CAMBRIDGE

CAMBRIDGE INTERNATIONAL EXAMINATIONS

International General Certificate of Secondary Education

MARK SCHEME for the November 2003 question papers

0606 AD	DITIONAL MATHEMATICS
0606/01	Paper 1, maximum raw mark 80
0606/02	Paper 2, maximum raw mark 80

These mark schemes are published as an aid to teachers and students, to indicate the requirements of the examination. They show the basis on which Examiners were initially instructed to award marks. They do not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

• CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2003 question papers for most IGCSE and GCE Advanced Level syllabuses.



Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the November 2003 examination.

	maximum	minimum	mark required	for grade:
	mark available	A	С	E
Component 1	80	63	31	21
Component 2	80	67	36	26

Grade A* does not exist at the level of an individual component.

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Mark Scheme Notes

- Marks are of the following three types:
 - M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0. B2, 1, 0 means that the candidate can earn anything from 0 to 2.
- The following abbreviations may be used in a mark scheme or used on the scripts:
 - AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
 - BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
 - CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
 - ISW Ignore Subsequent Working
 - MR Misread
 - PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)
 - SOS See Other Solution (the candidate makes a better attempt at the same question)

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Penalties

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$ " marks. MR is not applied when the candidate misreads his own figures this is regarded as an error in accuracy.
- OW –1, 2 This is deducted from A or B marks when essential working is omitted.
- PA –1 This is deducted from A or B marks in the case of premature approximation.
- S –1 Occasionally used for persistent slackness.
- EX –1 Applied to A or B marks when extra solutions are offered to a particular equation.



November 2003

INTERNATIONAL GCSE

MARK SCHEME

MAXIMUM MARK: 80

SYLLABUS/COMPONENT: 0606/01

ADDITIONAL MATHEMATICS Paper 1



UNIVERSITY of CAMBRIDGE Local Examinations Syndicate

Page 1 Mark Sch	neme	Syllabus Paper								
IGCSE EXAMINATIONS – NOVEMBER 2003 0606 1										
1. $x + 3y = k$ and $y^2=2x + 3$ Elimination of x or y $\rightarrow y^2 + 6y - (2k+3)=0$ or $\rightarrow x^2 - (2k + 18)x + (k^2 - 27) = 0$ Uses $b^2 - 4ac$ $\rightarrow k < -6$	M1 A1 M1 A1 [4]	x or y must go completely, but allow for simple arithmetic or numeric slips co Any use of b ² –4ac, even if =0 or >0 co								
2. $8^{-x} = 2^{-3x}$ $4^{\frac{1}{2}x} = 2^{x}$ Attempts to link powers of 2 $\rightarrow x -3 - (-3x) = 5 - (x)$ $\rightarrow x = 1.6 \text{ or } 8/5 \text{ etc}$ [$\log 8^{-x} = -3x\log 2$, $\log 4^{\frac{1}{2}x} = x\log 2$ equate coefficients of $\log 2$]	B1 B1 M1 A1 [B1B1 M1A1]	Wherever used Needs to use x ^a ÷x ^b =x ^{a-b} co								
3. $x^3 + ax^2 + bx - 3$ Puts x=3 → 27+9a+3b-3=0 Puts x=-2 → -8+4a-2b-3=15 (9a+3b=-24 and 4a-2b=26) Sim equations → a = 1 and b = -11	M1A1 M1A1 A1 [5]	Needs x=3 and =0 for M mark Needs x=-2 and =15 for M mark (A marks for unsimplified) co								
4. $(\sqrt{3}-\sqrt{2})^2 = 5 - 2\sqrt{6} \text{ or } 5 - 2\sqrt{2}\sqrt{3}$ Divides volume by length ² $\frac{4\sqrt{2} - 3\sqrt{3}}{5 - 2\sqrt{6}} \times \frac{5 + 2\sqrt{6}}{5 + 2\sqrt{6}}$ Denominator = 1 Numerator = $20\sqrt{2} - 15\sqrt{3} + 8\sqrt{12} - 6\sqrt{18}$ But $\sqrt{12} = 2\sqrt{3}$ and $\sqrt{18} = 3\sqrt{2}$ $\rightarrow 2\sqrt{2} + \sqrt{3}$	B1 M1 M1 M1 A1 [5]	Co anywhere V÷l ² used × by denominator with sign changed Correct simplification somewhere with either of these co								
5 $y=0 \text{ when } 3x + \frac{1}{4}\pi = \pi$ $\rightarrow x = \frac{1}{4}\pi$ $\int 6\sin(3x+\pi/4) dx = -6\cos(3x+\pi/4) \div 3$ Between 0 and $\frac{\pi}{4}$ $\rightarrow 2 + \sqrt{2} \text{ or } 3.41$	B1 M1 A2,1 DM1 A1 [6]	Co. Allow 45° Knows to integrate. Needs "cos". All correct, including ÷3, ×6 and -ve Uses limits correctly – must use x=0 In any form – at least 3sf								
 6 Wind 50i- 70j V(still air) = 280i -40j (i) Resultant velocity = v_{air} + w → 330i - 110j tan⁻¹(110/330) = 18.4° → Bearing of Q from P = 108° (ii) Resultant speed = √(330²+110²) Time = 273 ÷ resultant speed = 47 minutes Scale drawings are ok. 	M1 A1 DM1 A1 M1 A1√ [6]	Connecting two vectors (allow −) Co (Could get these 2 marks in (ii)) For use of tangent (330/110 ok) co Use of Pythagoras with his components For 273 ÷ √(a²+b²)								

