ADDITIONAL MATHEMATICS

Paper 0606/01

Paper 1

General comments

Well prepared candidates were able to answer the first few questions of the paper with ease, resulting in a good start to the examination giving them the confidence to cope with the more demanding questions later in the paper. The majority of candidates were able to make a reasonable attempt at the paper. Marks obtained covered the whole range, with a few candidates gaining full marks. Others obtained scores in single figures and were clearly unprepared for the paper.

Candidates should be reminded of the necessity of working to the appropriate accuracy throughout their solutions and of showing all working clearly and in appropriate detail.

Comments on specific questions

Question 1

(i) Most candidates attempted to equate f(a) to zero and hence obtain a value for a. Too many candidates gave an answer of a = 2 rather than the correct a = –2 from a completely correct equation.

(ii) Most candidates attempted a correct substitution of $x = -\frac{1}{2}$ into their f(x) and obtained a remainder. Solutions by algebraic long division were quite common and usually correct.

Answers: **(i)** –2 **(ii)** 21

Question 2

Solutions to this question were usually either completely correct, showing a good understanding of matrices and the necessity for compatibility, or else completely incorrect. This suggested that some candidates had not been taught matrices.

(i) Candidates were expected to write down one pair of compatible matrices which they were then expected to multiply together for the second part of the question. Many candidates wrote down a pair of compatible matrices but very often the larger matrix was the transpose of the matrix required. It was not uncommon to see four pairs of matrices offered as a solution; provided these were correct, candidates were not penalised unduly due to the interpretation of the wording of the question.

(ii) Provided a correct compatible pair of matrices had been obtained, candidates were usually able to obtain a correct resultant matrix. There were the odd slips in calculations and occasions when the results were not given in matrix form as expected.

Answers: **(i)** $\begin{pmatrix} 6 & 3 & 1 & 2 \\ 3 & 2 & 4 & 3 \\ 2 & 5 & 5 & 0 \\ 1 & 2 & 2 & 7 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ 2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 5 & 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 6 & 3 & 2 & 1 \\ 3 & 2 & 5 & 2 \\ 1 & 4 & 5 & 2 \\ 2 & 3 & 0 & 7 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 43 \\ 32 \\ 35 \\ 22 \end{pmatrix}$ or $\begin{pmatrix} 43 & 32 & 35 & 22 \end{pmatrix}$

Question 3

Most candidates realised that use of the discriminant was required. Many arithmetic errors occurred when obtaining the quadratic expression resulting from the use of the discriminant, and too many candidates attempted to solve a quadratic inequality rather than a quadratic equation showing a misunderstanding of what was required.

Answer: $-1, \frac{1}{4}$

Question 4

This question was attempted well by most candidates. There were very many completely correct solutions with few arithmetic or algebraic slips. Most candidates obtained a quadratic equation in terms of *y* and then later found corresponding values for *x,* although there were some who forgot to do this. Occasionally, as candidates are so used to solving equations for *x*, errors were made by incorrect substitutions for *x* instead of *y*.

Answer: $x = 4, y = 3 \quad x = 2.5, y = 3.5$

Question 5

(i) Correct applications of Pythagoras' theorem were common and most were able to solve their equation correctly. However there were still many candidates who were unable to simplify $\left(3+\sqrt{2}\right)^2$ and/or $\left(3-\sqrt{2}\right)^2$ correctly.

(ii) Many correct solutions were produced. Some candidates thought, incorrectly, that $\tan A = \frac{3+\sqrt{2}}{3-\sqrt{2}}$, but most were able to attempt to rationalise the denominator.

Answers: **(i)** $\sqrt{22}$ **(ii)** $\frac{11-6\sqrt{2}}{7}$

Question 6

(i) Most candidates were able to obtain the correct critical values and usually went on to obtain a correct set of values for *x*, although some thought that the values of *x* were discrete.

(ii) This part was often done less successfully. Many candidates were unable to solve $7 - 2x \le 1$ correctly, many giving an incorrect solution of $x \le 3$. This then led to an incorrect solution for the set of values for *x* for $A \cap B$. Again, many thought that the values of *x* were discrete and the incorrect answer of {3, 4} was a common one.

