
THINKING SKILLS

9694/31

Paper 3 Problem Analysis and Solution

October/November 2019

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).


GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

| Question | Answer | Marks | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-----------|--|----------|--------|-------|------|-------|---|---|----|----|----|---|-----|----|----|----|---|-----|----|----|----|---|-----|----|----|----|---|-----|----|----|----|---|-----|----|----|----|---|-----|----|----|----|---|-----|----|----|----|--|--|---------|--|--|--|---|---|---|---|-------|---|---|--|--|--|---|---|---|--|--|---|---|---|---|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|----------|
| 1(a) | $(20 + 12 + 10 + 12) \times 2 + 16 = 124$ cm AG | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1(b)(i) | $10 + 10 + 10 + 10 + 4 = \underline{44}$ cm | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1(b)(ii) | $2 + 10 + 10 + 10 + 10 = \underline{42}$ cm | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1(b)(iii) | <p><u>40</u> cm [1 mark]</p> <table border="1"> <thead> <tr> <th>Box</th> <th>starts</th> <th>Red</th> <th>Blue</th> <th>White</th> </tr> </thead> <tbody> <tr><td>1</td><td>0</td><td>44</td><td>40</td><td>40</td></tr> <tr><td>2</td><td>124</td><td>44</td><td>40</td><td>40</td></tr> <tr><td>3</td><td>248</td><td>42</td><td>42</td><td>40</td></tr> <tr><td>4</td><td>372</td><td>40</td><td>44</td><td>40</td></tr> <tr><td>5</td><td>496</td><td>40</td><td>44</td><td>40</td></tr> <tr><td>6</td><td>620</td><td>40</td><td>40</td><td>44</td></tr> <tr><td>7</td><td>744</td><td>40</td><td>40</td><td>44</td></tr> <tr><td>8</td><td>868</td><td>42</td><td>40</td><td>42</td></tr> </tbody> </table> <p><u>Boxes 4, 5, 6, 7</u> [3 marks]</p> <p><i>Award of marks for incorrect final answers :</i></p> <table border="1"> <thead> <tr> <th colspan="2" rowspan="2"></th> <th colspan="4">Correct</th> </tr> <tr> <th>1</th> <th>2</th> <th>3</th> <th>4</th> </tr> </thead> <tbody> <tr> <th rowspan="6">Given</th> <th>1</th> <td>0</td> <td></td> <td></td> <td></td> </tr> <tr> <th>2</th> <td>0</td> <td>1</td> <td></td> <td></td> </tr> <tr> <th>3</th> <td>0</td> <td>1</td> <td>2</td> <td></td> </tr> <tr> <th>4</th> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <th>5</th> <td>0</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <th>6</th> <td>0</td> <td>0</td> <td>0</td> <td>1</td> </tr> </tbody> </table> <p>OR</p> <p><i>For the 'box selection' marks, award 1 mark each for the following points seen in working (max 2), if the correct four boxes have not been given:</i></p> <ul style="list-style-type: none"> • the correct amount of red for at least two boxes (4 onwards) • box ribbon starting lengths for at least three ribbons beyond 124 (either relative to 0 or to the red sections) | Box | starts | Red | Blue | White | 1 | 0 | 44 | 40 | 40 | 2 | 124 | 44 | 40 | 40 | 3 | 248 | 42 | 42 | 40 | 4 | 372 | 40 | 44 | 40 | 5 | 496 | 40 | 44 | 40 | 6 | 620 | 40 | 40 | 44 | 7 | 744 | 40 | 40 | 44 | 8 | 868 | 42 | 40 | 42 | | | Correct | | | | 1 | 2 | 3 | 4 | Given | 1 | 0 | | | | 2 | 0 | 1 | | | 3 | 0 | 1 | 2 | | 4 | 0 | 1 | 2 | 3 | 5 | 0 | 0 | 1 | 2 | 6 | 0 | 0 | 0 | 1 | 4 |
| Box | starts | Red | Blue | White | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 0 | 44 | 40 | 40 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 124 | 44 | 40 | 40 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 248 | 42 | 42 | 40 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 4 | 372 | 40 | 44 | 40 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | 496 | 40 | 44 | 40 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 620 | 40 | 40 | 44 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 7 | 744 | 40 | 40 | 44 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 8 | 868 | 42 | 40 | 42 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | Correct | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | 1 | 2 | 3 | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Given | 1 | 0 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 2 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 3 | 0 | 1 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 4 | 0 | 1 | 2 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 5 | 0 | 0 | 1 | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 6 | 0 | 0 | 0 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1(c)(i) | <p>For one large box, height $3 \times 12 = 36$ cm, string used = $(30 + 2 \times 36) \times 2 + 16 = 220$ cm Difference = $(3 \times 124) - 220 = \underline{152}$ cm</p> <p><i>1 mark for a solution which omits the 16 cm for the bow, but is otherwise correct, giving 168 cm</i></p> <p>OR</p> <p>Saving on string will always be 2 bows + the length used on the faces that are together = $2 \times 16 + 4 \times (20 + 10) = 32 + 120 = \underline{152}$ cm</p> <p><i>1 mark for evidence of recognising that $2 \times (20 + 10)$ is saved between two boxes.</i></p> | 2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

