

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2013 series

9694 THINKING SKILLS

9694/31

Paper 3 (Problem Analysis and Solution),
maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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- 1 (a) What is the area of the largest shadow that can be projected on the screen using a single square? [1]

9 m².

- (b) Where should a square be placed, on track 1, in order for the shadow on the screen to be as close as possible to the left-hand side of the screen? [2]

7 metres from the left.

1 mark for 3.5 m seen or implied.
SC 1 mark for 13 metres

- (c) The lighting director wishes to project two non-touching square shadows on the screen. One square is placed at the centre of track 1 (i.e. 10 m from the left).

In how many positions can the second square be placed on track 2? [2]

6.

1 mark for 3
OR recognition that squares must lie between 5.25 and 9.75 (11.25 and 15.75).

- (d) Show that it is possible to project a shadow which has an area of 12 m² using just two squares. Give an example for the positions of the two squares. [2]

Track 1	8	8	10	10	12	12
Track 2	6	8	9	11	12	14

OR 5 m on track 2 and any on track 1 from 8 m to 13 m
OR 15 m on track 2 and any on track 1 from 7 m to 12 m.

1 mark for calculating the area of two overlapping shadows, one from track 1 and one from track 2 (including shadows placed at non-integer distances from the edge).

- (e) (i) Give a possible area for a shadow projected using two squares that is not a whole number of square metres. [1]

4.5 m² OR 3.5 m²

e.g. 5 m on track 2 and 1 m on track 3.

- (ii) What is the smallest total area of shadow that it is not possible to project onto the screen using just one or two squares, and that is a whole number of square metres? [2]

11 m²

1 mark for using 1, 3, 4, and 9 to ascertain which areas are possible.

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- 2 (a) What would be the value in all four pixels if 15 13 13 10 were pixellized? [1]

$$(15 + 13 + 13 + 10)/4 = 51/4 = \underline{12} \text{ (must be rounded down)}$$

- (b) What is the lowest original value possible for one of the pixellized pixels in the line below? [1]

7	6	4	3	13	13	13	13	10	12	12	15	14	12
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Highest value is 15 so lowest would be with no remainder
 $(15 + 15 + 15 + x)/4 = 13.$

$$x = \underline{7}$$

- (c) If one of the obscured pixels in the pictures above is 5, what possible combinations could there be for the other two? [3]

$12 \times 4 \leq \text{total} + 15 < 12 \times 4 + 3$ so total is in range 33 to 36
 $11 \times 4 \leq \text{total} + 14 < 11 \times 4 + 3$ so total is in range 30 to 33
 Total 33, one is 5 so other two must not be greater than 15 and add to 28.
 Only possible options are 15 & 13 (= 13 & 15) and 14 & 14.

3 marks for both pairs; 2 marks for one pair.
 If the correct pairs are not given, award 1 mark for two possible equations given, or one inequality. Award 2 marks for both inequalities or four equations.

- (d) What is the underlying picture? [3]

Possible values are 0 – no white, 3 one white (15/4 rounded down), 7 two whites, 11 for three whites, or 15 for four whites.

15	0	0	0	15	15	0	0	15	15	0	0
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1 mark for first two zeros, 1 mark for last two zeros, and 1 mark for section between.

- (e) (i) Consider repeating black and white patterns which are pixellized throughout. Give an example of a pattern which yields the same value in all the pixels when pixellization starts at one place, but not if it starts at another. [1]

An 8-long pattern such as 0 15 0 15 15 15 0 0 has the required property as it's 7 – 7 on one 'cut' and 11 – 3 on the next.
 A 6-long pattern would be 15 0 15 0 0 15.

- (ii) What is the shortest length of such a pattern? [1]

$$\underline{6}.$$

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- 3 (a) Show that Hyacinth will use 60 litres of fuel if she travels the entire journey to her sister's town on the highway. [1]

At 120 km/h, the car consumes 3 litres per 10 km. So the total amount of fuel consumed is:
 $3 \times 20 = 60$ litres.

