THINKING SKILLS **9694/31**

Paper 3 Problem Analysis and Solution

May/June 2017

2 hours

Additional Materials: Electronic Calculator

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.

Show your working. Marks may be awarded for correct steps towards a solution, even if the final answer is not correct. Marks may be lost if working needed to support an answer is not shown.

Calculators should be used where appropriate.

The number of marks is given in brackets [ ] at the end of each question or part question.

[Turn over

1 Alan is organising a business dinner for 46 people at the local hotel. Two kinds of table are available: round tables can seat 6 people and square tables can seat 8 people. He wants to seat all of the people so that there are no empty spaces at any table. He works out that there are two different combinations of round and square tables that will do this.

(a) What are the two possible combinations of round and square tables that will work?   [2]

Dinner is a buffet and is charged at $50 for a round table and $60 for a square table. Alan telephones the hotel to make the arrangements, but they inform him that they have only 4 of each kind of table. Alan realises that he can no longer seat all of the people so that there are no empty spaces at any table.

(b) Which combination of round and square tables will enable him to seat the 46 people for the lowest possible cost?   [2]

Alan considers not inviting two people, so as to bring the total down to 44 people.

(c) How much would Alan have to pay to seat 44 people?   [1]

There are additional charges for two extra items which Alan wants: drinks and table decorations. The costs are shown in the table below. Alan can choose, for each item, to pay a price per person or a price per table or a price for the whole room.

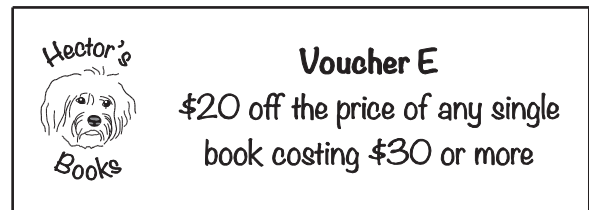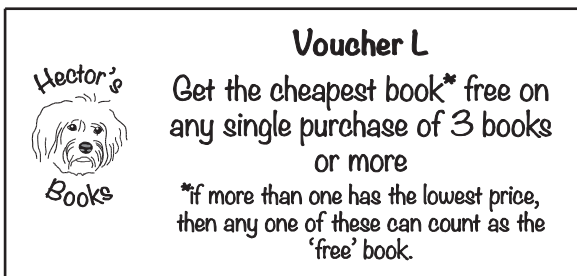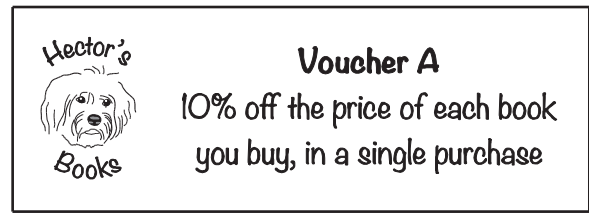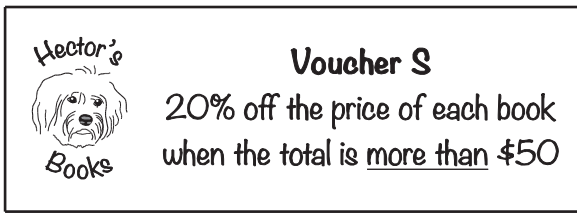|	Per person	Per table	Whole room
Drinks	$4	$20	$130
Table decorations	$1	$7	$50

(d) If Alan chooses the cheapest way to pay for each item, how much money could he save on drinks and table decorations altogether by inviting 44 people rather than 46?   [3]

The hotel now insists that Alan must choose the same payment method (price per person, price per table or price for the whole room) for both items.

(e) If Alan chooses the cheapest way overall, how much could he save on drinks and table decorations altogether by inviting 44 people rather than 46?   [2]

2 Hector owns a bookshop, and decides that he can attract new customers by issuing vouchers which offer a variety of discounts.

The vouchers issued are as follows:

Hector's Books — **Voucher S** — 20% off the price of each book when the total is <u>more than</u> $50	**Hector's Books** — **Voucher A** — 10% off the price of each book you buy, in a single purchase
Hector's Books — **Voucher L** — Get the cheapest book* free on any single purchase of 3 books or more *if more than one has the lowest price, then any one of these can count as the 'free' book.	**Hector's Books** — **Voucher E** — $20 off the price of any single book costing $30 or more

The prices of individual books range from $10 to $60 at the shop.

(a) What is the biggest discount (in $) that can be achieved using only one voucher, if three books are bought from the shop? [1]

Brodie has managed to acquire one of each of the vouchers.
He wants to buy 5 books, whose undiscounted prices are $17, $20, $33, $40 and $50.

(b) What is the biggest discount (in $) that Brodie can achieve if he uses just one of the vouchers, and buys some or all of the books he wants? [2]

Hector realises that he did not state that the vouchers cannot be used together, and so customers will be allowed to use as many of the vouchers as they wish on a single purchase. He decides that, if more than one voucher is used, the customer must give him the vouchers one at a time and the prices of the books will be adjusted by each voucher in the order they are given to him.

Brodie's brother Brock claims that, if he selects two of the vouchers, the order in which he uses them could affect the total discount.

(c) (i) Use some of Brodie's books to give an example which shows that Brock is correct. Give the two different discounts (in $) that can be achieved for your example. [2]

   (ii) Give one example of a pair of vouchers for which the order would never matter. [1]

(d) Show clearly how Brodie should use the four vouchers to buy his five books and pay the lowest price overall. (He may make more than one purchase.) [4]

3 The Cherries sports club are moving to a new stadium. The manager has invited members of the public to design a new flag, which will be given to supporters at their first game. The flag must be in the club colours of blue and white and it must have width 40 cm and length 30 cm.