| Question | Answer | Marks |
|----------|---|----------|
| 1(c)(ii) | The saving on string will always be 2 bows + the length used on the faces that are together. These faces do not include any lengths of string equivalent to the height of the box (nor do the bows). <i>Clear explanation required for the mark.</i> | 1 |

| Question | Answer | Marks |
|----------|---|----------|
| 2(a)(i) | There must be <u>4</u> . | 1 |
| 2(a)(ii) | All <u>12</u> distinct ordered pairs. | 1 |
| 2(b) | Both C and E need to trust others, and there are solutions with just 2: EA and CA (or CB or CD or CE), or CA and EC (or EB or ED) <i>2 marks for any complete and correct diagram (condone omission of given 5 arrows)</i> <i>1 mark for any diagram in which all trust relationships present, but also superfluous ones but with a maximum of 4 extra connections.</i> <i>SC: 1 mark for links identified but not shown in diagram.</i> | 2 |
| 2(c)(i) | One country has to trust all others, and all others have to trust it: a six-pointed star. $2 \times 6 = 12$ policies are required. <i>1 mark for any directed graph with desired property but which is not minimal</i> <i>SC: 1 mark for correct structure with 6 nodes instead of 7.</i> | 2 |
| 2(c)(ii) | The departure of the central country would <u>remove all trusted links for everyone</u> . | 1 |
| 2(d) | Rotational symmetry means that we only have to look at one case. Longest chain was 3, is now 5, so difference of 2 <i>Must be supported</i> | 1 |

| Question | Answer | Marks |
|----------|---|-------|
| 2(e) |  <p>All countries are the same (by symmetry). The minimum steps others are: Skip 1: 1, 1, 2, 2, 3, 3; Skip 2: 1, 2, 1, 2, 3, 2 So Skip 2 is better by 1 step in 6 passport checks.</p> <p><i>1 mark for Skip 2 with supporting evidence of decision.</i> <i>1 mark for satisfactory quantification of the difference: allow any clear, precise explanation, e.g. average number of checks, total checks for all chains for 1 particular country, etc.</i></p> | 2 |

| Question | Answer | Marks |
|----------|---|-------|
| 3(a) | <p>Peregrine has been walking 07:00 until 11:00 = 4 hours = 12 km other walker could have been walking from 06:00 until 11:00: 5 hours = 15 km <i>1 mark for either of 12 km or 15 km seen</i> $9 \times 3 = \underline{27}$ km</p> | 2 |
| 3(b) | <p>Effectively 8 am start for other walker : 9 km / 3 hour walk So Peregrine will reach his destination at 11am + 3 hours = <u>14:00</u></p> | 1 |
| 3(c) | <p>If he does not meet someone by 12:00 [1] he should turn back. The person he meets may have been walking since 06:00 (18 km travelled) and he may therefore have that far to go [1].</p> | 2 |
| 3(d) | <p>Furthest out and back would be 06:00 + 18 km = 12:00. If P left at 9 am he will have walked 9 km by then They walk towards each other and meet at <u>13:30</u> [2]</p> <p><i>Award 1 mark for considering the distance or time walked by someone starting at any time between 06:00 and 09:00</i></p> | 2 |
| 3(e) | <p>At 10:20 the first walkers will have walked for 4 hours 20 minutes = 13 km [1] He has walked 1 hour = 3 km. So 16 km [1] between hostels, so halfway point is 8 km in. This will take Peregrine 2 hours 40 minutes to accomplish: 12:00 [1] Picnickers leave at 10:20 in order to reach halfway point at 13:00 They will meet at <u>12:30</u></p> | 4 |
| 3(f)(i) | <p>The family must be at the hostel by nightfall: 14:00 to 18:00 = 4 hours = 8 km = <u>2 hours 40 minutes</u> for Peregrine</p> | 1 |

| Question | Answer | Marks |
|----------|---|-------|
| 3(f)(ii) | If the family sets off at 6 am, and walks without stopping, they walk '2t' km in the hours after 6 am and he walks 3(t – 3) km. These are equal when $t = 9$: at <u>15:00</u> | 1 |
| 3(g) | Maximum distance to next hostel = 7 hours = 21 km For the runner this is 21/4 [1] hours at 4 km/h. He must have set off no later than <u>12:45</u> | 2 |