- (b) Calculate the amount of time this journey would take. [2]

Distance spent on the 3 different types of road: 80 km highway; 60 km country road; 60 km minor road.

Time = $(80 \div 120) + (60 \div 80) + (60 \div 40) = 2.92$ hours = 2 hours 55 minutes

2 marks for the correct time.

If 2 marks cannot be awarded, award 1 mark for EITHER clear reference to the three different distances required OR one correct time calculation (5, 30, 45, 90, 5 minutes).

- (c) What is the minimum amount of fuel that she could use for the journey? State how long this journey would take. [3]

Answer: Fuel-minimising route: highway 10 km, country 100 km, minor 90 km.

Quantity of fuel used = $(1 \times 3) + (10 \times 2) + (9 \times 1) = 32$ litres

Length of journey = 3.58 hours = 3 hours 35 minutes.

If full marks cannot be awarded for the underlined answers, 1 mark should be awarded for each of the following aspects (awarded independently): correct apportionment of journey; calculation of time for their given journey (total 200 km, C>M>H>0); calculation of fuel for their given journey (total 200 km, C>M>H>0).

- (d) If she maximises the amount of time she spends on the highway, what is the minimum amount of time that the journey would take? State how much fuel this journey would use. [3]

Answer: Highway-maximising route = 50 km highway, 90 km country, 60 km minor.

Amount of time taken = $(50 \div 120) + (90 \div 80) + (60 \div 40) = 3$ hours 2.5 minutes

Amount of fuel required = $(5 \times 3) + (9 \times 2) + (6 \times 1) = 39$ litres

If full marks cannot be awarded for the underlined answers, 1 mark should be awarded for each of the following aspects (awarded independently): correct apportionment of journey; calculation of time for their given journey (total 200 km, C>M>H>4); calculation of fuel for their given journey (total 200 km, C>M>H>4).

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- (e) Show that it is possible to complete an enjoyable journey in less than 3 hours, using only 37 litres of fuel. [3]

Answer: if the journey is divided up as follows – 10 km on the highway, 150 km on the country roads, 40 km on the minor roads – the time required is $(10 \div 120) + (150 \div 80) + (40 \div 40) = 2 \text{ hours } 57.5 \text{ minutes}$ (177.5 minutes). The fuel required is $(1 \times 3) + (15 \times 2) + (4 \times 1) = 37$ litres.

3 marks if the correct apportionment of the journey is shown, and the amount of time that the journey would take.

If 3 marks cannot be awarded, award 2 marks for EITHER the correct times and quantities of fuel consumed for the suboptimal combinations given in the table below OR for a correct division of the journey, but with the precise time omitted/miscalculated.

Minor	Country	Highway	Time (hours)	Minutes	Fuel (litres)
50	120	30	3	0	38
50	130	20	3	2.5	37
40	140	20	2	55	38

If 2 marks cannot be awarded, award 1 mark for EITHER one of the suboptimal cases in the table above, but with the time or fuel consumption miscalculated OR one correctly calculated case journey from the rest of the valid combinations (given below).

Minor	Country	Highway	Time (hours)	Minutes	Fuel (litres)
90	100	10	3	35	32
80	90	30	3	22.5	35
80	100	20	3	25	34
80	110	10	3	27.5	33
70	80	50	3	10	38
70	90	40	3	12.5	37
70	100	30	3	15	36
70	110	20	3	17.5	35
70	120	10	3	20	34
60	90	50	3	2.5	39
60	100	40	3	5	38
60	110	30	3	7.5	37
60	120	20	3	10	36
60	130	10	3	12.5	35
50	110	40	2	57.5	39
50	140	10	3	5	36
40	130	30	2	52.5	39
30	150	20	2	47.5	39
30	160	10	2	50	38
20	170	10	2	42.5	39

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- (f) Show that it is possible to complete a journey that fulfils all three requirements and takes less than 3 hours. Give an example listing the different parts of the journey and the order in which they should be driven. [3]

Final constraint effectively means that the largest section cannot be more than 140 km: if it is 150 km, the remaining 50 km of road cannot divide the journey into sections 20 km or less (e.g. $20 + 10 + 20 + 10 + 20 + 10 + 20 + 10 + 20 + 10 + 20 + 10 + 50 = 200$).

