UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

THINKING SKILLS **9694/33**

Paper 3 Problem Analysis and Solution **October/November 2011**

1 hour and 30 minutes

Additional Materials: Answer Booklet/Paper
Electronic Calculator

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES
Calculators should be used where appropriate.

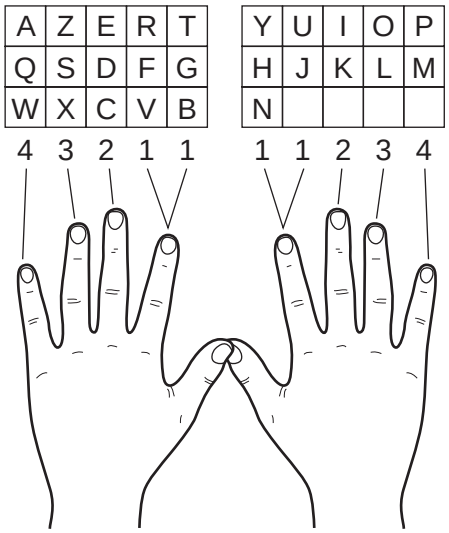
Answer **all** the questions.
Start each question on a new answer sheet.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **8** printed pages and **4** blank pages.

1 *Study the information below and answer the questions. Show your working.*

The layout of the letters on Sophie's standard French keyboard is:



She is a touch-typist (meaning she doesn't need to look at the keys) and uses the correct eight fingers for the letters, as shown above. The thumbs are not used.

She was asked to construct an eight-letter password by using a single letter from each finger, e.g. COMQXKVY.

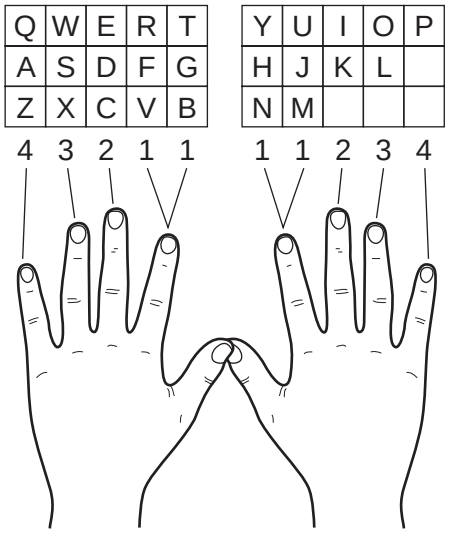
(a) Sophie selected VASOPHIE as her password, and used it until she was forced to change it after 90 days. She decides to use the same eight letters, and to keep VASO as the first four, in that order, but to type P, H, I and E in a different order, e.g. VASOPEIH.

How many passwords could she create in this way, excluding the one she has already used? [2]

Agnès also has a standard French keyboard, and selects a letter for each finger, going from left to right.
She tried what she thought was the simplest one: AZERUIOP, and was surprised to find that it was rejected because someone else had already used it.

(b) How many possible passwords could Agnès make (including any already used by others)? [2]

Fred has a standard English keyboard:



(c) Fred also makes up eight-letter passwords in the same way as Agnès (one per finger and left to right).

(i) How many possible passwords could Fred make? [1]

(ii) Give an example of a password that could be used by Agnès but not Fred. [1]

(iii) How many passwords could be made by Fred but not Agnès? [2]

Sophie's employer is worried that people will use passwords with simple patterns on the keyboard, and decides to give them eight-letter passwords randomly generated by a computer. These passwords use letters which require alternate use of the two hands.

Many users have trouble remembering and typing these random passwords. A particularly common error is a transposition of adjacent letters: i.e. to type two letters next to each other in the wrong order. For example, UD**WK**LGPZ instead of UD**KW**LGPZ. This has become so common that the system manager has decided to allow passwords with a single error of this type as being 'near enough'.

(d) (i) How many times larger is the collection of acceptable passwords which include such an error than the collection of those that could come from the random generator? [1]

(ii) An attacker knows the rules for generating and checking the passwords. What difference does allowing a transposition make to the (very small) chance of guessing the password of a particular user? [1]

2 *Study the information below and answer the questions. Show your working.*

An annuity is an arrangement where a person ('the policyholder') makes a single initial payment and then receives a fixed sum of money each year until he or she dies.

