Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

- The following abbreviations may be used in a mark scheme or used on the scripts:

  AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

  BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

  CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

  ISW Ignore Subsequent Working

  MR Misread

  PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

  SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

- MR −1 A penalty of MR −1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

- OW −1, 2 This is deducted from A or B marks when essential working is omitted.

- PA −1 This is deducted from A or B marks in the case of premature approximation.

- S −1 Occasionally used for persistent slackness.

- EX −1 Applied to A or B marks when extra solutions are offered to a particular equation.

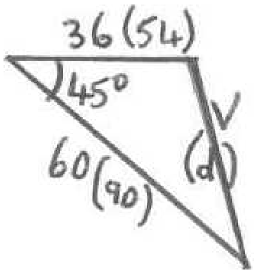
CAMBRIDGE
INTERNATIONAL EXAMINATIONS

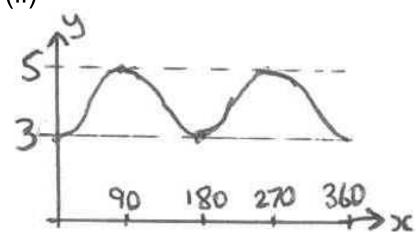
JUNE 2003

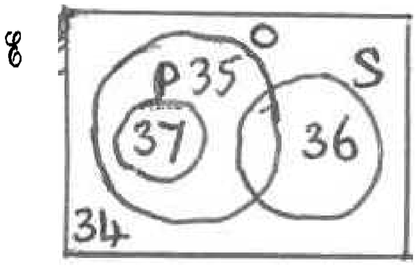
INTERNATIONAL GCSE

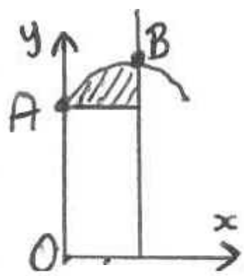
MARK SCHEME
MAXIMUM MARK: 80
SYLLABUS/COMPONENT: 0606/01 **ADDITIONAL MATHEMATICS** **Paper 1**

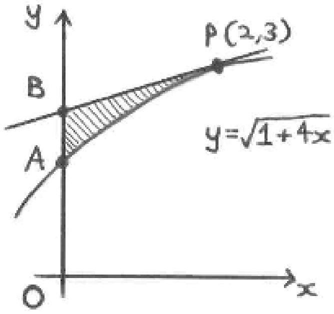
UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1.	x or y eliminated completely Uses the discriminant b^2-4ac on a quadratic set to 0 Arrives at k = 0 from 32k = 0 Correct answer $k \geq 0$.	M1 M1 A1 A1 [4]	Allow as soon as x or y eliminated. Condone poor algebra – quadratic must be set to 0 – b^2-4ac = 0, <0, >0 all ok. For k and 0. For $k \geq 0$.
2.	Length = $(1 + \sqrt{6}) \div (\sqrt{2} + \sqrt{3})$ Multiplying top and bottom by $\pm(\sqrt{3} - \sqrt{2})$ $\rightarrow \sqrt{3} + \sqrt{18} - \sqrt{2} - \sqrt{12}$ Reduces $\sqrt{18}$ to $3\sqrt{2}$ or $\sqrt{12}$ to $2\sqrt{3}$ $\rightarrow 2\sqrt{2} - \sqrt{3}$ $\rightarrow \sqrt{8} - \sqrt{3}$	 M1 M1 DM1 A1 [4]	 Multiply both top and bottom by $\pm(\sqrt{3} - \sqrt{2})$. Allow wherever this comes – not DM. Dependent on first M – collects $\sqrt{2}$ and $\sqrt{3}$. Co.
3.	(i) $32 - 80x + 80x^2$ (ii) $(k + x) \times$ (i) Coeff. of x is –80k + 32 Equated with $-8 \rightarrow k = ½$ or 0.5	B1 x 3 M1 A1√ [5]	Allow 2^5 for 32 (if whole series is given, mark the 3 terms). Must be 2 terms considered. For solution of k = $(-8 - a) \div (b)$
4.	Liner travels 54km or relative speed of lifeboat is 60km/h.  Correct vel./distance triangle Use of cosine rule in triangle $V^2 = 60^2 + 36^2 - 2.60.36\cos45$ or $d^2 = 90^2 + 54^2 - 2.90.54\cos45$. V = 42.9 or d = 64.4 $\rightarrow$ V = 42.9	B1 B1 M1 A1 A1 [5]	Anywhere. Triangle must be correct with 54, 45°, 90 or 36, 45°, 60 or even 36, 45°, 90. Allow for other angles. Unsimplified and allow for 135° as well as 45°. Co.

