THINKING SKILLS   **9694/03**

Paper 3 Problem Analysis and Solution   **For Examination from 2011**

SPECIMEN MARK SCHEME

1 hour 30 minutes

MAXIMUM MARK: 50

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International Examinations

[Turn over

1 (a) In the example above, what is the word given by 61 03 06 33 77 88 45? **[1]**

Looking up each pair of numbers gives the message **ALLEGRO**.

(b) Using the example above, give two different ways to encode UNO. **[2]**

Any two of the 64 possible ways to encode UNO are acceptable.

	U	**N**	**O**
1st digit	2 or 8	9 or 4	9 or 4
2nd digit	2 or 9	7 or 4	5 or 0

For example, 22 44 54 and 89 47 90   [1 mark for each correct encoding]

(c) Reconstruct all the correct pairs of numbers. Your answer should include the table below, completed fully. You do not need to worry about the order of the columns. **[4]**

Looking at the first digits of the pairs of digits in the correct and incorrect versions:

Correct	4	6	1	4	4	2	8	0	0	4	4	4	4
Incorrect	8	3	6	8	7	8	6	1	1	8	8	7	8

From which we can obtain:

Correct 1st digit	**4 or 2**	**6**	**1 or 8**	**0**	
Incorrect 1st digit	**8 or 7**	**3**	**6**	**1**	

[1 mark for this, columns in any order]

Using the second digits in the correct and incorrect versions, we have:

Correct	0	0	2	5	7	5	6	7	0	8	1	4	8
Incorrect	7	6	8	9	2	3	8	2	7	2	0	0	4

This gives:

Correct 2nd digit	*0*	*2 or 6*	*5*	*7 or 8*	*1 or 4*
Incorrect 2nd digit	*7 or 6*	*8*	*9 or 3*	*2 or 4*	*0*

[1 mark for this, columns in any order]

Remembering that an incorrect first digit is equal to a correct second digit, and vice-versa, we can combine these results and obtain:

Correct 1st digit	**4 or 2**	**6** *or 7*	**1 or 8**	**0**	*9 or 3*
Correct 2nd digit	**8 or 7**	**3** *or 0*	**6** *or 2*	**1** *or 4*	**5**

[3 marks for obtaining this]

The missing 9 and 5 can now be inserted:

Correct 1st digit	4 or 2	6 or 7	1 or 8	0 or 5	9 or 3
Correct 2nd digit	8 or 7	3 or 0	6 or 2	1 or 4	5 or 9

[**Max 4** marks for the complete solution]

(d) For four of the other characters in the message, we can be sure that each of them is either an A, a G, an O or a U. Which four positions are they in? [1]

These four letters (as well as the space) are on the diagonal of the table. Thus, in correcting the mistake, the numbers for their first and second digits will have simply swapped over.
There are four characters in the message where either their first digit has become their second, or vice-versa, indicating that they are definitely on the diagonal.

The four positions are the 2nd, 5th, 10th and 13th.

(e) Given that the first letters of the three words in the message are M, N and T respectively, write out the decoded message. (You can use the table below to help if you wish, but be sure to include your working with your answer.) [2]

Starting with the space, we can put [1,8] in the fifth row, and [6,2] in the fifth column. The M gives us [4,2] in the third row and [3,0] in the first column; however, since we have been told that the erroneous message used the number pairs in the right order, we can also now get the first row as [6,7] (which is the 'incorrect' version of [3,0]) and the third column as [8,7]. The T fixes the fourth row as [0,5], and consequently the fourth column as [1,4], leaving the second row as [9,3] and the second column as [5,9] by elimination.

MA NON TROPPO [2 marks]

[1 mark for a correct A, O, R or P in the correct place]

The table is given below for reference:

		0	5	8	1	2
		3	9	7	4	6
7	6	**A**	**B**	**C**	**D**	**E**
9	3	**F**	**G**	**H**	**I**	**L**
4	2	**M**	**N**	**O**	**P**	**Q**
0	5	**R**	**S**	**T**	**U**	**V**
1	8	**Z**	**.**	**,**	**-**	**(space)**

87	36	68	89	72	83	68	12	17	82	80	70	84
M	**A**		**N**	**O**	**N**		**T**	**R**	**O**	**P**	**P**	**O**
40	60	12	45	47	25	86	07	00	48	41	44	48

2 **(a)** **The following journeys all have a shortest path of 5. What is the total number of paths of length 5 in each case?**

    (i) **From (3,4) to (8,4)** **[1]**

        There is only <u>one path</u>: (4,4)(5,4)(6,5)(7,4).

    (ii) **From (4,5) to (8,4)** **[1]**

        The jump from (*,5) to (*,4) can occur in 5 places, so <u>five paths</u>.