Page 2 Mark Sch		Syllabus Paper
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$7 (0.6 0.2 0.5) \times \begin{pmatrix} 8 & 6 & 6 & 5 \\ 5 & 4 & 3 & 2 \\ 3 & 3 & 2 & 2 \end{pmatrix} \times \begin{pmatrix} 40 \\ 50 \\ 50 \\ 60 \end{pmatrix}$ (40)	B2,1,0	Wherever 3 matrices come – as row or column matrices – as 3 by 4 or 4 by 3 – independent of whether they are compatible for multiplication or not.
$= (7.3 \ 5.9 \ 5.2 \ 4.4) \times \begin{pmatrix} 10 \\ 50 \\ 50 \\ 60 \end{pmatrix}$	M1 A1	Correct method for multiplying any 2 of the 3 - co for A mark.
or $(0.6 \ 0.2 \ 0.5) \times \begin{pmatrix} 1220\\ 670\\ 490 \end{pmatrix}$	M1	Correct method for remaining two.
→ \$1111	B1 [6]	Co – even if from arithmetic.
8 (i) d/dx(lnx) = 1/x	B1	Anywhere, even if not used in "u/v"
$\frac{dy}{dx} = \frac{(2x+3) \times \frac{1}{x} - (\ln x) \times 2}{(2x+3)^2}$ (ii) $\delta y = (dy/dx) \times \delta x = 0.2p$ (iii) $dy/dt = dy/dx \times dx/dt$ $\rightarrow dx/dt = 0.6$	M1A1√ M1A1 M1 A1√ [7]	Uses correct formula. All ok. Could use product formula. A mark unsimplified. Allow if δy mixed with dy/dt. M mark given for algebraic dy/dx × p. Allow if dy/dt mixed with δy $\sqrt{for 0.12 \div his dy/dx}$. Condone use of δx etc
9 (a) Uses sec ² x = 1+tan ² x \rightarrow quad in sec or \times c ² then uses s ² +c ² =1 \rightarrow quad in cos \rightarrow 4sec ² x+8secx-5=0 \rightarrow -5cos ² x+8cosx+4=0 \rightarrow secx = -2.5 (or0.5) or cosx=-0.4 (or2) \rightarrow x = 113.6° or 246.4° (b) tan(2)+1) = 16/5 = 3.2	B1 M1 A1A1√	Co. Sets to 0 and uses correct method for solution of a 3 term quadratic in sec or cos. A1 co. A1√ for 360°−"first ans" only.
(b) $\tan(2y+1) = 16/5 = 3.2$ Basic angle associated with 3.2 = 1.27 Next angle = π + 1.27 and 2π + 1.27 (Value - 1) ÷ 2 \rightarrow 3.28 (others are 0.134 and 1.705)	B1 M1 M1A1 [8]	Anywhere (allow 72.6°) Realising the need to add on π and/or 2π Correct order used ie -1, then ÷2 for any correct value. Allow if all 3 values are given, providing none are over 4. (degrees – max 2/4 B1, M0, M1, A0)