The table below gives the annual sum received per year for life in return for an initial payment of \$100. Those starting younger receive less per year because they are expected to live longer.

Age when \$100 paid	*Annual sum received*	
	Male	*Female*
50	\$4.02	\$3.96
55	\$4.36	\$4.27
60	\$4.83	\$4.69

(Assume that the proportion of people who die between the ages of 50 and 60 is so small that it can be ignored.)

(a) If a 50-year-old man and a 60-year-old man both make a \$100 payment at the same time, how much longer will the 50-year-old have to wait than the 60-year-old before getting at least \$100 back? [2]

(b) Simon believes that if he dies at the average age for a male, he will have received the same amount in total whether he were to purchase an annuity at 50 or 60. What would the average age of death have to be for Simon to be right? [2]

Since inflation changes the value of a sum of money over time, the calculations are very complex, but we are assured that the system is such that the expected return 'in real terms' is calculated fairly.

(c) The figures in the table are lower for females than for males, and yet are fair, in that they have the same total expected payment. What must be the reason for this? [1]

Married couples also have the option to get a 'joint life policy', which continues to be paid while either the policyholder or their spouse is still alive.

Annual sum received per year for life in return for an initial payment of \$100:

Policyholder's age when \$100 paid	*Annual sum received*	
	Male and wife	*Female and husband*
50	\$3.87	\$3.90
55	\$4.16	\$4.19
60	\$4.55	\$4.59

(d) The rate is less for males than females for joint life policies, although it was greater for males than females for policies only covering the policyholder.

Suggest a reason for this. [2]

(e) Janet and John are married, and both aged 55. They have $200 to invest. They can either invest the $200 in one annuity, or they could invest $100 in each of two different annuities.

(i) Which option gives them the highest return next year? [1]

(ii) Of all the many different options for annuities available to them, the one which will give Janet and John the highest expected **total** return is the 'female and husband' annuity. Explain why this is the case. [2]

3 *Study the information below and answer the questions. Show your working.*

A newspaper report said that the average oarsman in a winning Olympic 'coxed four' (a rowing crew which involves five athletes) had altered dramatically between 1968 and 1988. In particular it said,

> "…the average weight had increased from 64 kg in 1968 to 72 kg in 1988, and the range of weights in a crew had exactly doubled over those years."

Another report, by a different newspaper, said,

> "…the average weight remained at 64 kg from 1968 to 1988, but the range of weights exactly doubled over those years."

You can assume that both newspapers had the same raw data (the weights of the athletes involved in 1968 and in 1988).

The apparent disagreement in the reports came about because of the different meanings of the word "average": the first newspaper calculated the means of the weights, while the second calculated the medians.

The rules of the competition state that no participant may be less than 55 kg. For the purposes of this investigation, you should only consider weights which are whole numbers of kilograms, and you should not consider weights greater than 100 kg.

(a) Give an example of five weights which have a **mean** of 64 kg. [1]

(b) What is the largest range that the weights from 1968 could have had, given that the claims made by the first newspaper are correct (and assuming that it used the mean in its calculation of the average)? [3]

(c) Give an example of five weights which have a mean of 72 kg, and a median of 64 kg. [2]

(d) Give an example of five weights for the year 1968 and five weights for the year 1988 which could have supported the claims by both of the newspapers (assuming the first newspaper had used the mean in its calculations, and the second had used the median). [3]

A third newspaper gave the following response to the two newspaper reports above:

> "The conflicting reports of the weight differences in the Olympic rowing crews were meaningless, because they treated all five athletes as equals; they are not. One is a coxswain who steers the boat, and is **always** as light as possible (within the rules of the competition), while the other four are the oarsmen who power the boat. It is the weights of these four oarsmen which might reveal a trend in the way athletes prepare for the sport. The average weight of the four oarsmen actually increased by more than 11 kg from 1968 to 1988, and the range of their weights more than tripled."

(e) Show that the third newspaper could **not** have been referring to the mean, when it refers to the "average". [2]

(f) Find five weights for the year 1968 and five weights for the year 1988 which are consistent with all three newspaper reports. [4]

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[Question 4 is printed on the next page]

4 *Study the information below and answer the questions. Show your working.*

This is the film schedule for the Red Carpet multi-screen cinema from this Wednesday to next Tuesday.