    (iii) **From (6,7) to (8,4)** **[1]**

        This is the same as from (0,0) to (2,3), and we can construct a table of the number of paths to each point along the way:

        1 3 6 10   Alternatively:
        1 2 3 4 $^5C_2$ or $^5C_3$ = 10
        0 1 1 1   Either way, there are <u>ten paths</u>.

(b) **List all the points on the sphere of distance 1 from (3,4,5).** **[2]**

    By adding or subtracting one from each coordinate in turn:

    <u>(2,4,5) (4,4,5) (3,3,5) (3,5,5) (3,4,4) (3,4,6)</u> [1 mark for any three correct]

(c) **How many shortest paths are there from (4,3,3) to (3,4,5)?** **[3]**

    This is the same as from (0,0,0) to (1,1,2). By extension of the first approach in **(a) (iii)**, with each 'square' representing number of paths that go to each of the four (x, y) points which are at a given z-coordinate:

    z=0 z=1 z=2   The "12" is comprised of the 3 paths which go
    1 2 2 6 3 12   through (0,1,2), the 3 paths through (1,0,2), and
    0 1 1 2 1 3   the 6 paths through (1,1,1).

    Alternatively, the permutations of (a move of 1 unit in) x, y, z and z can be listed:

    xyzz *yxzz* *zxyz* *zyzx*
    xzyz *yzxz* *zyxz* *zzxy*
    xzzy *yzzx* *zxzy* *zzyx*

    1 mark for evidence of a systematic approach.
    or 2 marks for obtaining 6 paths to (3,4,4), or listing 6 correct permutations.
    Max 3 marks for an answer of <u>twelve paths</u>.

(d) A four-dimensional hyper-taxi-driver needs to get from (3,1,3,1) to (2,2,2,2). How many shortest paths in hyperspace are there? **[2]**

Across one hyper-cube, this is the same as (0,0,0,0) to (1,1,1,1).

Since there is a move of just one unit in each dimension, the simplest solution is to find the number of permutations of 4 objects:

i.e. 4! = 4 × 3 × 2 × 1 = <u>24</u>.

Elaborating the reasoning behind this:

In 1 dimension, there is only 1 way to make a move of 1: (x).
In 2 dimensions, there are 2 ways: (xy), (yx).
In 3 dimensions, there are 3 possible positions for the move in z, for each of the 2 variations for 2 dimensions: 3 × 2 = 6.
In 4 dimensions, there are 4 possible positions for the move in w, for each of the 6 variations for 3 dimensions: 4 × 6 = 24.

Alternatively, the permutations of (a move of 1 unit in) w, x, y and z can be listed:

$wxyz$ $xwyz$ $ywxz$ $zwxy$
$wxzy$ $xwzy$ $ywzx$ $zwyx$
$wyxz$ $xywz$ $yxwz$ $zxwy$
$wyzx$ $xyzw$ $yxzw$ $zxyw$
$wzxy$ $xzwy$ $yzwx$ $zywx$
$wzyx$ $xzyw$ $yzxw$ $zyxw$

1 mark for 4 × 3 × 2.
or 1 mark for listing 12 of the permutations correctly.
Max 2 marks for an answer of <u>twenty-four paths</u>.

3 (a) If a cage with dimensions 10 m, 11 m and 15 m was to be constructed, what is the smallest total area of netting that would be needed? **[2]**

Answer: 535 metres^2 needed (if L = 15)
if length is 10 metres, 590 metres of netting is needed
if length is 11 metres, 575 metres of netting is needed
if length is 15 metres, 535 metres of netting is needed
One mark for a correct calculation of surface area that is not optimal [590 or 575]

Also one mark available for candidates who state that the sides must be as small as possible (or that the width and height must be as small as possible), if they have not managed to get any of the actual areas correct.

(b) Consider possible cages which are of the same height as the wall. What is the maximum length that the cage can be? Justify your answer **[3]**

Answer: 13 metres   award 1 mark
Justification: 13 metres gives 582 m^2 award 1 mark
                     14 metres gives 606 m^2 award 1 mark

If none of the above information is given, award one mark for considering any case involving a height of 15 m, a width of 9 metres and any length: any attempt to calculate the area of netting.

Full marks for an algebraic justification: equation + length = 13.75 m seen

(c) What is the largest volume cage that can be built, with a height of 14 metres? Justify your answer. **[3]**

Answer: 1890 metres^3
Three marks if EITHER the candidates gives this answer and consider the next solution up [14 × 16 × 9]
OR
Algebraic jusitifcation is given: 252 + 23x < 600   [one mark if this is sighted (regardless of a correct answer or not)]
23x < 348
X = 15.31…
If any of these lines of an algebraic solution are sighted, give a mark for justification.

Two marks if the candidate offers 1890 metres^3 as a solution without support.
One mark if the candidate considers frameworks with dimensions 14 × 9 × ?