5. Elimination of x or y. $\rightarrow 4x^2 + 6x - 4 = 0$ or $y^2 - 12y + 11 = 0$ Solution of quadratic = 0. $\rightarrow$(0.5, 11) and (-2, 1) Length = $\sqrt{(2.5^2 + 10^2)} = 10.3$	M1 A1 DM1 A1 M1A1 [6]	x or y eliminated completely. Correct equation – not necessarily = 0 Usual method for solving quadratic = 0 All correct. Condone incorrect pairing if answers originally correct. Must be correct formula correctly applied.
6. $A^2 = \begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 4 & -9 \\ 0 & 1 \end{pmatrix}$ $A^{-1} = ½ \times \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$ $B = A^2 - 4A^{-1} = \begin{pmatrix} 2 & -15 \\ 0 & -3 \end{pmatrix}$	M1A1 B1B1 M1A1 [6]	Do not allow M mark if all elements are squared. If correct, allow both marks. If incorrect, some working is needed to give M mark. B1 for ½, B1 for matrix. M mark is independent of first M. Allow M mark for $4A^{-1} - A^2$.
7. f(x) = 4 – cos2x (i) amplitude = $\pm$1. Period = 180^o or π (ii)  Max (90^o, 5) and (270^o, 5)	 B1B1 B2,1 B1B1 [6]	 Independent of graph. Do not allow "4 to 5". Must be two complete cycles. 0/2 if not. Needs 3 to 5 marked or implied. Needs to start and finish at minimum. Needs curve not lines. Independent of graph (90, 270 gets B1). Allow radians or degrees.

8. 		
(i) O, P, S correct	B2,1	Give B1 if only one is correct.
(ii) 34, 35, 36, 37 correct	B2,1	These 2 B marks can only be awarded only if B2 has been given for part (i).
$O \cap S$ = odd squares $\rightarrow$ 4	B1	Co.
$O \cup S$ = odd and even squares $\rightarrow 49 + 5 = 54$	M1A1 [7]	Any correct method. Co.
9. (i) $\log_4 2 = \frac{1}{2}$ $\log_8 64 = 2$	B1B1	Anywhere.
$\rightarrow 2x + 5 = 9^{1.5}$ $\rightarrow x = 11$	M1A1	Forming equation and correctly eliminating "log". Co.
(ii) Quadratic in 3^y	M1	Recognising that the equation is quadratic.
Solution of quadratic = 0	DM1	Correct method of solving the equation = 0.
$\rightarrow 3^y = 5$ or -10		
Solution of $3^y = k$	M1	Not dependent on first M1. Correct method.
y = 1.46 or 1.47	A1 [8]	Co. (not for $\log 5 \div \log 3$). Ignore ans from $3^y = -10$.