So possible combinations are:

<i>Minor</i>	<i>Country</i>	<i>Highway</i>	<i>Time (hours)</i>	<i>Minutes</i>	<i>Fuel (litres)</i>
40	140	20	2	55	38
50	110	40	2	57.5	39
40	130	30	2	52.5	39

An example journey:

20	10	20	10	20	10	20	10	20	10	20	10	20
C	M	C	M	C	M	C	M	C	H	C	H	C

3 marks for a correct journey.

2 marks a correct combination of distances (e.g. in the table) but no correct journey given.

1 mark some evidence that the candidate appreciates the maximum length that the longest type of road can be (140 km).

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- 4 (a) (i) What is the longest possible time that a round of Celerity can take to complete, from the pressing of the START button? [2]

A maximum of 30 turns can take up to 10 seconds each.

300 seconds/5 mins

If 2 marks cannot be awarded, award 1 mark for 150 seconds OR 30 turns seen.

- (ii) Assuming a maximum pause of 2 minutes between rounds, what is the longest time it can take to play a full game of Celerity? [1]

54 mins

Accept an incorrect answer to (i) $\times 8 + 14$ minutes.

- (b) What is the largest number and what is the smallest number that can appear on the central display, when the players have had 3 turns each and the round has not finished? [2]

Two of the turns must be 2 to produce the highest number, and two of the turns must be 4 to produce the lowest number, in order to avoid multiples of 3.

Largest 82 [1 mark]

Smallest 62 [1 mark]

- (c) List all the possibilities for the next turn for Ric and the subsequent turn for Lee that will not cause the display to freeze (e.g. Ric 3, Lee 3). [3]

Ric 1	Lee 3
Ric 1	Lee 5
Ric 3	Lee 1
<i>Ric 3</i>	<i>Lee 3</i>
Ric 3	Lee 4
Ric 4	Lee 3
Ric 4	Lee 5

Award 1 mark for every 2 correct answers (but ignore Ric 3 Lee 3, which was given as an example).

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(d) (i) Why did Lee choose to press his 4 button? [1]

He gave away 15 points. Pressing 1 would have given away 18 points, and pressing a button for the fourth time or not pressing a button would have given away 19 points.

To concede as few points as possible.

(ii) Which of his buttons could Lee have pressed that would not have caused the display to freeze? [1]

He had only pressed his 3 button twice.

(iii) Show how the round could have continued from 19 until the display reached 0. [2]

Any one of the following 3 possibilities:

Lee	Ric
3	2
4	2
1	2
1	4

Lee	Ric
3	2
1	2
4	2
1	4

Lee	Ric
3	2
1	2
1	2
4	4

If 2 marks cannot be awarded, award 1 mark for any combination of 1, 1, 3 and 4 for Lee and 2, 2, 2 and 4 for Ric, even if a multiple of 3 is produced at some point, e.g.

Lee	Ric
3	2
1	4
4	2
1	2

which would produce 9 after Ric's 4.

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(e) (i) How many points were scored in round 4? [2]

34

If 2 marks cannot be awarded, award 1 mark for appreciation that 15 points were scored in round 5 and/or that 112 points had been scored altogether.

(ii) Which of the first four rounds were won by Lee and which by Ric? [1]

Lee won rounds 1 and 4 ($29 + 34 = 63$)

Ric won rounds 2 and 3 (and 5) ($11 + 23 + 15 = 49$)

The stimulus gives the information that after 5 rounds Ric had won one more round than Lee, so $29 + 11 + 23 = 63$ for Lee and $34 + 15 = 49$ for Ric is not an option.