(d) Show that it is possible to fulfil the desired animal welfare outcome of a minimum volume of 1900 m^3. Give the length, width and height of the cage which does this. **[3]**

Answer: any of the shaded and emboldened solutions offered in the table below

Two marks for a correct solution, in which either the dimensions, the area of netting or the volume is omitted.
OR
Two marks for one of the suboptimal solutions given in the table below, as long as it does not have height or width 14 m [since these solutions are prompted by the question in part **(c)**].

One mark for a suboptimal solution, in which either the dimensions, the area of netting or the volume is omitted.

height	length	width	volume	netting
9	14	14	1764	574
9	15	14	1890	597
9	16	13	1872	586
9	17	12	1836	573
9	**18**	**12**	**1944**	**594**
9	19	11	1881	578
9	**20**	**11**	**1980**	**598**
10	14	13	1820	582
10	15	12	1800	570
10	**16**	**12**	**1920**	**592**
10	17	11	1870	577
10	**18**	**11**	**1980**	**598**
10	**19**	**10**	**1900**	**580**
10	20	9	1800	560
11	13	13	1859	598
11	14	12	1848	586
11	15	11	1815	572
11	**16**	**11**	**1936**	**594**
12	13	12	1872	600

(e) It is decided that the wall is bad for the birds' welfare, and is to be demolished. Plans are drawn up for the new cage [which must have 5 netted sides, now the wall has gone].

Four 10 metre struts are found, and it is decided that these should be used for the cage [i.e. at least one of the dimensions should be 10 metres]. The other struts are subject to the same restrictions as before.

What is the minimum extra amount of netting needed if the cage is still to fulfill the 1900 m^3 requirement? **[4]**

ANSWER: 152 m^2 of netting
Four marks for this answer

If candidate does not reach this answer award as following:
one mark for a solution in which the height measurement is the smallest of the three.
AND
One mark for subtracting 600 m^2 from the amount of netting they find, in order to calculate the extra netting needed.
AND
One mark for calculating any of the suboptimal solutions given below (not the shaded and emboldened, optimal solution).

length	width	height	volume	netting
22	10	9	1980	796
19	10	10	1900	770
18	11	10	1980	778
16	**12**	**10**	**1920**	**752**
15	13	10	1950	755
14	14	10	1960	756

4 (a) State the arrival times at the Castle bus stop of all the buses that will continue from Linkham to Ferrin on 24 July. (The bus makes the journey whether or not the Castle is open). **[3]**

Answer: 09:40, 10:45 and 18:45 – award 3 marks
Deduct 1 mark for each extra time (minimum 0 marks).
If a candidate has selected the three correct buses, but consistently given the times for one of the other bus stops (e.g. 09:05, 10:10 and 18:10) award 1 mark.

Possible one mark combinations [any row of this table]

First	Second	Last
09.05	10.10	18.10
09.10	10.15	18.15
09.24	10.29	18.15
09.26	10.31	18.31
09.40	10.45	18.45
10.05	11.05	18.45
10.19	11.19	18.59
10.21	11.21	19.01
10.35	11.35	19.15
10.40	11.40	19.30

(b) At what time will the Castle open, and at what time will it close:

(i) On 17 July? **[2]**

Answer: Castle will open at 09:55 – award 1 mark
Castle will close at 16:55 – award 1 mark

(ii) On 20 July? **[2]**

Answer: Castle will open at 12:10 – award 1 mark
Castle will close at 18:00 – award 1 mark

Skill: Extract relevant data.

(c) **The first date in July when the Castle will be closed all day is 6 July.**
On what date will the Castle next open? Show clearly how you reach your conclusion.
[2]

Answer: 9 July – award 2 marks

If a candidate does not state the correct answer, but shows evidence of having calculated how long the castle is open on 7, 8, 9 or 10 July, then award 1 mark:

Working:

Date	Time which the castle could open (given causeway openings)
7	2 hours
8	2 hours 40 minutes
9	3 hours 15 minutes
10	3 hours 45 minutes

(d) **Barry and Clare will be staying in Barwell from 2 July to 12 July. During their holiday they would like to take the bus to Ferrin. However, they will only do so if they will be able to spend at least five hours exploring the Island, including a visit to the Castle.**

On which date(s) would this be possible?

Show all your working, and for each date that is possible state the earliest arrival time at, and the latest departure time from, the Castle bus stop. **[6]**

Answer: 4 July – award 1 mark
Earliest arrival 12:45 – award 1 mark
Latest departure 18:45 – award 1 mark
9 July – award 1 mark
Earliest arrival 09:40 – award 1 mark
Latest departure 18:45 – award 1 mark

Deduct 1 mark for each extra date given in addition to 4 July and 9 July (minimum 0 marks).
If only one correct date is given do not deduct mark unless two or more extra dates are given.
Each of the marks for bus times can only be awarded if linked to a correct date.
If a candidate gives the answer 7 July or 8 July to question **(c)**, then do not deduct marks for these dates appearing in question **(d)**. Just ignore them

Skill: Analyse complex data and draw conclusions.

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