10.		
<table><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>y</td><td>9.2</td><td>8.8</td><td>9.4</td><td>10.4</td><td>11.6</td></tr><tr><td>xy</td><td>18.4</td><td>26.4</td><td>37.6</td><td>52.0</td><td>69.6</td></tr><tr><td>x^2</td><td>4</td><td>9</td><td>16</td><td>25</td><td>36</td></tr></table>	M1 A2,1	Knows what to do. Points accurate – single line with ruler
(i) Plots xy against x^2 or x^2 against xy to get a line		
c = 12 to 12.5 or -7.25 to -7.75 m = 1.55 to 1.65 or 0.62 to 0.63	B1 B1	 Allow if y = mx + c used.
xy = $1.6x^2$ + 12 or x^2 = 0.625xy - 7.5 $\rightarrow$ y = 1.6x + 12/x	 M1 A1	 Allow if y = mx + c used. Must be xy = mx^2 + c or x^2 = mxy + c.
(ii) Reads off at xy = 45 $\rightarrow$ x = 4.5 to 4.6	M1A1 [9]	Algebra is also ok as long as xy = 45 is solved with an equation given M1 above.
11. $y = xe^{2x}$		
(i) $d/dx(e^{2x}) = 2e^{2x}$	B1	Anywhere – even if $dy/dx = 2x\ e^{2x}$ or $2\ e^{2x}$.
$dy/dx = e^{2x} + x.2\ e^{2x}$ sets to 0 $\rightarrow$ x = -0.5	M1 M1A1	Use of correct product rule. Not DM mark. Allow for stating his dy/dx = 0.
(ii) $d^2y/dx^2 = 2\ e^{2x} + [2\ e^{2x} + 4x\ e^{2x}]$ $= 4\ e^{2x}(1 + x) \rightarrow k = 4$	M1A1 A1	Use of product rule needed. Allow if he reaches $4e^{2x}(1 + x)$.
(iii) when x = -0.5, d^2y/dx^2 is +ve (0.74) $\rightarrow$ Minimum	M1A1 [9]	No need for figures but needs correct x and correct d^2y/dx^2.
12. **EITHER**		
		
At A, y = 4	B1	Anywhere.
dy/dx = 2cosx - 4sinx	M1A1	Any attempt at differentiation.
dy/dx = 0 when tanx = ½	M1A1	Sets to 0 and recognises need for tangent.
At B, x = 0.464 or 26.6°	A1	Co. Accept radians or degrees here.

$\int(2\sin x + 4\cos x)dx = -2\cos x + 4\sin x$	M1A1	Any attempt with trig. functions.
Area under curve = $[\ ]_{0.464} - [\ ]_0$ $\rightarrow -(-2) = 2$.	DM1	x-limits used correctly. If "0" ignored or automatically set to 0, give DM0.
Reqd area = $2 - (4 \times 0.464) = 0.144$ (5 or 6).	M1A1 [11]	Plan mark – must be radians for both M and A.
12. **OR** 		
$dy/dx = \frac{1}{2}(1 + 4x)^{-\frac{1}{2}} \times 4$ At P, $m = {}^2/_3$	M1A1	Any attempt with dy/dx – not for $\sqrt{(1 + 4x)} = 1 + 2\sqrt{x}$. A mark needs everything.
Eqn of tangent $y - 3 = {}^2/_3(x - 2)$ At B, $x = 1{}^2/_3$	M1A1	Not for normal. Not for "$y + y_1$" or for m on wrong side. Allow A for unsimplified.
$\int\sqrt{(1 + 4x)}dx = (1 + 4x)^{1.5} \times {}^2/_3 \div 4$	M1A1 A1	Any attempt at integration with (1 + 4x) to a power. Other fn of x included, M1 only.
Area under curve = $[\ ]^2 - [\ ]^0 = 4{}^1/_3$	DM1A1	Use of limits 0 to 2 only. Must attempt a value at 0.
Shaded area = Area of trapezium - $4{}^1/_3 = {}^1/_3$	M1	Plan mark independent of M marks.
Or Area under $y = {}^2/_3x + 1{}^2/_3 - 4{}^1/_3 = {}^1/_3$	A1	A1 co.
[or $\int x dy = \int(\frac{1}{4}y^2 - \frac{1}{4})dy$ $= y^3/12 - y/4$	[M1A1 A1	Attempt at differentiation. A1 for each term.
area to left of curve = $[\ ]_3 - [\ ]_1 = 1{}^2/_3$ shaded area = $1{}^2/_3$ – triangle $(\frac{1}{2}.2.1{}^1/_3)$ $= {}^1/_3$]	DM1A1 M1 A1] [11]	Must be limits 1 to 3 used correctly. Plan mark independent of other Ms.

