CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS

Paper 4 (Extended)

0580/04, 0581/04

May/June 2009

2 hours 30 minutes

Candidates answer on the question paper.

Additional Materials: Electronic calculator   Geometrical instruments
                      Mathematical tables (optional)   Tracing paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.

If working is needed for any question it must be shown below that question.

Electronic calculators should be used.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures.

Give answers in degrees to one decimal place.

For π use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 130.

For Examiner's Use

This document consists of **19** printed pages and **1** blank page.

[Turn over

1 Marcus receives $800 from his grandmother.

(a) He decides to spend $150 and to divide the remaining $650 in the ratio

$$\text{savings} : \text{holiday} = 9 : 4.$$

Calculate the amount of his savings.

Answer(a) $ .... [2]

(b) (i) He uses 80% of the $150 to buy some clothes.

Calculate the cost of the clothes.

Answer(b)(i) $ .... [2]

(ii) The money remaining from the $150 is $37\frac{1}{2}$ % of the cost of a day trip to Cairo.

Calculate the cost of the trip.

Answer(b)(ii) $ .... [2]

(c) (i) Marcus invests $400 of his savings for 2 years at 5 % per year **compound** interest.

Calculate the amount he has at the end of the 2 years.

Answer(c)(i) $ .... [2]

(ii) Marcus's sister also invests $400, at r% per year **simple** interest.
At the end of 2 years she has exactly the same amount as Marcus.

Calculate the value of r.

Answer(c)(ii) $r =$ .... [3]

2 A normal die, numbered 1 to 6, is rolled 50 times.

The results are shown in the frequency table.

Score	1	2	3	4	5	6
Frequency	15	10	7	5	6	7

(a) Write down the modal score.

Answer(a) .... [1]

(b) Find the median score.

Answer(b) .... [1]

(c) Calculate the mean score.

Answer(c) .... [2]

(d) The die is then rolled another 10 times.
The mean score for the 60 rolls is 2.95.
Calculate the mean score for the extra 10 rolls.

Answer(d) .... [3]

         [Turn over

3

P

NOT TO
SCALE

10 cm

14 cm

Q

R
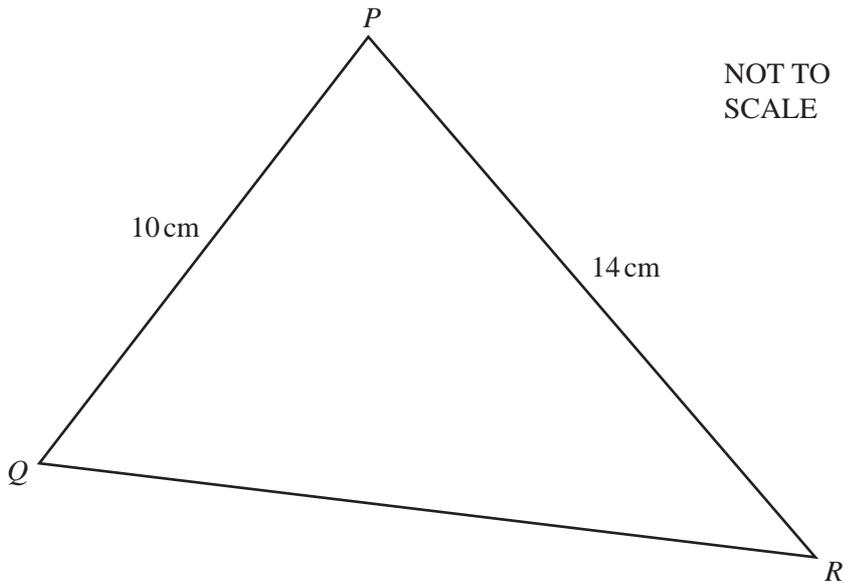
In triangle *PQR*, angle *QPR* is acute, *PQ* = 10 cm and *PR* = 14 cm.

(a) The area of triangle *PQR* is 48 cm².

Calculate angle *QPR* and show that it rounds to 43.3°, correct to 1 decimal place.
You must show all your working.

Answer (a)

[3]

(b) Calculate the length of the side *QR*.