Answers: **(i)** $-\frac{2}{3} \le x \le 4$ **(ii)** $3 \le x \le 4$

Question 7

(i) Many correct solutions were produced. Candidates were either able to do this correctly or not at all.

(ii) Again, many correct solutions were seen, the great majority of candidates being able to produce a correct method and hence a correct result. Few attempted to use permutations, most realising that combinations were needed for both parts of the question.

Answers: **(i)** 1287 **(ii)** 531

Question 8

This was probably the most demanding question on the paper with very few completely correct solutions being seen. Few candidates realised that part **(ii)** involved calculus.

(i) Most candidates were able to produce the required result although incorrect answers of 1000e and 1 were common. Very often the mark for this part was the only mark obtained for this question.

(ii) As previously mentioned, few candidates realised that calculus was involved. Those that did invariably equated their $\frac{dN}{dt}$ to 20 rather than – 20 and so answers of $k = -\frac{1}{50}$ were often seen. This then affected the result in part **(iii)**. Others assumed a linear rate of change and often equated, mistakenly, $1000e^{kt}$ to 980 which did produce a result which was fortuitously close to the correct result.

(iii) This was attempted marginally better than the previous part with many candidates using a correct method to try to work out the required time, thus earning method marks even though incorrect values for k were used.

Answers: **(i)** 1000 **(ii)** $\frac{1}{50}$ **(iii)** 34.7

Question 9

Most candidates were able to make a good attempt at all parts of this question and very often scored quite highly.

(i) Most candidates realised that $20\,(1-2x)^{19}$ was part of the solution, but there were quite a few candidates who omitted the –2 or omitted the negative sign from their answer.

(ii) Most candidates recognised the need for differentiation of a product. Problems occurred for those candidates who were unable to differentiate $\ln x$ with respect to x. However, credit was given for the attempt to deal with a product correctly.

(iii) Most candidates recognised the need for differentiation of a quotient (although many did attempt this correctly as a product just as successfully). Again, problems arose if candidates did not know how to differentiate tan ($2x$ + 1) with respect to x, although credit was given for the attempt to differentiate a quotient or appropriate product.

Answers: **(i)** $-40(1 - 2x)^{19}$ **(ii)** $x + 2x\ln x$ **(iii)** $\frac{x\left(2\sec^2(2x+1)\right) - \tan(2x+1)}{x^2}$

Question 10

(i) The given equation of the tangent to the curve did not help some candidates who used it to obtain fortuitous solutions. Most realised that differentiation was needed to obtain the gradient of the tangent, but very often failed to continue appropriately. However, there were still many candidates who were able to provide completely correct solutions to this part.

(ii) Provided candidates had read the question carefully, most were able to make a reasonable (and often correct) attempt at equating the tangent equation with that of the curve together with the solution of the resulting cubic equation. Many failed to realise that they already 'knew' a root of this cubic equation and used valuable time looking for a first root. Many candidates mistakenly equated the tangent equation with the gradient function of the curve and were thus unable to gain any credit from this approach. For many candidates who employed a correct method to find the x-coordinate of the required point, errors were often made in the calculation of the corresponding y-coordinate.

Answer: **(ii)** $\left(-\frac{4}{2}, -\frac{40}{3}\right)$

Question 11

(a) Many completely correct solutions to this part were seen showing that candidates were able to manipulate trigonometric terms and expressions correctly.

(b) (i) Most candidates were able to reach the result of $\cos x = \frac{1}{3}$ and obtain correct solutions for this equation. The great majority failed to include $\sin x = 0$ as part of their solution, having divided through their original equation by $\sin x$ and thus losing the solution from this equation. There were also too many candidates who thought, incorrectly, that $\tan x = \frac{\cos x}{\sin x}$. Candidates should be reminded that angles are to be given correct to 1 decimal place rather than 3 significant figures.