DM1 for quadratic equation. Equation must be set to 0.
Formula - must be correctly used. Allow arithmetical errors such as errors over squaring a negative number.
Factors – must be an attempt at two brackets. Each bracket must then be equated to 0 and solved.
Completing the square – must result in $(x\pm k)^2 = p$. Allow if only one root considered.

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

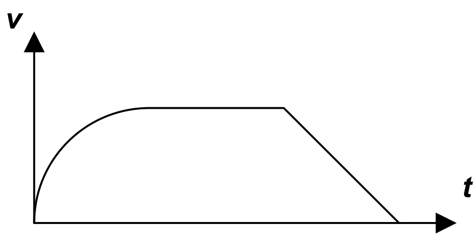
JUNE 2003

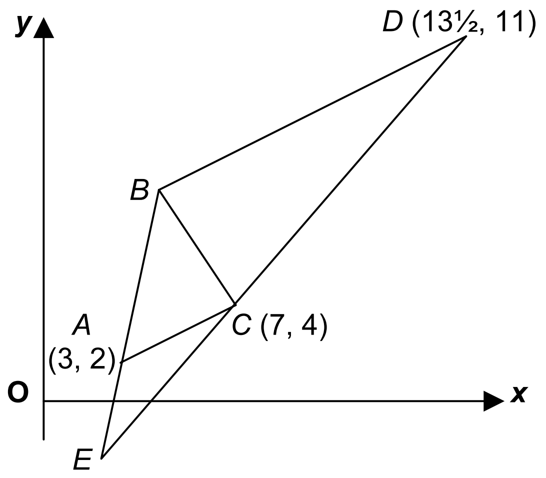
INTERNATIONAL GCSE

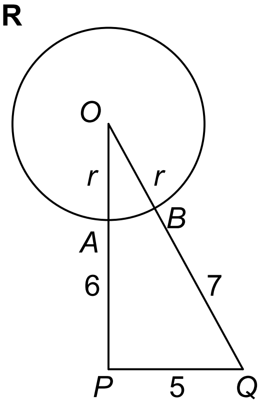
MARK SCHEME
MAXIMUM MARK: 80
SYLLABUS/COMPONENT: 0606/02 **ADDITIONAL MATHEMATICS** **Paper 2**