A local manufacturer produces large rolls of a blue and white striped fabric, with width 40 cm. The stripes on the fabric are alternate white stripes and blue stripes, each of length 20 cm. The manufacturer supports The Cherries and he is willing to sell the fabric to the club at a discounted price. The club therefore decides that all flags must be made from this particular fabric.

Andy designs a flag which has a 20 cm white stripe with a 5 cm blue stripe on either side.
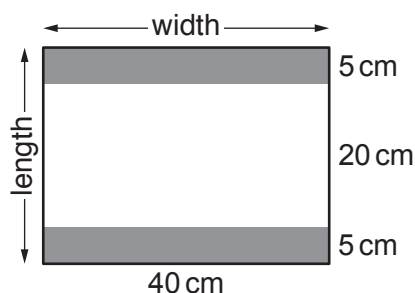
A new roll of fabric always starts with a 20 cm white stripe.

Consider the first 3 m length cut from the roll. You should assume that each flag is made from a continuous piece of fabric.

(a) (i) How many of Andy's flags can be made from this 3 m length? [1]

(ii) What length of blue fabric, in total, is not used? [2]

Now consider a complete roll of 50 m of fabric.

(b) How many of Andy's flags can be made from this complete roll? [2]

To avoid wasting fabric, the manager decides that the fabric can be cut along the lines where the colours change, or lines parallel to these lines, and that flags can have joins, so long as each stripe of colour is free of joins. So, for example, one of Andy's flags can be made from three pieces of fabric: one 20 cm white piece and two 5 cm blue pieces.

(c) With joins allowed, how many of Andy's flags can be made from the first 3 m of fabric in a roll? [1]

Having seen Andy's design, Bill designs a flag with three blue stripes and two white stripes. His design is such that no fabric is wasted when equal numbers of his and Andy's flags are made from the first 2 m length cut from a new roll of fabric. Joins are allowed, as described earlier.

(d) Sketch a design that Bill might use. Indicate the lengths of the stripes. [2]

Fabric is priced at a rate of $10 per metre and any continuous length can be bought. The manager is going to buy a single length of fabric from the beginning of a new roll. The cost of stitching and finishing one flag with no more than one join is $2. The cost of stitching and finishing one flag with more than one join is calculated as $1.25 per join.

(e) Find whether it is cheaper to make four of Andy's flags with no joins or four of Andy's flags in each of which there are exactly two joins. State the difference in total cost. [3]

A supporter of The Cherries donates $500.

(f) Find the greatest number of Andy's flags that can be made using this donation. [4]

4 Square Deal is a game for two players, played over a number of rounds. In each round both players have a 4 × 4 grid onto which numbered tiles are placed.

There are 34 tiles, numbered as follows:

0 0 1 1 1 2 2 2 2 3 3 3 3 3 4 4 4 4 4 4

5 5 5 5 5 6 6 6 6 7 7 7 8 8

At the beginning of a round the tiles are placed in a bag. The players then take turns to withdraw two tiles at a time from the bag, at random. At each turn, one of the two tiles must be placed on the player's own grid and the other one placed on the opponent's grid. Each player attempts to create rows and columns of four numbers that add up to a total that is a square number, and tries to prevent the other player from doing so. The round continues until both grids are full.

A player's score for the round is the sum of the highest value row and the highest value column.

- The value of a row or column that adds up to a total which is a square number is the sum of the squares of the individual numbers.
- A row or column that does not add up to a square number has a value of zero.

For example, in the grid below, two rows add up to totals which are square numbers:
4 + 1 + 7 + 4 = 16 and 2 + 5 + 0 + 2 = 9.
The values of these rows are $4^2 + 1^2 + 7^2 + 4^2 = 82$ points and $2^2 + 5^2 + 0^2 + 2^2 = 33$ points.
There are no columns with totals which are square numbers.
The player's score for this round is 82 (highest value row) + 0 (highest value column) = 82 points.

7	3	3	5
4	1	7	4
8	5	2	4
2	5	0	2

The game is normally won by the first player to reach an overall total of 900 points. However, a player whose grid in any round has all four rows and all four columns adding up to totals which are square numbers is said to have made a Square Deal. No points are scored in this round: instead, the player making the Square Deal wins the game immediately.

Russell and Gordon are playing a game of Square Deal. Russell's grid at the end of the first round was as follows:

5	7	0	8
4	6	3	3
1	7	6	2
4	5	8	4

(a) What was Russell's score in the first round? [3]

In a later round, Gordon had a chance of making a Square Deal on the final turn of the round. He knew that the four tiles still in the bag were 0, 2, 4 and 7, and his grid was as follows:

4	6	3	3
4		5	7
5	1	2	1
3	2	6	5

However, when he took two of the tiles from the bag, the best score that he could make on his own grid was 158 points, made up of 74 points for the highest value row and 84 points for the highest value column.

(b) Which two tiles did Gordon take from the bag on the final turn of this round? Explain your answer. [3]

(c) **(i)** What is the highest possible score that a player could achieve in a single round? [3]

   (ii) Draw a completed grid that would produce this score. [2]

In the round currently in progress the two grids are as follows:

5	0	3	5
8	6	7	
6		3	1
0	2	5	4

Russell's grid

4	4	2	3
2		7	8
7	1	4	3
5	2		6

Gordon's grid

It is Russell's turn, and he has taken tiles numbered 1 and 5 from the bag.

(d) Taking into account the four tiles left in the bag ahead of Gordon's turn, explain in detail why Russell should place the 1 on his own grid and the 5 on Gordon's grid and on which squares the tiles should be placed. [4]

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