

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The expression $f(x) = 3x^3 + 8x^2 - 33x + p$ has a factor of $x - 2$.

(i) Show that $p = 10$ and express $f(x)$ as a product of a linear factor and a quadratic factor. [4]

(ii) Hence solve the equation $f(x) = 0$. [2]

2 A committee of four is to be selected from 7 men and 5 women. Find the number of different committees that could be selected if

(i) there are no restrictions, [1]

(ii) there must be two male and two female members. [2]

A brother and sister, Ken and Betty, are among the 7 men and 5 women.

(iii) Find how many different committees of four could be selected so that there are two male and two female members which must include either Ken or Betty but not both. [4]

3 Points A and B have coordinates $(-2, 10)$ and $(4, 2)$ respectively. C is the mid-point of the line AB .

Point D is such that $\overrightarrow{CD} = \begin{pmatrix} 12 \\ 9 \end{pmatrix}$.

(i) Find the coordinates of C and of D . [3]

(ii) Show that CD is perpendicular to AB . [3]

(iii) Find the area of triangle ABD . [2]

- 4 The profit $\$P$ made by a company in its n th year is modelled by

$$P = 1000e^{an+b}.$$

In the first year the company made $\$2000$ profit.

- (i) Show that $a + b = \ln 2$. [1]

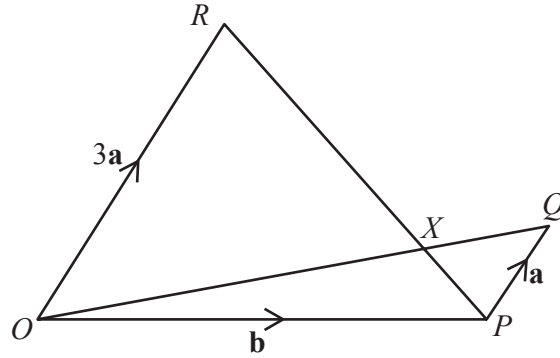
In the second year the company made $\$3297$ profit.

- (ii) Find another linear equation connecting a and b . [2]

- (iii) Solve the two equations from parts (i) and (ii) to find the value of a and of b . [2]

- (iv) Using your values for a and b , find the profit in the 10th year. [2]

5



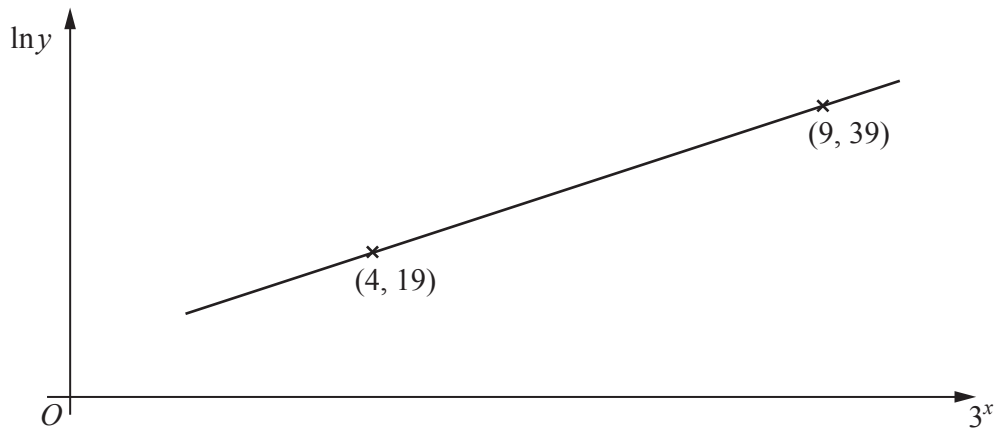
In the diagram $\vec{OP} = \mathbf{b}$, $\vec{PQ} = \mathbf{a}$ and $\vec{OR} = 3\mathbf{a}$. The lines OQ and PR intersect at X .

(i) Given that $\vec{OX} = \mu\vec{OQ}$, express \vec{OX} in terms of μ , \mathbf{a} and \mathbf{b} . [1]

(ii) Given that $\vec{RX} = \lambda\vec{RP}$, express \vec{OX} in terms of λ , \mathbf{a} and \mathbf{b} . [2]

(iii) Hence find the value of μ and of λ and state the value of the ratio $\frac{RX}{XP}$. [3]

- 6 Variables x and y are such that, when $\ln y$ is plotted against 3^x , a straight line graph passing through $(4, 19)$ and $(9, 39)$ is obtained.



- (i) Find the equation of this line in the form $\ln y = m3^x + c$, where m and c are constants to be found. [3]

- (ii) Find y when $x = 0.5$. [2]

(iii) Find x when $y = 2000$.

[3]

7 The functions f and g are defined for real values of x by

$$f(x) = \frac{2}{x} + 1 \text{ for } x > 1,$$

$$g(x) = x^2 + 2.$$

Find an expression for

(i) $f^{-1}(x)$, [2]

(ii) $gf(x)$, [2]

(iii) $fg(x)$. [2]

(iv) Show that $ff(x) = \frac{3x+2}{x+2}$ and solve $ff(x) = x$.

[4]

8 A particle moving in a straight line passes through a fixed point O . The displacement, x metres, of the particle, t seconds after it passes through O , is given by $x = 5t - 3 \cos 2t + 3$.

(i) Find expressions for the velocity and acceleration of the particle after t seconds. [3]

(ii) Find the maximum velocity of the particle and the value of t at which this first occurs. [3]

- (iii) Find the value of t when the velocity of the particle is first equal to 2 ms^{-1} and its acceleration at this time. [3]

- 9 (i) Determine the coordinates and nature of each of the two turning points on the curve $y = 4x + \frac{1}{x-2}$.

[6]

- (ii) Find the equation of the normal to the curve at the point $(3, 13)$ and find the x -coordinate of the point where this normal cuts the curve again. [6]

10 (i) Prove that $\frac{1}{1 - \cos x} + \frac{1}{1 + \cos x} = 2 \operatorname{cosec}^2 x$. [3]

(ii) Hence solve the equation $\frac{1}{1 - \cos x} + \frac{1}{1 + \cos x} = 8$ for $0^\circ < x < 360^\circ$. [4]

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