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10 f(x) = 5-3e	^{1/2} X					
(i) Range	is <5	B1	Allow ≤ or <			
· · ·	$e^{5} = 0 \rightarrow e^{\frac{1}{2}x} = \frac{5}{3}$ or calculator $\rightarrow x = 1.02$	M1A1	Normally 2,0 but if working shown, car get M1 if appropriate			
(iii)	(1.02, 0) and (0, 2)	B1 B1√	Shape in 1 st qu Both shown or		statement.	
(iv) e ^{½x} = (x/2 = I f ¹ (x) =	5 - y)÷3 n[(5-y)/3] : 2ln[(5-x)/3]	M1 M1 A1 [8]	Using logs.	Reasonable attempt $e^{\frac{1}{2}x}$ as the subject. Using logs. All ok, including x, y interchanged.		
11						
	(i) y=½x and y=3x-15 → C(6,3)	M1 A1	Soln of simulta Co (or step me			
	OB=OC+CB	M1	Vectors, step	or soln of y=	½x+5 and	
	→ B(8,9)	A1√	y=3x-15 From his C			
m of OC = ½, eqn of AD is y	m of AD = −2 ⁄-6=−2(x−2) or y=−2x+10	M1 A1	use of m1m2=-1 (M0 if perp to y=3x) Co – unsimplified.			
Soln of y=½x	and eqn of AD \rightarrow D(4,2)	M1A1	Sol of simultaneous eqns. co.			
	$c = \sqrt{45}$, OA = $\sqrt{40}$ OABC = 2($\sqrt{45} + \sqrt{40}$)	M1 M1 A1	Once. Adding OA,AB,BC,CO Co.			
		[11]				
12 EITHER						
	πr + 2x + 2(5r/4) = ½(125 – πr – 5r/2)	M1 A1	Attempt at 4/5 Co.	lengths.		
	h = 3r/4	M1	Anywhere in th			
Area of triar	ngle = $\frac{1}{2} \times 2r \times 3r/4 = 3r^2/4$	M1	independent o Use of ½bh wi			
A = ½πr² + ½ = 125r - 1	2rx + ½πr² -7r²/4	B1 A1	Correct ½πr² - Answer given		rtuitous ans.	
(ii) dA/	dr = 125 – πr –7r/2	M1A1	Any attempt to differentiate. Co.			
Solv	red = 0 to give	DM1	Setting his diff	erential to 0		
→ r = 250) / (2π + 7) or 18.8	A1	Any correct for	rm.		
		[10]				

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40.05					
12 OR (i)	h / (12-r) = 30 / 12	M1	Use of similar lengths correc		eeds ¾
	\rightarrow h = 5(12-r) / 2	A1	Correct in any subject		ls h as
	Uses V=πr²h to give	M1	Needs correct	formula	
	\rightarrow V = $\pi(30r^2 - 5r^3/2)$	A1	Beware fortuit	ous answers	s (AG)
(ii) dV/dr =	= π(60r − 15r²/2)	M1A1	Any attempt to	o differentiate	e. co
= 0 wł	hen r = 8 \rightarrow h = 10	DM1	Setting his dV/dr to 0 + attempt.		
\rightarrow V :	= 640π or 2010	A1	Correct to 3 or more sig figures		
	ne of cone = ⅓π×12²×30 40π or 4520	M1	Anywhere		
Ratio	of 4 : 9 or 1 : 2.25 (3 sf)	A1 [10]	Exactly 4:9 or	2.25 to 3 sig	figures
DM1 for quade	ratic equation				
Sets t Formu correc	ormula. he equation to 0 ala must be correct and stly used. one simple slips in sign.		(2) Factors Sets the equ Attempts to c Solves each	obtain bracke	



