Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

ADDITIONAL MATHEMATICS **0606/12**

Paper 1 **May/June 2016**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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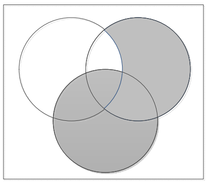
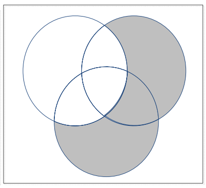
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

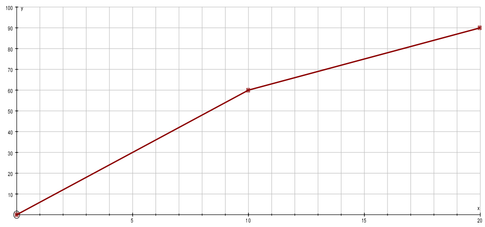
Question	Answer	Marks	Guidance
1 **(a)**	$Y \subset X$ or $Y \subseteq X$ only $Y \cap Z = \varnothing$ or $\{\ \}$ only	**B1** **B1**	
(b)	**(i)**  **(ii)** 	**B1** **B1**	
2 **(i)**	$32 - \frac{20}{x} + \frac{5}{x^2}$	**B3**	**B1** for each correct term – must be integers
(ii)	$(3 \times 32) + \left(-\frac{20}{x} \times 4x\right) = 16$ Accept $16x^{\text{o}}$	**M1** **A1**	for $(3 \times \textit{their}\ 32) + \left(\frac{\textit{their}(-20)}{x} \times 4x\right)$
3 **(i)**	$\mathbf{b} - \mathbf{c} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ $4 + y^2 = 36 + 4$ $y = \pm 6$	**B1** **M1** **A1**	may be implied by further correct working for one correct attempt at using the modulus
(ii)	$\mu + 4 = 2\lambda$ or $-4\mu + 24 = 7\lambda$ $\mu - 4 = -\lambda$ or $8\mu - 8 = \lambda$ leading to $\mu = \frac{4}{3}$, $\lambda = \frac{8}{3}$ oe allow 1.33 and 2.67 or better	**B1** **B1** **DB1**	for one correct equation in μ and λ for a second correct equation in μ and λ for both, must have both previous **B** marks

Question	Answer	Marks	Guidance
4	$\left(4+\sqrt{5}\right)x^2+\left(2-\sqrt{5}\right)x-1=0$		**You must be convinced that a calculator is not being used.**
	$x=\dfrac{-\left(2-\sqrt{5}\right)\pm\sqrt{\left(2-\sqrt{5}\right)^2-4\left(4+\sqrt{5}\right)(-1)}}{2\left(4+\sqrt{5}\right)}$	M1 A1	for use of quadratic formula (allow one sign error), allow $b^2=9-4\sqrt{5}$ all correct
	$x=\dfrac{-\left(2-\sqrt{5}\right)\pm\sqrt{9-4\sqrt{5}+16+4\sqrt{5}}}{2\left(4+\sqrt{5}\right)}$	DM1	for attempt to simplify the discriminant (minimum of 4 terms must be seen in discriminant, 2 terms involving √5 and 2 constant terms)
	$=\dfrac{-\left(2-\sqrt{5}\right)+5}{2\left(4+\sqrt{5}\right)}$		
	$=\dfrac{3+\sqrt{5}}{2\left(4+\sqrt{5}\right)}$	A1	for $\dfrac{3+\sqrt{5}}{2\left(4+\sqrt{5}\right)}$ or $\dfrac{3+\sqrt{5}}{8+2\sqrt{5}}$, ignore negative solution if included
	$=\dfrac{\left(3+\sqrt{5}\right)\left(4-\sqrt{5}\right)}{2\left(4+\sqrt{5}\right)\left(4-\sqrt{5}\right)}$	M1	for attempt to rationalise an expression of the form $\dfrac{a\pm b\sqrt{5}}{c\pm d\sqrt{5}}$ as part of their solution of the quadratic Must obtain an integer denominator
	$=\dfrac{7+\sqrt{5}}{22}$	A1	**Final A1 can only be awarded if all previous marks have been obtained**
5 (i)	$(1-\cos\theta)(1+\sec\theta)$		
	$=1-\cos\theta+\dfrac{1}{\cos\theta}-\dfrac{\cos\theta}{\cos\theta}$ $=\sec\theta-\cos\theta$ $=\dfrac{1}{\cos\theta}-\cos\theta$	M1	M1 for expansion and use of $\sec\theta=\dfrac{1}{\cos\theta}$ consistently, allow one sign error
	$=\dfrac{1-\cos^2\theta}{\cos\theta}$	DM1	for attempt at a single fraction, dependent on first **M1**
	$=\dfrac{\sin^2\theta}{\cos\theta}$	A1	
	$=\sin\theta\tan\theta$ www	A1	