Answer(b) QR = ... cm [4]

4

North

North

A

126°

NOT TO
SCALE

B

North

250 m

23°

P
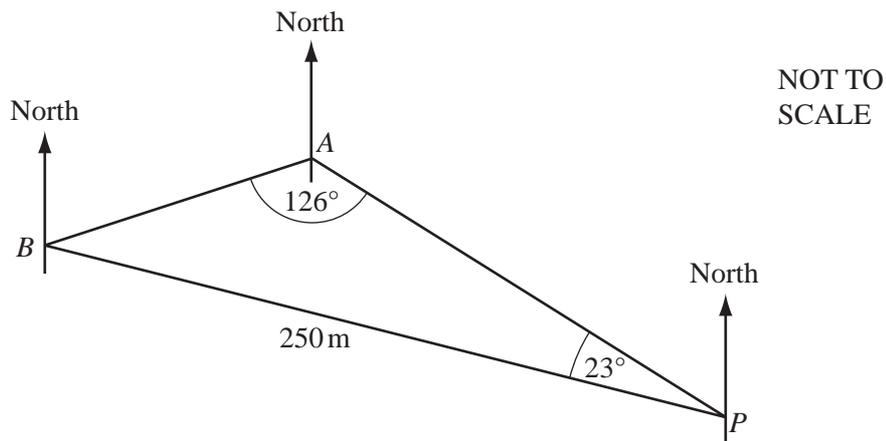
The diagram shows three straight horizontal roads in a town, connecting points *P*, *A* and *B*.

PB =250 m, angle *APB* = 23° and angle *BAP* = 126°.

(a) Calculate the length of the road *AB*.

Answer(a) AB = ... m [3]

(b) The bearing of *A* from *P* is 303°.

Find the bearing of

(i) *B* from *P*,

Answer(b)(i) .... [1]

(ii) *A* from *B*.

Answer(b)(ii) .... [2]

5 **(a)** The table shows some values for the equation $y = \dfrac{x}{2} - \dfrac{2}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.

x	-4	-3	-2	-1.5	-1	-0.5	0.5	1	1.5	2	3	4
y	-1.5	-0.83	0	0.58			-3.75		-0.58	0	0.83	1.5

(i) Write the missing values of y in the empty spaces. [3]