(ii) There were many instances of use of correct identities, manipulation of trigonometric terms and solution of 3 term quadratic equations which resulted in the equation $\sin y = -0.5$. Few candidates however were able to obtain the required angles from this result. Many lost marks through premature approximation and candidates should be reminded that angles in radians are to be given to 3 significant figures and thus intermediate working should be to a greater accuracy. Few candidates worked in terms of π.

Answers: **(b) (i)** 70.5°, 180°, 289.5° **(ii)** 3.76 rad, 5.76 rad or $\frac{7\pi}{6}$ rad, $\frac{11\pi}{6}$ rad

Question 12 EITHER

Most candidates attempted this part.

(i) A surprising number of candidates did not know the formula for the volume of a cylinder. This of course did affect part **(ii)**.

(ii) Credit was given if candidates were able to state the surface area of the cylinder, but many were able to provide the relevant proof convincingly.

(iii) Many correct attempts were seen. Most candidates were able to differentiate correctly and attempt to solve their result equated to zero. A surprising number of candidates were unable to solve a correct equation involving r^3. There were candidates who for some reason attempted to solve $A = 0$. Again, there were candidates who lost marks through premature approximation and working to less than the required degree of accuracy.

(iv) Too many candidates lost the mark for finding the stationary value of A, making no attempt to do so. Most were able to employ a correct method of determining the nature of the stationary point, usually using the second derivative.

Answers: **(i)** $h = \frac{1000}{\pi r^2}$ **(iii)** 5.42 **(iv)** 554, minimum

Question 12 OR

Very few candidates attempted this part of the question, thus making it difficult to comment upon it fully, as most of the solutions seen were usually correct apart from arithmetic or sign slips.

(i) Most candidates were able to differentiate successfully and equate to zero. Full credit was given where appropriate, to those candidates who chose to leave their answers in degrees. These were often then penalised in part **(ii)** if they were not changed to radians.

(ii) Integration was usually carried out successfully, but few of those solutions seen were manipulated to give an answer in terms of π as requested.

Answers: (i) $\frac{\pi}{12}, \frac{5\pi}{12}$ or equivalent (ii) $\frac{\pi^2}{12}$

ADDITIONAL MATHEMATICS

Paper 0606/02

Paper 2

General comments

There were some candidates who produced high quality, well presented work, displaying an impressive knowledge and range of skills for this level of study. There were some large Centres where almost all the candidates scored very highly. However, there were others where most candidates achieved very low scores.

The overall level of difficulty was comparable with previous examinations, although there were some part questions which caused problems for all but the strongest candidates. There were few completely correct solutions to **Question 1**, **Question 5** part **(iii)**, **Question 11 EITHER** part **(i)** and **Question 11 OR** which were found particularly difficult.

The very weakest candidates found it very hard to score, the majority of their marks were gained on **Question 2**, **Question 3** part **(i)**, **Question 4**, **Question 5** parts **(i)** and **(ii)** and **Question 8**. However most candidates could make a reasonable attempt at some part of all the questions, only the very weak left out any of the questions.

Weaker candidates should take note of the marks allocated for a question as producing several sides of working when an answer can gain at most two marks, is not sensible.

There were a significant number of candidates in some Centres who rejected negative solutions inappropriately, for example when looking for coordinates of maximum and minimum points.

Comments on specific questions

Question 1

This proved to be a difficult question for the start of the paper. The idea of range and domain were not understood by many candidates, very few appreciated that the answers to parts **(i)** and **(iii)** were necessarily the same. In general part **(ii)** was answered better although some candidates gave answers of 1+ ln y or $1+\frac{\lg x}{\lg e}$.

Answers: **(i)** $f(x) > e^{-1}$ **(ii)** $1 + \ln x$ **(iii)** $x > e^{-1}$

Question 2

Although most candidates had a reasonable knowledge of the binomial expansion, there were errors in its application to the given expression with a significant number of candidates having errors in the sign of the coefficients. There were some who made no attempt to evaluate the coefficients, having written down a correct expression for the expansion.