November 2003

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MARK SCHEME

MAXIMUM MARK: 80

SYLLABUS/COMPONENT: 0606/02

ADDITIONAL MATHEMATICS Paper 2



Pa	ge 1	Mark Scheme	Syllabus	Paper
		IGCSE EXAMINATIONS – NOVEMBER 2003	0606	2
1 [4]		Eliminate x or y		M1
		$\Rightarrow y^2 - 8y + 15 = 0$ $x^2 - 10x + 9 = 0$		
		Factorise or formula \Rightarrow (1, 3) and (9, 5)		DM1 A1
		Midpoint is (5, 4)		B1 √
2 [4]		$\cos \theta \left(\frac{1+\sin\theta - (1-\sin\theta)}{1-\sin^2\theta} = \cos\theta \left(\frac{2\sin\theta}{1-\sin^2\theta}\right) = \frac{2\sin\theta\cos\theta}{1-\sin^2\theta}$)	M1 A1
		Use of Pythagoras $\Rightarrow \frac{2\sin\theta\cos\theta}{\cos^2\theta} = 2\tan\theta \Rightarrow k = 2$		B1 A1
3 [4]		$\log_2 x = 2\log_4 x$ or $\log_4 (x - 4) = \frac{1}{2} \log_4 x$	$g_2(x-4)$	B1
		$2\log_4 x - \log_4 (x - 4) = 2$ or $\log_2 x - \frac{1}{2} \log_2 (x - 4)$	4) = 2	
		Eliminate logs $\frac{x^2}{x-4} = 16$ or $\frac{x}{\sqrt{x-4}} = 4$		M1 A1
		Solve for $x \implies x = 8$		A1
4 [4]	(i)	Contraction of the second seco		B2 B1 B1
	(ii)	$A \cap B' \cap C'$		
	(iii)	$B \cup (A \cap C)$		
5 [5]	(i)	$243x^5 - 405x^4 + 270x^3$		B1 B1 B1
	(ii)	Coefficient of $x^4 = (-405 \times 1) + (270 \times 2) = 135$		M1 A1
6 [6]		At B, $v = 40$ (e ^{-t} - 0.1) = 0 \Rightarrow e ^{-t} = 0.1 \Rightarrow t = ln 1	0 (=2.30)	M1 A1
		$\int 40 (e^{-t} - 0.1) dt = 40 (-e^{-t} - 0.1t)$		M1 A1
		$AB = \int_{0}^{\log 10} = 40 \left[\left(-\frac{1}{10} - \frac{\ln 10}{10} \right) - \left(-1 \right) \right] = 4(9 - \ln 10) \approx 26.8$		DM1 A1

Pa	ge 2	Mark Scheme Syllabus	Paper					
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7 [7]		Dealing with elements $\begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix}$ and $\begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix}$	M1					
		$\mathbf{A}^{-1} = -\frac{1}{2} \begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix} \qquad \mathbf{B}^{-1} = \frac{1}{8} \begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix}$	A1 A1					
	(i)	C = B - 2 A ⁻¹ = $\begin{pmatrix} 2 & 1 \\ -2 & 3 \end{pmatrix} + \begin{pmatrix} 1 & -2 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -5 & 7 \end{pmatrix}$	M1 A1					
	(ii)	$\mathbf{D} = \mathbf{B}^{-1}\mathbf{A} = \frac{1}{8} \begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix} = \frac{1}{8} \begin{pmatrix} 9 & 5 \\ 14 & 6 \end{pmatrix}$	M1 A1					
8 [7]	(i)	$\frac{10!}{6!4!} = \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4} = 210$						
	(ii) No pink selected i.e. any 6 from (5 + 2) = 7							
	(iii)	All selections contain at least 1 red						
		No yellow selected i.e. any 6 from $(3 + 5) = \frac{8!}{6!2!} = 28$	M1 A1					
		At least 1 of each colour – 120 – (7 + 28) = 175	M1 A1					
9 [8]	(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{4x-3}\right) = \left(4x-3\right)^{-\frac{1}{2}} \times \frac{1}{2} \times 4$	M1 A1					
		$\frac{\mathrm{d}}{\mathrm{d}x}\left\{(2x+3)\sqrt{4x-3}\right\} = \left(2x+3\right)\left(\frac{2}{\sqrt{4x-3}}\right) + 2\sqrt{4x-3}$	M1 A1√					
		$=\frac{12x}{\sqrt{4x-3}} \Longrightarrow k=12$	A1					
	(ii)	$\int \frac{x}{\sqrt{4x-3}} \mathrm{d}x = (2x+3)\sqrt{4x-3} \times \frac{1}{12}$	M1 A1					
		$\int \frac{x}{\sqrt{4x-3}} \mathrm{d}x = (2x+3)\sqrt{4x-3} \times \frac{1}{12}$ $\int_{1}^{7} = \frac{1}{2} (85-5) = 6\frac{2}{3}$	A1					
10 [10]		(i) ∠AOB = 19.2 + 16 = 1.2	M1 A1					
		(ii) <i>DE</i> = 8 sin 1.2 ≈ 7.46	M1 A1					
	1	(iii) $\angle DOE = \sin^{-1} (7.46 \div 16) \approx 0.485 (AG)$	M1 A1					
	16	(iv) Sector $DOB = \frac{1}{2} \times 16^2 \times 0.485 = 62.08$	M1					
	/	Length $OE = \sqrt{(16^2 - 7.46^2)} \approx 14.2$	M1					
	* 4	$\Delta DOE = \frac{1}{2} \times 7.46 \times 14.2 \approx 52.97$	M1					
		Shaded area $\approx 9.1 - 9.3$ (9.275)	A1					

