UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/01, 0606/01**

Paper 1 **October/November 2009**

MARK SCHEME

Maximum Mark : 80

IMPORTANT NOTICE

Mark Schemes have been issued on the basis of **one** copy per Assistant examiner and **two** copies per Team Leader.

1(i) $2a^3 - 7a^2 + 7a^2 + 16 = 0$ leading to $a^3 = -8$, $a = -2$ **(ii)** $2\left(-\frac{1}{2}\right)^3 - 7\left(-\frac{1}{2}\right)^2 - 14\left(-\frac{1}{2}\right) + 16$ $= 21$	M1 A1 [2] M1 A1 [2]	M1 for use of $x = a$ M1 for substitution of $x = -\frac{1}{2}$
2 (i) **(ii)** $\begin{pmatrix} 6 & 3 & 1 & 2 \\ 3 & 2 & 4 & 3 \\ 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 43 \\ 32 \\ 35 \\ 22 \end{pmatrix}$	B1,B1 [2] B2,1,0 [2]	B1 for each matrix, must be in correct order -1 for each error
3 $4(2k+1)^2 = 4(k+2)$ $4k^2 + 3k - 1 = 0$ leading to $k = \frac{1}{4}, -1$	M1 A1 M1 A1 [4]	M1 for use of '$b^2 - 4ac$' Correct quadratic equation M1 for correct attempt at solution A1 for both l values
4 $(13-3y)^2 + 3y^2 = 43$ (or $x^2 + \frac{(13-x)^2}{3} = 43$) $6(2y^2 - 13y + 21) = 0$ (or $2(2x^2 - 13x + 20) = 0$) $(2y-7)(y-3) = 0$ (or $(2x-5)(x-4) = 0$ $y = 3$ or $\frac{7}{2}$ $\left(x = \frac{5}{2} \text{ or } 4\right)$ (or $x = 4$ or $\frac{5}{2}$ $\left(y = \frac{7}{2} \text{ or } 3\right)$)	M1 A1 M1 A1,A1 [5]	M1 for eliminating one variable A1 for correct quadratic M1 for correct attempt at solving quadratic A1 for each correct pair
5 (i) $\left(3+\sqrt{2}\right)^2 + \left(3-\sqrt{2}\right)^2 = 22$ AC = $\sqrt{22}$ **(ii)** $\tan A = \frac{3-\sqrt{2}}{3+\sqrt{2}}$ $\frac{(3-\sqrt{2})(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} = \frac{11-6\sqrt{2}}{7}$	M1 A1 [2] M1 M1, A1 [3]	M1 for use of Pythagoras M1 for correct ratio M1 for rationalising

6 (i) $3x^2-10x-8=0$ $(3x+2)(x-4)=0$ critical values $-\frac{2}{3},4$ $A=\{x:-\frac{2}{3}\le x\le 4\}$ **(ii)** $B=\{x:x\ge 3\}$ $A\cap B=\{x:3\le x\le 4\}$	M1 A1 A1 [3] B1 B1 [2]	M1 for attempt to solve quadratic A1 for critical values B1 for values of *x* that define *B*. B1 for attempt to combine the sets correctly and correct use of notation
7 (i) ${}^{13}C_8=1287$ **(ii)** 7 teachers, 1 student : 6 6 teachers, 2 students ${}^7C_6\times{}^6C_2$ **:**105 5 teachers, 3 students ${}^7C_5\times{}^6C_3$:420 531	M1, A1 [2] B1 B1 B1 B1 [4]	M1 for correct C notation
8 (i) When $t=0$, $N=1000$ **(ii)** $\frac{dN}{dt}=-1000ke^{-kt}$ when $t=0$, $\frac{dN}{dt}=-20$ leading to $k=\frac{1}{50}$ **(iii)** $500=1000e^{-kt}$ $t=-50\ln\frac{1}{2}$ leading to 34.7 mins	B1 [1] M1 B1 A1 [3] M1 M1 A1 [3]	M1 for differentiation B1 for $\frac{dN}{dt}=-20$ stated M1 for attempt to formulate equation using half life M1 for a correct attempt at solution
9 (i) $20\times-2(1-2x)^{19}$ **(ii)** $x^2\frac{1}{x}+2x\ln x$ **(iii)** $\frac{x\left(2\sec^2(2x+1)\right)-\tan(2x+1)}{x^2}$	B1,B1 [2] M1 B1 A1 [3] M1 B1 A1 [3]	B1 for 20 and $(1-2x)^{19}$ B1 for -2 M1 for attempt to differentiate a product. B1 for $\frac{1}{x}$ M1 for attempt to differentiate a quotient. B1 for differentiation of $\tan(2x+1)$

