi}{4}\right) = 1$ for $0 < x < 2\pi$, giving your answer in terms of π. [3]

$$\tan\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$$

$$x + \frac{\pi}{4} = \frac{\pi}{6}, \frac{7\pi}{6}$$

$$x = \frac{\pi}{6} - \frac{\pi}{4}, \frac{7\pi}{6} - \frac{\pi}{4}$$

$$x = -\cancel{\frac{\pi}{12}}, \frac{11\pi}{12}$$



Examiner comment

Question 11(a)

(i) The candidate has correctly worked from the left hand side of the identity to the right hand side. This shows good understanding of the structure of an identity and is very good practice. Some candidates treat identities as equations and move terms from one side to the other. This is not valid and is not good practice. This candidate has also used the 'third letter rule' to help them correctly identify the relationship $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$. This is helpful. All the key steps have been included in the solution, as required. The first mark is given for the first statement in the proof and the second mark for the second statement. The final steps are all complete and no bracketing or grouping errors, such as short fraction bars, have been made and so the final mark can also be given. **3/3**

(ii) The candidate has used their answer to part (a)(i), as expected. The M1 is earned for the statement $\cos\phi = \frac{1}{2}$ or $-\frac{1}{2}$ as it is at that point where all the requirements for the first mark are met, by implication. The candidate has used a CAST diagram to help them find all the angles that are within the required range for ϕ. They have their calculator in the correct mode and, as indicated by the given range for ϕ, have given their answers in degrees. As each answer is an integer number of degrees, no decimal places are needed and none should be stated.
This is a logical and well-presented solution. **3/3**

Mark awarded = 6 out of 6

Question 11(b)

As indicated by the range of values for x, the answers to this part are angles measured in radians. As the answers need to be exact in terms of π, it is important not to decimalise here. The candidate applies the correct order of operations and earns M1 for the second line of working. They realise that the negative angle is not part of the solution as it is outside the range required for x. They do not check the next angle to see if it falls within the range. The next angle will be $\pi + \frac{7\pi}{6} = \frac{13\pi}{6}$ and since $\frac{13\pi}{6} - \frac{\pi}{4} = \frac{23\pi}{12}$ which is in the required range, it should also be included. This candidate may have earned all three marks if they had carried on checking angles until they found the first angle that was outside the range by being too great.

Mark awarded = 2 out of 3

Total mark awarded = 8 out of 9

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