Screen	*Film*	*Daily at:*	*Extra Shows*
1	Colossal (124 minutes)	14:15 17:25 20:35	11:05 (Sat only)
2	Wolf Gang (98 minutes)	13:45 (not Mon) 16:00 18:15 20:30	11:30 (Sat & Sun)
3	Illinois (102 minutes)	13:20 (not Mon) 15:40 18:00 20:20	11:00 (Sat only)
4	Mahatma (83 minutes)	13:50 (not Mon) 15:45 17:40 19:35 21:30	12:05 (Sat & Sun)
5	Lupine Jives (109 minutes)	14:10 16:30 18:50 21:10	11:50 (Sat & Sun)

All cinema tickets are valid throughout the cinema for the whole of the day of purchase.

The Red Carpet is in Ivystone. It runs a courtesy bus service for the benefit of customers living in neighbouring villages. Two buses operate, travelling a circular route in opposite directions.

Bus Timetable

Red Carpet (Ivystone) → Paltrow → Winslet → Red Carpet

	Sat only	Not Mon						
Red Carpet	09:40	11:50	13:30	15:10	16:50	18:30	20:10	22:10
Eastwood	09:48	11:58	13:38	15:18	16:58	18:38	20:18	22:18
Sarandon	09:59	12:09	13:49	15:29	17:09	18:49	20:29	22:29
Paltrow	10:05	12:15	13:55	15:35	17:15	18:55	20:35	22:35
Heston	10:14	12:24	14:04	15:44	17:24	19:04	20:44	22:44
Winslet	10:21	12:31	14:11	15:51	17:31	19:11	20:51	22:51
Redford	10:32	12:42	14:22	16:02	17:42	19:22	21:02	23:02
Hepburn	10:40	12:50	14:30	16:10	17:50	19:30	21:10	23:10
Red Carpet	10:47	12:57	14:37	16:17	17:57	19:37	21:17	23:17

Red Carpet (Ivystone) → Winslet → Paltrow → Red Carpet

	Sat & Sun only							
Red Carpet	10:10	12:40	14:20	16:00	17:40	19:20	21:00	23:20
Hepburn	10:17	12:47	14:27	16:07	17:47	19:27	21:07	23:27
Redford	10:25	12:55	14:35	16:15	17:55	19:35	21:15	23:35
Winslet	10:36	13:06	14:46	16:26	18:06	19:46	21:26	23:46
Heston	10:43	13:13	14:53	16:33	18:13	19:53	21:33	23:53
Paltrow	10:52	13:22	15:02	16:42	18:22	20:02	21:42	00:02
Sarandon	10:58	13:28	15:08	16:48	18:28	20:08	21:48	00:08
Eastwood	11:09	13:39	15:19	16:59	18:39	20:19	21:59	00:19
Red Carpet	11:17	13:47	15:27	17:07	18:47	20:27	22:07	00:27

(a) At what time will the last film finish each day? [2]

(b) How many times will each of the five films be shown during this seven-day period? [3]

(c) Dustin lives in Winslet but works in Ivystone. Every Thursday, after work, he goes to the Red Carpet, arriving between 17:30 and 17:35. His routine never varies: he watches the first film that starts after his arrival, then the next different film that starts after his first one has finished.

(i) Which two films will Dustin watch this Thursday? [2]

(ii) How long after the second film finishes will he have to wait for a bus to take him home? [2]

(d) Nicole lives in Redford. She has arranged to meet her friend Meryl outside the Red Carpet at 15:30 on Saturday.

(i) What is the time of the latest bus that Nicole can catch from Redford? [1]

Nicole and Meryl want to see Colossal, Illinois and Mahatma. They don't mind which order they see them in, but they do want to watch each of the three films all the way through.

(ii) What is the earliest time that they will be able to leave the Red Carpet? [3]

The two buses travelling in opposite directions pass each other a number of times each day. On Saturday, for example, the first time they pass is between Winslet and Redford at approximately 10:28.

(e) Between which two villages and at approximately what time do the buses pass for the last time each day? [2]

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