10 (i) $\frac{dy}{dx}=9x^2-4x+2$ at P grad = 7 tangent $y-3=7(x-1)$	M1 A1 M1 A1 [4]	M1 for differentiation and attempt to find gradient M1 for attempt to find tangent equation, allow unsimplified
(ii) at Q, $7x-4=3x^3-2x^2+2x$ leading to $3x^3-2x^2-5x+4=0$ $(x-1)(3x^2+x-4)-0$ $(x-1)(3x+4)(x-1)=0$ leading to $x=-\frac{4}{3}, y=-\frac{40}{3}$	M1 B1,M1 M1 A1 [5]	M1 for equating tangent and curve equations B1 for realising (x – 1) is a factor and attempt to factorise M1 for factorisation and attempt to solve quadratic A1 for both
11 (a) $\tan\theta+\cot\theta=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$ $=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$ $=\frac{1}{\cos\theta\sin\theta}$ $=\operatorname{cosec}\theta\sec\theta$	M1 M1 A1 [3]	M1 for attempt to get in terms of sin and cos and attempt to get one fraction M1 for use of identity
(b)(i) $\tan x=3\sin x$ $\frac{\sin x}{\cos x}=3\sin x$ $\sin x-3\sin x\cos x=0$ leading to $\cos x=\frac{1}{3}$, $\sin x=0$ $x=70.5^\circ, 289.5^\circ$ and $x=180^\circ$	M1 A1√A1 B1 [4]	M1 for use of $\tan x=\frac{\sin x}{\cos x}$ and correct attempt to solve √A1 on their $x=70.5^\circ$ B1 for $x=180^\circ$
(ii) $2\cot^2 y+3\operatorname{cosec} y=0$ $2(\operatorname{cosec}^2 y-1)+3\operatorname{cosec} y=0$ $2\operatorname{cosec}^2 y+3\operatorname{cosec} y-2=0$ $(2\operatorname{cosec} y-1)(\operatorname{cosec} y+2)=0$ leading to $\sin y=-\frac{1}{2}$, $y=\frac{7\pi}{6},\frac{11\pi}{6}$	M1 M1 M1 A1,A1 [5]	M1 for use of correct identity M1 for attempt to solve quadratic M1 for dealing with cosec

12 EITHER **(i)** $\pi r^2 h = 1000$, leading to $h = \dfrac{1000}{\pi r^2}$	M1 A1 [2]	M1 for attempt to use volume
(ii) $A = 2\pi rh + 2\pi r^2$ leading to given answer $A = 2\pi r^2 + \dfrac{2000}{r}$	M1 A1 [2]	M1for attempt to use surface area GIVEN ANSWER
(iii) $\dfrac{dA}{dr} = 4\pi r - \dfrac{2000}{r^2}$ when $\dfrac{dA}{dr} = 0$, $4\pi r = \dfrac{2000}{r^2}$ leading to $r = 5.42$	M1 A1 DM1 A1 [4]	M1 for attempt to differentiate and set to 0 DM1 for solution
(iv) $\dfrac{d^2A}{dr^2} = 4\pi + \dfrac{4000}{r^3}$ + ve when $r = 5.42$ so min value $A_{\min}$=554	M1 A1 A1 [3]	M1 for second derivative method or gradient method' A1 for minimum, can be given if r incorrect but + ve
12 OR (i) $y = x + \cos 2x$ $\dfrac{dy}{dx} = 1 - 2\sin 2x$ when $\dfrac{dy}{dx} = 0$, $\sin 2x = \dfrac{1}{2}$ leading to $x = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$	M1 A1 M1 M1 A1,A1 [6]	M1 for attempt to differentiate M1 for setting to 0 and attempt to solve M1 for correct order of operations
(ii) Area = $\displaystyle\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} x + \cos 2x \, dx$ $= \left[\dfrac{x^2}{2} + \dfrac{1}{2}\sin 2x\right]_{\frac{\pi}{12}}^{\frac{5\pi}{12}}$ $= \dfrac{\pi^2}{12}$	M1 A1,A1 DM1 A1 [5]	M1 for attempt to integrate A1for each term correct DM1 for correct use of limits (Trig terms cancel out)