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11	[10]		V	5	10	15	20	25	(i) Plotting lg R	against lg v	M1	
			R	32	96	180	290	420	Accuracy of poir	nts: Straight I	ine A2,	1, 0
			lg v	0.70	1.00	1.18	1.30	1.40	(ii) $R = k v^{\beta} \Rightarrow lg$	$gR = \lg k + \beta$	lg v B1	
			lg R	1.51	1.98	2.26	2.46	2.61	β = gradien	t ≈ 1.55 - 1.6	0 M1	A1
							lg <i>k</i> =	= lg <i>R</i> i	ntercept $\approx 0.4 =$	⇒ <i>k</i> ≈ 2.4 - 2	.6 M1	A1
		(iii)	lg R :	= lg 75	i ≈ 1.88	$B \Rightarrow from$	m grapł	h lg <i>v</i> ≈	0.92 - 0.96 ⇒	v ≈ 8.3 - 9.1	M1	A1
			[Or b	y solvi	ng e.g	., 75	= 2.5 <i>v</i> ¹	^{.58} or	1.88 = 0.4 +	1.58 lg <i>v</i>]		
EIT	2 HER 1]	(i)	$gf(x) = \frac{4}{2 - (3x - 2)}$						B1			
			Solve	$e \frac{4}{4-3}$	$\frac{1}{x} = 2$		[or so	olve fg()	$x)=3\left(\frac{4}{2-x}\right)-$	2 = 2]	M1	
			\Rightarrow X :	= 2/3							A1	
		(ii)	f(<i>x</i>) =	= g(x) =	⇒ 3 <i>x</i> –	$2 = \frac{4}{2}$	$\frac{1}{x} \Rightarrow 3$	$8x^2 - 8x$	(+ 8 = 0			
			Discr	iminar	nt = 64	- 96 <	0	\Rightarrow	No real roc	ots	M1	A1
		(iii)	f ^{−1} : x	$x\mapsto (x)$	+ 2) ÷	3					B1	
			<i>y</i> = 4	/ (2 –	<i>x</i>)	\Rightarrow)	x = 2 - 4	4/y	\Rightarrow g ⁻¹ : x \vdash	→ 2 – 4/x	M1	A1
		(iv)	1	1	1	tr					B1	B1
				1	X				Lines inte	rsect at (1, ⁻	1) B1	

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12 OR [11]	(i)	$1 - x^2 + 6x \equiv a - (x + b)^2 \equiv a - x^2 - 2bx - b^2 \Rightarrow a - b^2 \equiv 1 \text{ and } -2b \equiv 6$	M1 A	.1
		[or $1 - x^2 + 6x \equiv 1 - (x^2 - 6x) \equiv 1 - \{(x - 3)^2 - 9\}$]		
		\Rightarrow b = -3, a = 10	A1	
	(ii)	$1 - x^2 + 6x \equiv 10 - (x - 3)^2 \implies$ Maximum at (3, 10)		
		\therefore Single-valued for $x \ge 3$ and hence for $x \ge 4$	M1 A	.1
	(iii)	$y = 10 - (x - 3)^2 \implies (x - 3)^2 = 10 - y \implies x - 3 = \sqrt{(10 - x)}$	M1	
		$\Rightarrow f^{-1}: x \mapsto 3 + \sqrt{(10 - x)}$	A1	
	(iv)	When $x = 2$, $g(x) = 9$ and when $x = 7$, $g(x) = -6$	B1	
		Range of g is $-6 \le g \le 10$	B1	
	(v)		B 2, 1	, 0