Question	Answer	Marks	Guidance
	Alternative method: $(1-\cos\theta)\left(\frac{\cos\theta+1}{\cos\theta}\right)$	M1	for attempt at a single fraction for second factor and use of $\sec\theta=\frac{1}{\cos\theta}$
	$=\frac{1-\cos^2\theta}{\cos\theta}$	DM1	for expansion
	$=\frac{\sin^2\theta}{\cos\theta}$	A1	
	$=\sin\theta\tan\theta$ www	A1	
(ii)	$\sin\theta\tan\theta=\sin\theta$ $\sin\theta(\tan\theta-1)=0$		
	$\tan\theta=1,\ \theta=\frac{\pi}{4}$, allow 0.785 or better	B1	for $\theta=\frac{\pi}{4}$ from $\tan\theta=1$
	$\sin\theta=0,\ \theta=0,\ \pi$ or 3.14 or better	B1 B1	for $\theta=0$ from $\sin\theta=0$ for $\theta=\pi$ from $\sin\theta=0$
6	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x}(4x+1)^{\frac{1}{2}}\right)$ $=\mathrm{e}^{3x}\frac{1}{2}\times4(4x+1)^{-\frac{1}{2}}+3\mathrm{e}^{3x}(4x+1)^{\frac{1}{2}}$	B1	for $r\mathrm{e}^{3x}(4x+1)^{-\frac{1}{2}}$ must be part of a sum, $r=\frac{1}{2}$ or 2 or $\frac{1}{2}\times4$
		B1	for $s\mathrm{e}^{3x}(4x+1)^{\frac{1}{2}}$ must be part of a sum, s is 1 or 3
	$=\frac{2\mathrm{e}^{3x}}{(4x+1)^{\frac{1}{2}}}+3\mathrm{e}^{3x}(4x+1)^{\frac{1}{2}}$	B1	for all correct, allow unsimplified
	$=\frac{\mathrm{e}^{3x}}{(4x+1)^{\frac{1}{2}}}(2+12x+3)$	DM1	for $\frac{\mathrm{e}^{3x}}{(4x+1)^{\frac{1}{2}}}(a+bx)$, dependent on first 2 **B** marks , must be using a correct method, collecting terms in the numerator correctly
	$=\frac{\mathrm{e}^{3x}}{(4x+1)^{\frac{1}{2}}}(12x+5)$	A1	
7 (i)	$\cos3x=\frac{1}{2},\ x=\frac{\pi}{9}$ or $0.349,\ 20^\circ$, allow 0.35	M1 A1	for correct attempt to solve the trigonometric equation
(ii)	$B\left(\frac{\pi}{3},\ 3\right)$ or $(1.05,\ 3)$, $(60^\circ,\ 3)$	B1B1	**B1** for each, must be in correct position or in terms of $x=$ and $y=$

Question	Answer	Marks	Guidance
(iii)	$\int_{\frac{\pi}{9}}^{\frac{\pi}{3}} 1-2\cos 3x \, dx = \left[x - \frac{2}{3}\sin 3x\right]_{\frac{\pi}{9}}^{\frac{\pi}{3}}$	M1	for $x \pm a\sin 3x$ attempt to integrate at least one term
		A1	for correct integration
	$= \frac{\pi}{3} - \left(\frac{\pi}{9} - \left(\frac{2}{3} \times \frac{\sqrt{3}}{2}\right)\right)$	DM1	for correct use of limits from **(i)** and **(ii)**, must be in radians
	$= \frac{2\pi}{9} + \frac{\sqrt{3}}{3}$ oe or 1.28	A1	
8 (i)	$\lg y = x^2 \lg b + \lg A$		
	$\lg b = \pm 0.21$	B1	for $\lg b = \pm 0.21$ may be implied
	$b = 0.617$ allow 0.62, $10^{-0.21}$	B1	
	$\lg A = 0.94$ allow 0.93 to 0.95	B1	
	$A = 8.71$ allow awrt 8.5 to 8.9	B1	
	Alternative method		
	5.37 or $10^{0.73} = Ab$ 1.259 or $10^{0.1} = Ab^4$	B1	for both equations, allow correct to 2 sf
	$b^3 = 10^{-0.63}$	B1	
	$b = 0.617$ allow 0.62, $10^{-0.21}$	B1	
	$A = 8.71$ allow awrt 8.5 to 8.9	B1	
(ii)	$x = 1.5$, $x^2 = 2.25$	M1	for correct use of graph $y = theirA \times theirb^{1.5^2}$ or $\lg y = \lg theirA + \left(1.5^2 \lg theirb\right)$
	$y = 2.93$, allow awrt 2.9 or 3.0	A1	
(iii)	$\lg y = 0.301$, or $2 = \text{'}8.71\left(0.617\right)^{x^2}\text{'}$	M1	for correct use of graph to read off x^2 $2 = theirA\left(theirb\right)^{x^2}$ or $\lg 2 = \left(\lg theirb\right)x^2 + \lg\left(theirA\right)$
	$x = 1.74$, allow $\sqrt{3}$ or awrt 1.7, 1.8	A1	
9 (i)	$y = \frac{2}{3}\left(3x+10\right)^{\frac{1}{2}}$ $\left(+c\right)$	B1	for $p\left(3x+10\right)^{\frac{1}{2}}$ where p is a constant
		B1	for $\frac{2}{3}\left(3x+10\right)^{\frac{1}{2}}$ oe unsimplified
	passes through $\left(2, -\frac{4}{3}\right)$, so $c = -4$	M1	for attempt to find c, must have attempt to integrate, must have the first **B1**
	$y = \frac{2}{3}\left(3x+10\right)^{\frac{1}{2}} - 4$ oe	A1	