Those candidates who had answered part **(i)** confidently (if not completely correctly) were usually able to demonstrate a correct method in part **(ii)**. A common mistake was the use of $1 + x^2$ instead of $(1 + x)^2$.

The most frequent error in part **(ii)** was to add just two terms instead of three. More work than necessary was often done, twelve terms being found rather than just the three needed.

Answers: **(i)** $64 - 96x + 60x^2 - 20x^3$ **(ii)** 4

Question 3

Nearly all candidates knew to calculate values of x^2y and to plot against x although a few weaker ones plotted y against x. There were relatively few instances of unacceptably inaccurate plotting and a lot of candidates scored the first 3 marks in this question, but could go no further.

Not all candidates rearranged the given equation to realise that the gradient was b and the intercept a.

A large proportion of the candidates attempted part **(ii)** by finding the gradient from their graph, though not all continued to realise that the intercept gave a, choosing to find a by substitution.

A few ignored the instruction to *Use your graph* and used simultaneous equations, sometimes using the original equation, usually unsuccessfully.

Answer: $a = 5$ $b = 10$

Question 4

This question was generally found to be straightforward and candidates frequently gained full marks, although a large number failed to score the final two marks for the nature of the stationary values. Another error sometimes seen was to substitute the x-values found into the expression for the derivative, when trying to find y, leading to (3,0) and (–5,0). There were several cases of candidates misreading 45 as 4, which then gave a far more complicated equation to solve. Whenever the solutions appear as awkward decimals it is worth checking that the question has been read correctly. Some candidates solved the equation $6x + 6 = 0$ and then decided that their 2 stationary points were both minima since the value of x was negative.

Answer: (–5, 235) maximum, (3, –21) minimum

Question 5

The first two parts of this question were generally answered correctly. However, only a handful of correct solutions to part **(iii)** were seen and, in general, candidates displayed very little understanding of the difference between vectors and scalars or of what is implied by equal vectors. This was a question where several pages of algebra were sometimes seen, in spite of only 3 marks being available. A quick sketch showing the line ABC, the origin, the coordinates and the lengths would have made this part straightforward. However no such sketches were seen. It was common to see statements such as $\overrightarrow{OA} = \overrightarrow{AC}$ or $\overrightarrow{OA} + \overrightarrow{OC} = 25$ and the answer was frequently given as $\begin{pmatrix} 14 \\ 48 \end{pmatrix}$. Clearly, more time spent working on the topic of vectors would be of value to many candidates.

Answers: **(i)** 25 **(ii)** 5 **(iii)** $\begin{pmatrix} 22 \\ 4 \end{pmatrix}$

Question 6

This was a question that discriminated well and it was encouraging to see how many able candidates scored full marks.

In part **(i)** many were able to make a good attempt at differentiating $\sqrt{4x+12}$ although some did forget the multiplication by 4. There were many good answers with the correct use of the product rule and the subsequent simplification. Many candidates were able to make a single fraction and find k. Some decided that $k = 3$ because of part **(ii)**.

There were a lot of interesting ideas about integration. Despite the *hence*, too few of the candidates seemed to be aware that the solution to this part was connected to part **(i)**. There were many hopeless attempts to integrate the numerator and denominator separately. Whatever they had made of the integration, most candidates were aware of the necessary substitutions required to evaluate the integrated expression. The negative sign inevitably caused a problem for many.

Answers: **(i)** $k = 6$ **(ii)** 20

Question 7

Few candidates scored highly on this question although many picked up a mark or marks for the graph of sin $2x$ in part **(i)**. Many candidates did little else or tried to solve the given equation algebraically, incorrectly introducing tangent into their equation. Of the candidates who did rearrange the equation to make sin $2x$ the subject and hence obtain the equation of the second curve to be drawn, the sketches of the second curve were sometimes poor, more often looking like a quadratic than a cosine curve. Having said this, some beautiful, carefully drawn graphs were seen. In part **(iii)** it was not unusual for the values of x at the intersection to be given rather than the number of the points of intersection as required in the question.

