

CANDIDATE  
NAME

--

CENTRE  
NUMBER

--	--	--	--	--

CANDIDATE  
NUMBER

--	--	--	--

**FURTHER MATHEMATICS**

**9231/01**

Paper 1

**For Examination from 2017**

SPECIMEN PAPER

**3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

**DO NOT WRITE IN ANY BARCODES.**

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **21** printed pages and **1** blank page.



1 The curve  $C$  is defined parametrically by

$$x = 2 \cos^3 t \quad \text{and} \quad y = 2 \sin^3 t, \quad \text{for } 0 < t < \frac{1}{2}\pi.$$

Show that, at the point with parameter  $t$ ,

$$\frac{d^2y}{dx^2} = \frac{1}{6} \sec^4 t \operatorname{cosec} t. \quad [4]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

2 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 7 - 2t^2. \quad [6]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

3 Given that  $a$  is a constant, prove by mathematical induction that, for every positive integer  $n$ ,

$$\frac{d^n}{dx^n}(xe^{ax}) = na^{n-1}e^{ax} + a^nxe^{ax}. \qquad [6]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 4 The sequence  $a_1, a_2, a_3, \dots$  is such that, for all positive integers  $n$ ,

$$a_n = \frac{n+5}{\sqrt{(n^2-n+1)}} - \frac{n+6}{\sqrt{(n^2+n+1)}}.$$

The sum  $\sum_{n=1}^N a_n$  is denoted by  $S_N$ .

- (i) Find the value of  $S_{30}$  correct to 3 decimal places. [3]

.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....

- (ii) Find the least value of  $N$  for which  $S_N > 4.9$ . [4]

.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....

5 The cubic equation  $x^3 + px^2 + qx + r = 0$ , where  $p$ ,  $q$  and  $r$  are integers, has roots  $\alpha$ ,  $\beta$  and  $\gamma$ , such that

$$\alpha + \beta + \gamma = 15,$$
$$\alpha^2 + \beta^2 + \gamma^2 = 83.$$

(i) Write down the value of  $p$  and find the value of  $q$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Given that  $\alpha$ ,  $\beta$  and  $\gamma$  are all real and that  $\alpha\beta + \alpha\gamma = 36$ , find  $\alpha$  and hence find the value of  $r$ . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

A series of 25 horizontal dotted lines spanning the width of the page, providing a template for writing.

6 The matrix  $\mathbf{A}$ , where

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 10 & -7 & 10 \\ 7 & -5 & 8 \end{pmatrix},$$

has eigenvalues 1 and 3.

(i) Find corresponding eigenvectors.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

It is given that  $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$  is an eigenvector of  $\mathbf{A}$ .

(ii) Find the corresponding eigenvalue.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





7 The linear transformation  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$  is represented by the matrix  $\mathbf{M}$ , where

$$\mathbf{M} = \begin{pmatrix} 1 & -2 & -3 & 1 \\ 3 & -5 & -7 & 7 \\ 5 & -9 & -13 & 9 \\ 7 & -13 & -19 & 11 \end{pmatrix}.$$

(i) Find the rank of  $\mathbf{M}$  and a basis for the null space of  $T$ .

[6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



8 The curve  $C$  has equation  $y = \frac{2x^2 + kx}{x + 1}$ , where  $k$  is a constant.

- (i) Find the set of values of  $k$  for which  $C$  has no stationary points. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) For the case  $k = 4$ , find the equations of the asymptotes of  $C$  and sketch  $C$ , indicating the coordinates of the points where  $C$  intersects the coordinate axes. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

9 It is given that  $I_n = \int_1^e (\ln x)^n dx$  for  $n \geq 0$ .

(i) Show that

$$I_n = (n - 1)[I_{n-2} - I_{n-1}] \text{ for } n \geq 2. \qquad [6]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



10 (i) Using de Moivre’s theorem, show that

$$\tan 5\theta = \frac{5 \tan \theta - 10 \tan^3 \theta + \tan^5 \theta}{1 - 10 \tan^2 \theta + 5 \tan^4 \theta}. \quad [5]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Hence show that the equation  $x^2 - 10x + 5 = 0$  has roots  $\tan^2(\frac{1}{5}\pi)$  and  $\tan^2(\frac{2}{5}\pi)$ . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**(iii)** Deduce a quadratic equation, with integer coefficients, having roots  $\sec^2(\frac{1}{5}\pi)$  and  $\sec^2(\frac{2}{5}\pi)$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

11 Answer only **one** of the following two alternatives.

**EITHER**

The points  $A$ ,  $B$  and  $C$  have position vectors  $\mathbf{i}$ ,  $2\mathbf{j}$  and  $4\mathbf{k}$  respectively, relative to an origin  $O$ . The point  $N$  is the foot of the perpendicular from  $O$  to the plane  $ABC$ . The point  $P$  on the line-segment  $ON$  is such that  $OP = \frac{3}{4}ON$ . The line  $AP$  meets the plane  $OBC$  at  $Q$ .

(i) Find a vector perpendicular to the plane  $ABC$  and show that the length of  $ON$  is  $\frac{4}{\sqrt{(21)}}$ . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Find the position vector of the point  $Q$ . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) Show that the acute angle between the planes  $ABC$  and  $ABQ$  is  $\cos^{-1}\left(\frac{2}{3}\right)$ . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

OR

The curve  $C$  has polar equation  $r = a(1 - \cos \theta)$  for  $0 \leq \theta < 2\pi$ .

(i) Sketch  $C$ . [2]

(ii) Find the area of the region enclosed by the arc of  $C$  for which  $\frac{1}{2}\pi \leq \theta \leq \frac{3}{2}\pi$ , the half-line  $\theta = \frac{1}{2}\pi$  and the half-line  $\theta = \frac{3}{2}\pi$ . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) Show that

$$\left(\frac{ds}{d\theta}\right)^2 = 4a^2 \sin^2\left(\frac{1}{2}\theta\right),$$

where  $s$  denotes arc length, and find the length of the arc of  $C$  for which  $\frac{1}{2}\pi \leq \theta \leq \frac{3}{2}\pi$ . [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**BLANK PAGE**

---

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.