(ii) On the grid, draw the graph of $y = \dfrac{x}{2} - \dfrac{2}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.
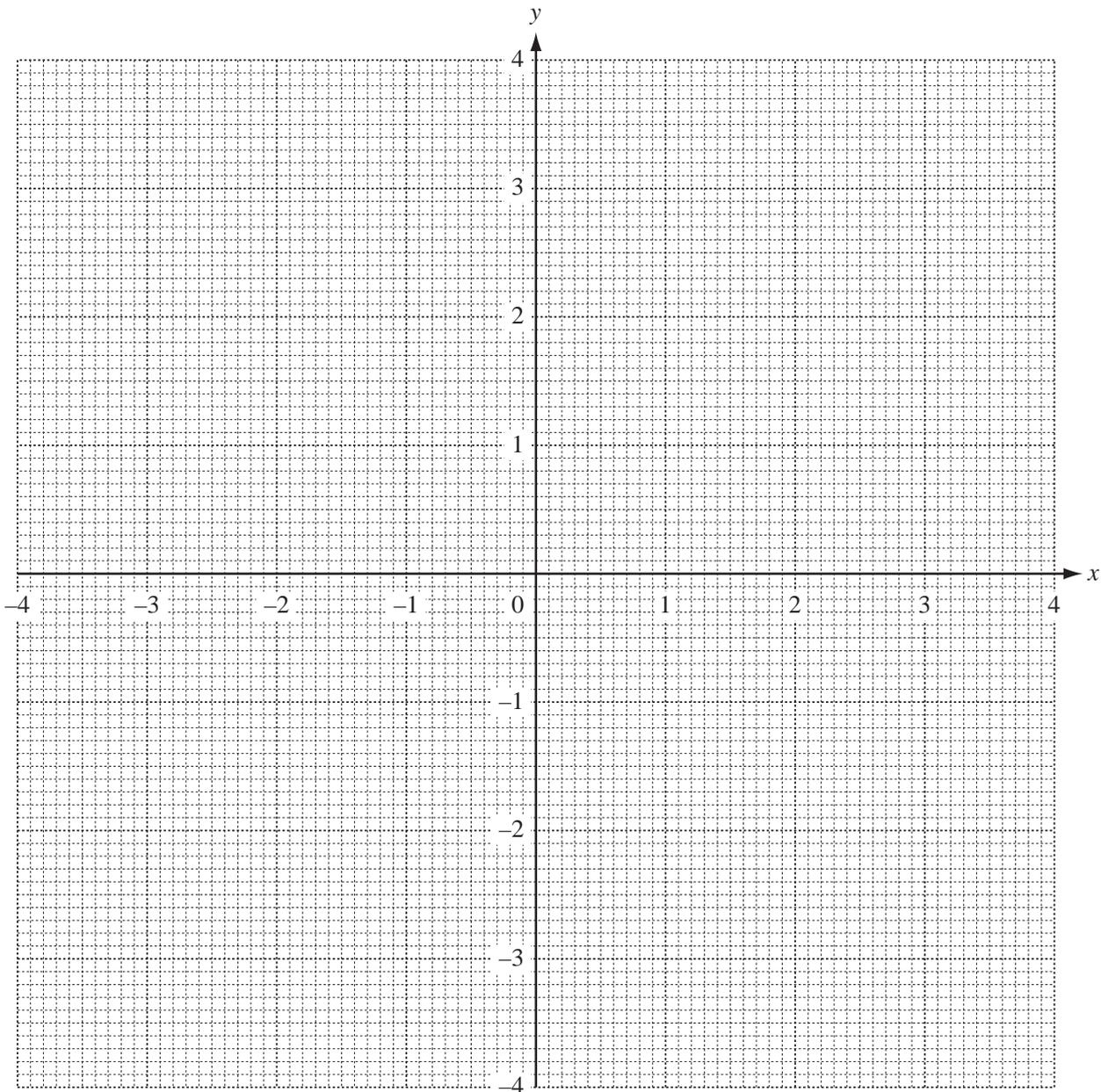
[5]

(b) Use your graph to solve the equation $\dfrac{x}{2} - \dfrac{2}{x} = 1$.

Answer(b) x = ... or *x =* .... [2]

(c) **(i)** By drawing a tangent, work out the gradient of the graph where $x = 2$.

Answer(c)(i) .... [3]

(ii) Write down the gradient of the graph where $x = -2$.

Answer(c)(ii) .... [1]

(d) **(i)** On the grid, draw the line $y = -x$ for $-4 \leqslant x \leqslant 4$. [1]

(ii) Use your graphs to solve the equation $\dfrac{x}{2} - \dfrac{2}{x} = -x$.

Answer(d)(ii) *x =* .... or *x =* .... [2]

(e) Write down the equation of a straight line which passes through the origin and does **not** intersect the graph of $y = \dfrac{x}{2} - \dfrac{2}{x}$.

Answer(e) .... [2]

6 (a)

NOT TO
SCALE

B

$(x + 1)$ cm

A $(x + 6)$ cm   D $(x + 2)$ cm   C
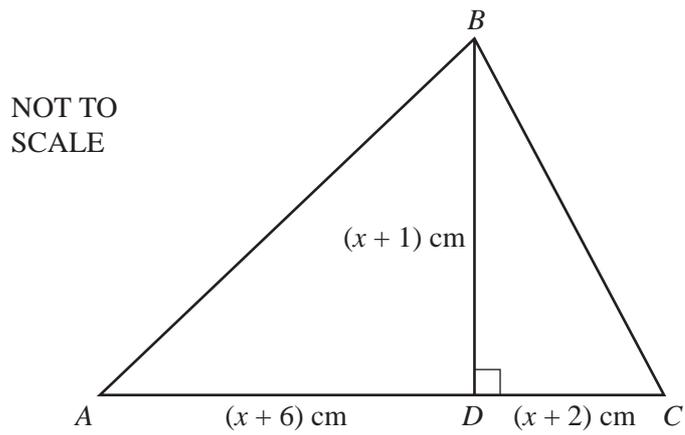
In triangle *ABC*, the line *BD* is perpendicular to *AC*.