Answer: **(iii)** 2

Question 8

Candidates tended to do this question well, although in parts **(i)** and **(ii)** many candidates lost marks due to making arithmetic errors, often because of the negative signs involved. In part **(iii)** most candidates were able to find A^{-1}, but some did not follow the instruction *and hence find the matrix* X *such that AX = B* and so lost marks.

Another common error in part **(iii)** was to evaluate BA^{-1}. Those who could not multiply matrices could usually find A^{-1}.

Answers: **(i)** $\begin{pmatrix} 0 & -6 \\ 10 & -12 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 11 \\ 10 \end{pmatrix}$ **(iii)** $\frac{1}{10}\begin{pmatrix} 5 & -9 \\ 0 & 12 \end{pmatrix}$

Question 9

This question proved a very good discriminator. Strong candidates scored high marks with average ones picking up part marks but weak ones made very little progress. All three parts had candidates who scored full marks. In many cases candidates did realise that they had to differentiate to find the acceleration and integrate to find the distance but then they could not perform these operations accurately. In part **(i)** there were a few candidates who did not realise that $t = 0$ at O. In part **(ii)** some candidates made the question more difficult by differentiating using the quotient rule. In part **(iii)** some candidates tried to find the distance by using integer values of t from 1 to 8 instead of putting limits into the integral.

Answers: **(i)** 1.25 ms^{-1} **(ii)** -0.08 ms^{-2} **(iii)** 2 m

Question 10

This question was quite well done, and of the later questions, the one most likely to yield full marks. The second part was done slightly better by average ability candidates than the first part, although a few forgot to convert back from u to x.

In part **(a)** there were numerous misreads with $\lg(x+3)$ being written as $\lg(x-3)$ and a high number of miscopies, $\lg(x-3)$ again appearing in candidates equations. Some candidates handled the laws of logarithms correctly but then lost the final accuracy mark through poor rearrangement of their equation. Most could write 2lg 5 as lg 25 but the need to change 2 to lg 100 caused more difficulty, although a number of candidates overcame this problem by evaluating 2 – 2lg 5 on their calculator and then using 10^x to get a numerical value on eliminating logarithms.

Weak candidates found the second part much harder. Some made the substitution correctly but then failed to recognise the resulting equation as a quadratic.

Answers: **(a)** 5 **(b)** –1 and 2

Question 11

Candidates who answered **11 EITHER** were generally more successful than those who chose **11 OR**.

EITHER

The first part was only done correctly in a handful of cases, with most candidates deciding that angle *PAQ* was a right angle, or that the opposite angles of the kite were supplementary. Part **(ii)** was usually answered correctly, although some premature approximation was seen. Part **(iii)** was less well answered as not all candidates had a correct plan for finding the shaded area. The most common error was simply to subtract the area of one sector from the other. There were also instances of the formula for the area of a triangle being used but with the angles taken as $\frac{\pi}{6}$ or $\frac{\pi}{3}$ degrees.

Answers: **(ii)** 6.77 **(iii)** 2.66

OR

Most of the candidates who attempted this question made it far too hard for themselves by forming equations of lines and solving simultaneously. Using this method the vast majority failed to get the coordinates of *D* or *E*. Those using midpoints had the most success, finding first the midpoint of *AC* and using this as the midpoint of *BD* to find *D*. However some, after finding the midpoint, did not know how to find *D*.

Others used a variant of this, adding the coordinates of *A* and *C* and equating them to the coordinates of *B* added to the coordinates of *D*.

The failure of many to find *D* and *E* meant that often part **(iii)** was not attempted. When it was, lengths and equations of lines seemed to be the way most candidates were thinking, though what plan they had was not always evident. The area of the parallelogram was not always attempted and when it was it was often unsuccessful; use of the slant heights or incorrect use of array method were often seen. The formula for the area of a trapezium, even if quoted, was seldom used correctly. The height was often wrong and *AB* was not always recognised as 10.

Answers: **(i)** (–4, 9) **(ii)** (–1, 7) **(iii)** (29, 7)