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1	Put $x = -b/2$ (or synthetic or long division to remainder) $\Rightarrow 3b^3 + 7b^2 - 4 = 0$ AG	M1 A1
	Search $\Rightarrow b = -1$ [or $b = -2$] (1st root or factor)	M1 A1
	Attempt to divide $\Rightarrow 3b^2 + 4b - 4$ (or $3b^2 + b - 2$) or further search $\Rightarrow b = -2$ [or $b = -1$]	M1
[7]	Factorise (or formula) [3 term quadratic] or method for 3rd value $\Rightarrow b = -2, -1$ or $^2/_3$	DM1 A1
2 (i)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \pm(9\mathbf{i} + 12\mathbf{j})$	M1
	Unit vector = $\overrightarrow{AB} \div \sqrt{9^2 + 12^2} = \pm(0.6\mathbf{i} + 0.8\mathbf{j})$ [Accept any equivalent unsimplified version of column vectors, $\pm\begin{pmatrix}9\\12\end{pmatrix}$, $\pm\begin{pmatrix}0.6\\0.8\end{pmatrix}$]	M1 A1
(ii)	$\overrightarrow{AC} = {}^2/_3\overrightarrow{AB} = 6\mathbf{i} + 8\mathbf{j}$ (or $\overrightarrow{CB} = {}^1/_3\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j}$)	M1
[6]	$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ (or $\overrightarrow{OB} - \overrightarrow{CB}$) = $12\mathbf{i} + 5\mathbf{j}$ (or equivalent)	M1 A1
3	$\int(3x^{0.5} + 2x^{-0.5})\,dx = 3x^{1.5}/1.5 + 2x^{0.5}/0.5$ (one power correct sufficient for M mark)	M1 A1 A1
	$\int_1^8 = (2 \times 8\sqrt{8} + 4\sqrt{8}) - (2 + 4)$ Must be an attempt at integration	M1
[6]	Putting $\sqrt{8} = 2\sqrt{2}$ (i.e. one term converted $\sqrt{}$ to $k\sqrt{2}$) $\Rightarrow -6 + 40\sqrt{2}$	B1√ A1
4	$16^{x+1} = 2^{4x+4}$ or 16×2^{4x} or 16×4^{2x} or 16×16^x $20(4^{2x}) = 20(2^{4x})$ or $5(2^{4x+2})$ or 20×16^x	B1 B1
	$2^{x-3}\,8^{x+2} = 2^{x-3}\,2^{3x+6} = 2^{4x+3}$ or 8×2^{4x} or 8×4^{2x} or 8×16^x	B1
[4]	Cancel 2^{4x+2} or 2^{4x} and simplify $\Rightarrow 4.5$ or equivalent	B1
5 (i)	$f(0) = ½$ $f^2(0) = f(½) = (\sqrt{e} + 1)/4 \approx 0.662$ (accept 0.66 or better)	B1 M1 A1
(ii)	$x = (e^y + 1)/4$ $\Rightarrow e^y = 4x - 1$ $\Rightarrow f^{-1} : x \mapsto \ln(4x - 1)$	M1 A1
(iii)	Domain of f^{-1} is $x \geq ½$ Range of f^{-1} is $f^{-1} \geq 0$	B1 B1
[7]		

6 (i)	$x^2 - 8x + 12 = 0$ Factorise or formula $\Rightarrow$ Critical values $x = 2, 6$ $x^2 - 8x + 12 > 0$ $\Rightarrow \{x : x < 2\} \cup \{x : x > 6\}$	M1 A1 A1
(ii)	$x^2 - 8x = 0$ $\Rightarrow$ Must be an attempt to find *2* solutions $x^2 - 8x < 0$ $\Rightarrow \{x : 0 < x < 8\}$	M1 A1
(iii) [7]	Solution set of $\lvert x^2 - 8x + 6 \rvert < 6$ is combination of **(i)** and **(ii)** $\{x : 0 < x < 2\}$ $\{x : 6 < x < 8\}$	B1 B1 (one for each range)
7 (i)	$6! = 720$	B1
(ii)	M … $\Rightarrow$ $5! = 120$	M1 A1
(iii)	4! 48	M1 A1
(iv)	$6!/4!\,2! = 15$ Accept ${}_6C_4$ or ${}_6C_2 = 15$	B1
(v) [8]	$5!/3!\,2! = 10$ (or, answer to (iv) less ways M can be omitted) (Listing – ignoring repeats $\geqslant 8$ [M1] $\Rightarrow$ 10 [A1])	M1 A1
8 (i)	Collect sin *x* and cos x $\Rightarrow$ $\sin x = 5 \cos x$ Divide by cos *x* $\Rightarrow$ $\tan x = 5$ (accept $^1/_5$ – for M only) x = 78.7° or (258.7°) i.e. 1st solution + 180°	M1 M1 A1 A1√
(ii) [8]	Replace $\cos^2 y$ by $1 - \sin^2 y$ $3\sin^2 y + 4\sin y - 4 = 0$ Factorise (or formula) (3 term quadratic) $\Rightarrow \sin y = {}^2/_3$ (or -2) y = 0.730 (accept 0.73 or better) or (2.41) i.e. π (or $\frac{22}{7}$) less 1st solution	B1 M1 A1 A1√
9 (i)	$\int (12t - t^2)\,dt = 6t^2 - {}^1/_3t^3$ From $t = 0$ to $t = 6$ distance = $\int_0^6 = 144$ Max. speed = 36 $\Rightarrow$ from $t = 6$ to $t = 12$ distance = 36 x 6 (= 216) During deceleration distance = $(0^2 - 36^2) \div 2(-4) = 162$ Area of Δ is fine for M mark but value of *t* must be from *constant* acceleration *not* $12 - 2t = \pm 4$ Total distance = 144 + 216 + 162 = 522	M1 A1 A1 B1 M1 A1
(ii) [8]		B2, 1, 0