$AD = (x + 6)$ cm, $DC = (x + 2)$ cm and the height $BD = (x + 1)$ cm.

The area of triangle *ABC* is 40 cm^2.

(i) Show that $x^2 + 5x - 36 = 0$.

Answer (a)(i)

[3]

(ii) Solve the equation $x^2 + 5x - 36 = 0$.

Answer(a)(ii) $x =$ .... or $x =$ .... [2]

(iii) Calculate the length of *BC*.

Answer(a)(iii) $BC =$ .... cm [2]

(b) Amira takes 9 hours 25 minutes to complete a long walk.

(i) Show that the time of 9 hours 25 minutes can be written as $\dfrac{113}{12}$ hours.

Answer (b)(i)

[1]

(ii) She walks $(3y + 2)$ kilometres at $3\,$km/h and then a further $(y + 4)$ kilometres at $2\,$km/h.

Show that the total time taken is $\dfrac{9y + 16}{6}$ hours.

Answer(b)(ii)

[2]

(iii) Solve the equation $\dfrac{9y + 16}{6} = \dfrac{113}{12}$.

Answer(b)(iii) $y =$ .... [2]

(iv) Calculate Amira's average speed, in kilometres per hour, for the whole walk.

Answer(b)(iv) .... km/h [3]

7

NOT TO SCALE

250 cm

x cm

x cm
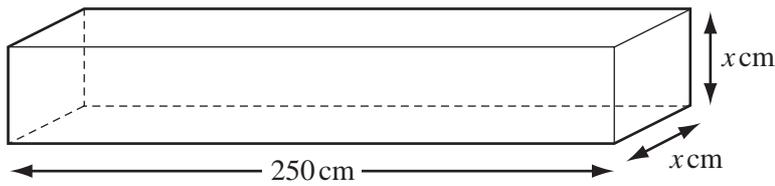
A solid metal bar is in the shape of a cuboid of length of 250 cm.
The cross-section is a square of side x cm.
The volume of the cuboid is 4840 cm^3.

(a) Show that $x = 4.4$.

Answer (a)

[2]

(b) The mass of 1 cm^3 of the metal is 8.8 grams.
Calculate the mass of the whole metal bar in kilograms.

Answer(b) .... kg [2]

(c) A box, in the shape of a cuboid measures 250 cm by 88 cm by h cm.
120 of the metal bars fit exactly in the box.
Calculate the value of h.

Answer(c) h = ... [2]

(d) One metal bar, of volume $4840 \, \text{cm}^3$, is melted down to make 4200 identical small spheres.

All the metal is used.

(i) Calculate the radius of each sphere. Show that your answer rounds to 0.65 cm, correct to 2 decimal places.

[The volume, V, of a sphere, radius r, is given by $V = \frac{4}{3}\pi r^3$.]

Answer(d)(i)

[4]

(ii) Calculate the surface area of each sphere, using 0.65 cm for the radius.

[The surface area, A, of a sphere, radius r, is given by $A = 4\pi r^2$.]

Answer(d)(ii) .... cm^2 [1]

(iii) Calculate the total surface area of all 4200 spheres as a percentage of the surface area of the metal bar.

Answer(d)(iii) .... % [4]

8

First   Second   Third
Calculator   Calculator   Calculator

F = faulty
NF = not faulty
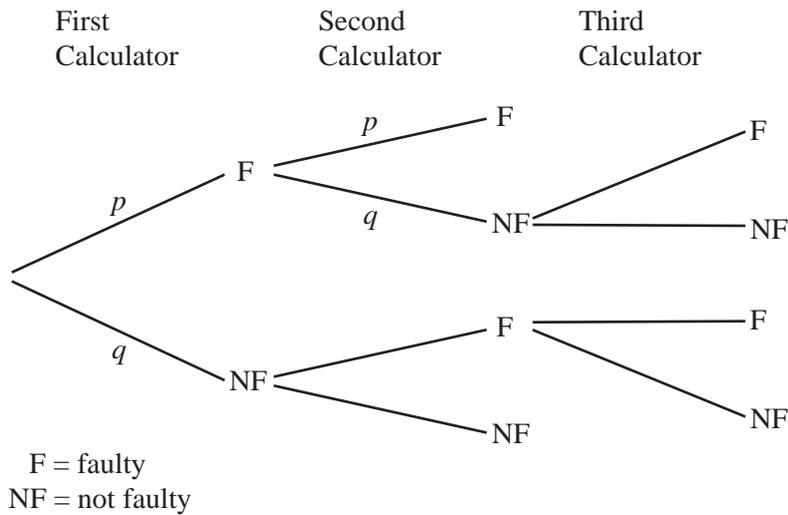
The tree diagram shows a testing procedure on calculators, taken from a large batch.