Question	Answer	Marks	Guidance
(ii)	When $x = 5$, $y = -\frac{2}{3}$	B1	
	perpendicular gradient $= -5$	B1	
		M1	for attempt at the normal using *their* perpendicular gradient and *their y* value (but not $y = -\frac{4}{3}$ or $-\frac{5}{3}$).
	Equation of normal: $y + \frac{2}{3} = -5(x-5)$	A1	
	When $y = -\frac{5}{3}$,	DM1	for use of $y = -\frac{5}{3}$ in their normal equation to get as far as $x = ...$
	$x = 5.2$ oe	A1	
10 (i)	Area: $20 = \pi x^2 + xy$	B1	
	$y = \frac{20 - \pi x^2}{x}$	B1	
	$P = 2\pi x + 2x + 2y$ $= 2\pi x + 2x + 2\left(\frac{20}{x} - \pi x\right)$	M1	for attempt to use perimeter and obtain in terms of x only
	$= 2x + \frac{40}{x}$	A1	all steps seen, www AG
	Alternative method: $20 = \pi x^2 + xy$	B1	
	$P = 2\pi x + 2y + 2x$ $= \frac{2}{x}\left(\pi x^2 + xy\right) + 2x$	M1	for attempt to use perimeter and write in $\frac{\pi x^2 + xy}{x}$
	$= \frac{2}{x}(20) + 2x$	B1	for replacing $\pi x^2 + xy$ with 20
	$= 2x + \frac{40}{x}$	A1	all steps seen, www AG

Question	Answer	Marks	Guidance
(ii)	$\frac{\mathrm{d}P}{\mathrm{d}x} = 2 - \frac{40}{x^2}$	M1	for attempt to differentiate
	When $\frac{\mathrm{d}P}{\mathrm{d}x} = 0$,	DM1	for equating to zero and attempt to solve at least as far as $x^2 =$
	$x = 2\sqrt{5}$ allow 4.47, $\sqrt{20}$	A1	
	leading to $P = 8\sqrt{5}$, allow 17.9	A1	
	$\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{80}{x^3}$, always positive so a minimum	A1	for this statement or use of gradient inspection either side of correct x
11 (a) (i)	Distance = area under graph	M1	for attempt to find the area, one correct area seen (triangle, rectangle or trapezium) as part of a sum.
	= 1275	A1	
(ii)	deceleration is 1.5 oe	B1	
(b)		B1	for a straight line between $(0,0)$ and $(10,60)$
		B1FT	**FT** a straight line between $(10, 60)$ and $(20, 90)$, a displacement vector $\begin{pmatrix}10\\30\end{pmatrix}$ from *their* $(10, \textit{their}\,60)$
(c) (i)	e^{2t} is always positive or oe	B1	
(ii)	$a = 8\mathrm{e}^{2t}$ $\mathrm{e}^{2t} = \frac{3}{2}$	M1	for attempt to differentiate, must be of the form $p\mathrm{e}^{2t}$, equate to 12 and solve.
	$t = \frac{1}{2}\ln\frac{3}{2}$, $\ln\sqrt{\frac{3}{2}}$ or $\frac{1}{2}\ln 1.5$	A1	Allow fractions equivalent to $\frac{3}{2}$
(iii)	$s = \left[2\mathrm{e}^{2t} + 6t\right]_{0.4}^{0.5}$	M1 A1	for attempt to integrate to get $q\mathrm{e}^{2t} + 6t$ all correct
	$= (2\mathrm{e} + 3) - (2\mathrm{e}^{0.8} + 2.4)$ $(= 8.436 - 6.851)$	DM1	for correct use of limits or considering distances separately, ignore attempts at c
	$= 1.59$, allow 1.58	A1	