10 (i)	$\frac{dy}{dx} = \frac{(x-2)2-(2x+4)1}{(x-2)^2} = \frac{-8}{(x-2)^2} \Rightarrow k = -8$	M1 A1
	Must be correct formula for M mark (accept $\frac{-8}{(x-2)^2}$ as answer)	
(ii)	When $y = 0$, $x = -2$ (B mark is for *one* solution only) NB. $x = 0$, $y = -2$	B1
	$m_{tangent} = -8/16 = -1/2 \Rightarrow m_{normal} = +2$ (M is for use of m_1 m_2 = -1, whether numeric or algebraic)	M1
	Equation of normal is $y - 0 = 2(x + 2)$ (candidate's m_{normal} and $[x]_{y=0}$ for M mark)	M1 A1
(iii)	When $y = 6$, $x = 4$	B1
	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = \frac{-8}{(x-2)^2} \times 0.05 = \frac{-8}{4} \times 0.05 = -0.1$ (accept $\pm$)	M1 A1√
	i.e. $\left[\frac{dy}{dx}\right]_{x=4}$ x 0.05 for M mark.	
[9]	√ is for error in k only. (Condone S $\approx \frac{dy}{dx}$ x S)	
11	**EITHER**  	
	(i) $m_{AC} = (4 - 2)/(7 - 3) = ½$	B1
	$m_{BD} = ½$	B1√
	$m_{BC} = -2$	B1√
	Equation of *BD* is $y - 11 = ½(x - 13.5)$ i.e. $4y = 2x + 17$	M1
	Equation of *BC* is $y - 4 = -2(x - 7)$ i.e. $y = -2x + 18$	M1
	Solving $y = 7$, $x = 5.5$	M1 A1

[10]	**(ii)** $\frac{\Delta EBD}{\Delta EAC}$ = (ratio of corresponding sides or *x*- or *y*- steps)^2 = 4/1 Quadrilateral *ABDC*/ Δ *EBD* = 3/4 [Or, find *E*(1/2, -3) and then use array method to find *one* of: area quadrilateral *ABDC* = 22.5 area Δ *EBD* = 30 Find other area and hence ratio = 3/4 or equivalent]	M1 A1 A1 M1 A1 A1
11	**OR**  	
	(i) $(r + 6)^2 + 5^2 = (r + 7)^2$	M1
	Solve $\Rightarrow$ r = 6	M1 A1
	tan *AOB* = 5/12 *AOB* = 0.395 or 22.6°	M1
	Length of arc *AB* = 6 x 0.395 = 2.37 or better	M1 A1
	(ii) Sector *AOB* = ½ x 6^2 x 0.395 = 7.11	M1
	Shaded area = ½ x 5 x 12 - 7.11	M1
	All figures in sector and triangle correct √	A1√
[10]	22.9 or better	A1

Grade thresholds taken for Syllabus 0606 (Additional Mathematics) in the June 2003 examination.

	maximum mark available	minimum mark required for grade:		
		A	C	E
Component 1	80	54	29	20
Component 2	80	60	34	23

Grade A* does not exist at the level of an individual component.