Each time a calculator is chosen at random, the probability that it is faulty (F) is $\dfrac{1}{20}$.

(a) Write down the values of p and q.

Answer(a) p = ... and *q =* .... [1]

(b) Two calculators are chosen at random.

Calculate the probability that

(i) both are faulty,

Answer(b)(i) .... [2]

(ii) exactly one is faulty.

Answer(b)(ii) .... [2]

(c) If **exactly one** out of two calculators tested is faulty, then a third calculator is chosen at random.

Calculate the probability that exactly one of the first two calculators is faulty **and** the third one is faulty.

Answer(c) .... [2]

(d) The whole batch of calculators is rejected
      either   if the first two chosen are both faulty
      or   if a third one needs to be chosen and it is faulty.

Calculate the probability that the whole batch is rejected.

Answer(d) .... [2]

(e) In one month, 1000 batches of calculators are tested in this way.

How many batches are expected to be rejected?

Answer(e) .... [1]

9 The heights of 100 students are measured.

The results have been used to draw this cumulative frequency diagram.

Cumulative frequency
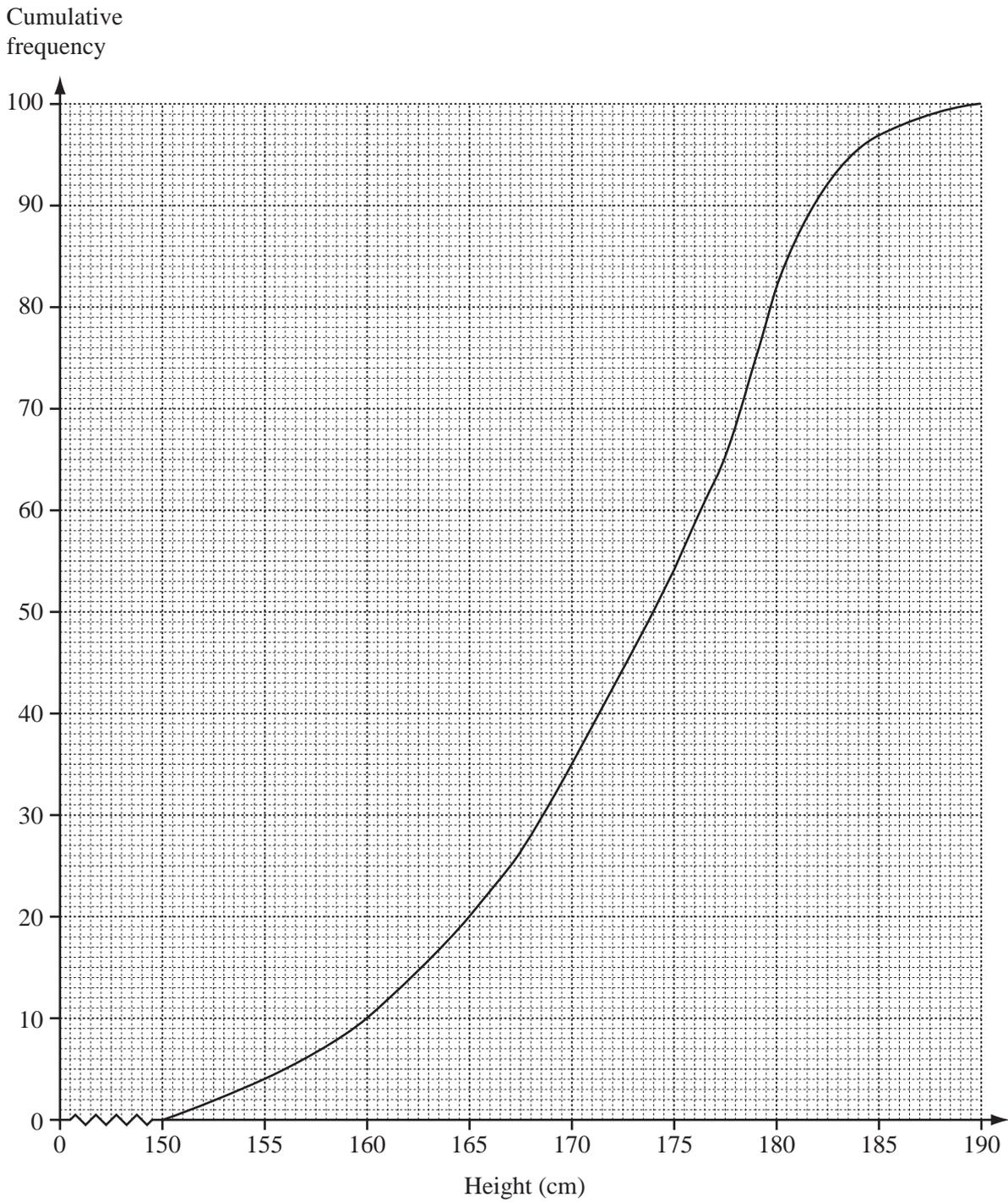
Height (cm)