| Question | Answer | Marks |
|----------|--|-------|
| 4(a) | Via Detlie, earliest arrival time is 09:41 + 31 mins = 10:12. Via Toth, earliest is 09:43 + 21 mins = <u>10:04</u> , so earliest is 10:04. <i>1 mark for either time correct AND an appropriate conclusion from their times if two are given.</i> <i>SC: 1 mark for 10:18 or 10:10 (+6 minutes, uses departure times)</i> <i>SC: 1 mark for 10:06 or 09:58 (–6 minutes, uses sum of travel times from Munro)</i> <i>SC :1 mark for 10:25 or 10:30 (using the departure times from Carleton)</i> | 2 |
| 4(b) | Earliest arrival times are 16:30 + 21 mins = 16:51 aboard <i>Frances</i> and 16:01 (from Ockelman Island) + 12 mins = 16:13 aboard <i>Marie</i> , so <u>38 mins</u> . <i>1 mark for either time correct OR 44 minutes (6 minutes on Ockelman Island omitted)</i> | 2 |
| 4(c) | By 13:00, <i>Marie</i> had stopped five times at the island. According to the record, 23 + 28 + 37 + 45 + 34 = 167 visitors had arrived and (0 +) 7 + 11 + 18 + 26 = 62 had departed, so there were <u>105</u> visitors on the island at that time. <i>1 mark for recognition that the first five rows of figures (and only the first five) are involved in the calculation (indicated by '167 arrived' or '62 departed'), or for calculating the number on the island after the 3rd (70), or 4th (97) or 6th (109) ferries have left.</i> | 2 |
| 4(d) | Cost of <i>Day Roamer</i> tickets = $(2 \times \$15) + (2 \times \$9) = \$48$ Extraction of ticket prices \$5.10 and \$3.00 (Munro – Toth) and \$6.30 and \$3.80 (Ockelman Is.) $2 \times (\$5.10 + \$3.00 + \$6.30 + \$3.80) = \$36.40$ Difference <u>\$11.60</u> <i>1 mark each for any of the following (max 2):</i> <i>\$48 [whole family day roamer]</i> <i>\$18.20/\$36.20 [half family/whole family ticket by ticket]</i> <i>\$3.60/\$7.20 [adult(s) difference]</i> <i>\$2.20/\$4.40[child(ren) difference]</i> <i>SC: 2 marks for final answer of \$5.80</i> <i>(uses one-way tickets OR only buys tickets for 1 × adult + child)</i> | 3 |

| Question | Answer | Marks |
|----------|--|----------|
| 4(e) | $52 \times \$15 + 31 \times \$9 = \$1059$ Number of adult tickets = <u>52</u> <i>Search method:</i> <i>The criteria for the search are tickets = 83 and income = \$1059</i> <i>1 mark for an initial search that meets either one of the criteria</i> <i>AND for an adjustment that gets closer to the solution</i> <i>Algebraic method (1 mark for parsing algebraically)</i> $a + c = 83$ AND $15a + 9c = 1059$ OR $15x + 9(83 - x) = 1059$ | 2 |
| 4(f) | <p>Detlie first [CDMTC] Depart C 10:25, depart D 13:27, arrive M 13:53 Depart M 17:20 3 hours 27 minutes / 207 minutes at Munro</p> <p>Toth first [CTMDC] Depart C 10:30, depart T 12:57, arrive M 13:14 Depart M 16:24 3 hours 10 minutes / 190 minutes at Munro</p> <p>She should therefore visit the towns in the order Detlie, Munro, Toth.</p> <p><i>4 marks for 3 hours 27 minutes / 207 minutes and 3 hours 10 minutes / 190 minutes AND statement of the order Detlie, Munro, Toth.</i></p> <p><i>3 marks for 3 hours 27 minutes / 207 minutes OR 3 hours 10 minutes / 190 minutes.</i></p> <p><i>2 marks for three or more correct arrival/departure times in Munro.</i> <i>1 mark for two correct arrival/departure times in Munro (allowing the 6 minute discrepancy)</i></p> <p><i>SC: using the departure times from the table rather than the journey times (i.e. 6 minutes later):</i> <i>1 mark for arrival and departure times from Munro [13:59 and 17:20, 13:20 and 16:24] OR the time spent on Munro for either route [3 h 21 m or 3 h 04 m]</i> <i>2 marks for 3 h 21 m AND 3 h 04 m</i></p> | 4 |