(a) Find

  (i) the median height,

Answer(a)(i) .... cm [1]

  (ii) the lower quartile,

Answer(a)(ii) .... cm [1]

  (iii) the inter-quartile range,

Answer(a)(iii) .... cm [1]

  (iv) the number of students with a height greater than 177 cm.

Answer(a)(iv) .... [2]

(b) The frequency table shows the information about the 100 students who were measured.

Height (*h* cm)	$150 < h \leqslant 160$	$160 < h \leqslant 170$	$170 < h \leqslant 180$	$180 < h \leqslant 190$
Frequency			47	18

  (i) Use the cumulative frequency diagram to complete the table above. [1]

  (ii) Calculate an estimate of the mean height of the 100 students.

Answer(b)(ii) .... cm [4]

10 $f(x) = 2x - 1$ $g(x) = x^2 + 1$ $h(x) = 2^x$

(a) Find the value of

(i) $f\left(-\frac{1}{2}\right)$,

Answer(a)(i) .... [1]

(ii) $g(-5)$,

Answer(a)(ii) .... [1]

(iii) $h(-3)$.

Answer(a)(iii) .... [1]

(b) Find the inverse function $f^{-1}(x)$.

Answer(b) $f^{-1}(x) =$ .... [2]

(c) $g(x) = z$.
Find x in terms of z.

Answer(c) $x =$ .... [2]

(d) Find $gf(x)$, in its simplest form.

Answer(d) $gf(x) =$ .... [2]

(e) h(x) = 512.

Find the value of x.

Answer(e) x = ... [1]

(f) Solve the equation   2f(x) + g(x) = 0, giving your answers correct to 2 decimal places.

Answer(f) x = ... or *x =* .... [5]

(g) Sketch the graph of

(i) y = f(x),

(ii) y = g(x).

(i) y = f(x)

(ii) y = g(x)

[3]

11

Diagram 1   Diagram 2   Diagram 3   Diagram 4
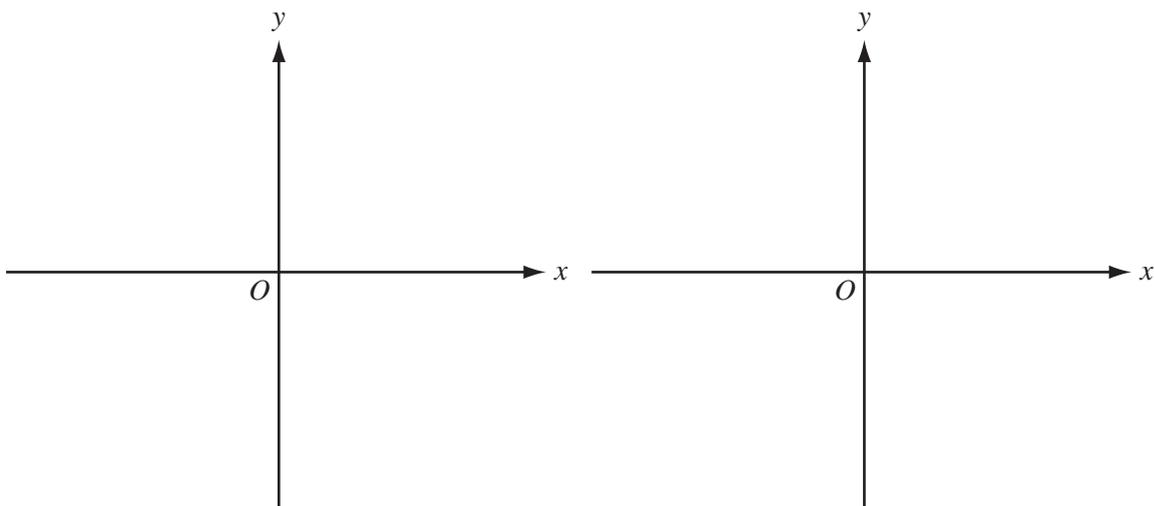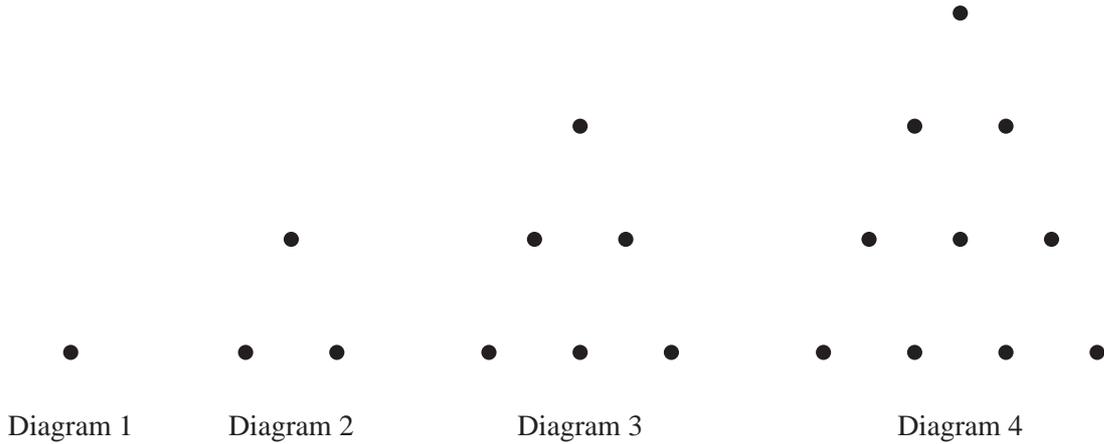
The first four terms in a sequence are 1, 3, 6 and 10.
They are shown by the number of dots in the four diagrams above.

(a) Write down the next four terms in the sequence.

Answer(a) .... , ... , ... , ... [2]

(b) (i) The sum of the two consecutive terms 3 and 6 is 9.
The sum of the two consecutive terms 6 and 10 is 16.

Complete the following statements using different pairs of terms.

The sum of the two consecutive terms .... and .... is ....

The sum of the two consecutive terms .... and .... is .... [1]

(ii) What special name is given to these sums?

Answer(b)(ii) .... [1]

(c) (i) The formula for the *n*th term in the sequence 1, 3, 6, 10… is $\dfrac{n(n+1)}{k}$,

where *k* is an integer.

Find the value of *k*.

Answer(c)(i) *k* = ... [1]

(ii) Test your formula when *n* = 4, **showing your working**.

Answer (c)(ii)

[1]

(iii) Find the value of the 180th term in the sequence.

Answer(c)(iii) .... [1]

(d) (i) Show clearly that the sum of the *n*th and the (*n* + 1)th terms is $(n + 1)^2$.

Answer (d)(i)

[3]

(ii) Find the values of the two consecutive terms which have a sum of 3481.

Answer(d)(ii) .... and .... [2]

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