CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2013 series

9231 FURTHER MATHEMATICS

9231/13 Paper 1, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Qu No	Commentary	Solution	Marks	Part Mark	Total
1	Simplifies.	$\text{f}(r+1)-\text{f}(r) = r(r+1)!-(r-1)r!$	M1		
		$= r!(r^2+r-r+1) = r!(r^2+1)$	A1		
	Uses difference method.	$\sum_1^n = \text{f}(2)-\text{f}(1)+\text{f}(3)-\text{f}(2)+\ldots\text{f}(n+1)-\text{f}(n)$	M1		
		$= n(n+1)!-0 = n(n+1)!$	A1		
	Obtains result.	$\therefore \sum_{n+1}^{2n} = 2n(2n+1)!-n(n+1)!$	A1	5	[5]
		(Or directly using $\sum_{n+1}^{2n} = \text{f}(2n+1)-\text{f}(n+1)$ from the method of differences.)			
2	Makes substitution.	$y^2-4y+3y^{\frac{1}{2}}-2=0$	M1		
	Squares.	$\Rightarrow 9y = 4+y^4+16y^2-4y^2+16y-8y^3$ (N.B. Must see both terms in y^2.)	M1		
	Obtains result.	$\Rightarrow y^4-8y^3+12y^2+7y+4=0$ (AG)	A1	3	
		$S_2 = 0^2-2\times(-4)=8$	B1		
		$S_8 = 8S_6-12S_4-7S_2-16$	M1		
		$\Rightarrow S_8 = 8S_6-12S_4-56-16 = 8S_6-12S_4-72$ (AG)	A1	3	[6]
		Alternatively – for final two marks.			
		$S_2 = 8$, $S_3 = -9$, $S_4 = 40$, $S_5 = -60$, $S_6 = 203$, $S_7 = -378$ $S_8 = 1072$ (generated by substitution of roots in equations and summing.) Then $8S_6-12S_4-72 = 1624-480-72 = 1072 = S_8$ M1 requires a complete method, A1 if all correct.			
3	Differentiates once.	$\frac{\text{d}}{\text{d}x}\left(\text{e}^x\sin x\right) = \text{e}^x\sin x+\text{e}^x\cos x$	B1		
	Rearranges.	$= \sqrt{2}\text{e}^x\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)$	M1		
	Shows true for $n=1$.	$= \sqrt{2}\text{e}^x\sin\left(x+\frac{1}{4}\pi\right) \Rightarrow \text{H}_1$ true.	A1		
	States inductive hypothesis. (May be seen by implication.)	$\text{H}_k : \frac{\text{d}^k}{\text{d}x^k}(\text{e}^x\sin x) = \left(\sqrt{2}\right)^k\text{e}^x\sin\left(x+\frac{1}{4}k\pi\right)$	B1		
	Differentiates.	$\frac{\text{d}^{k+1}}{\text{d}x^{k+1}} = \left(\sqrt{2}\right)^k\left(\sin\left(x+\frac{1}{4}k\pi\right)\text{e}^x+\text{e}^x\cos\left(x+\frac{1}{4}k\pi\right)\right)$	M1		
	Rearranges.	$= \left(\sqrt{2}\right)^{k+1}\text{e}^x\left(\frac{1}{\sqrt{2}}\sin\left(x+\frac{1}{4}k\pi\right)+\frac{1}{\sqrt{2}}\cos\left(x+\frac{1}{4}k\pi\right)\right)$	A1	7	[7]

	Shows $H_k \Rightarrow H_{k+1}$ and states conclusion.	$= \left(\sqrt{2}\right)^{k+1} e^x \sin\left(x + \frac{1}{4}(k+1)\pi\right) \quad \Rightarrow H_{k+1}$ true. $\therefore$,by PMI, true for all positive integers n. (CWO)	A1		
4	Differentiates. Differentiates again. (One mark for each pair of terms.) Substitutes values. Obtains result.	$3y^2y' - (3x^2y' + 6xy) = 0$ (B1 for 1st term and = 0, but allow recovery) At (1,–2) $12y' - (3y' - 12) = 0$ $\Rightarrow 9y' = -12 \Rightarrow y' = -\frac{4}{3}$ (AG) $3y^2y'' + 6y(y')^2 - (6xy' + 3x^2y'' + 6xy' + 6y) = 0$ At (1,–2) B1 for each pair of terms. 3rd mark includes = 0, but allow recovery. $12y'' - 12 \times \frac{16}{9} - \left(-8 + 3y'' + 6 \times \frac{-4}{3} - 12\right) = 0$ $\Rightarrow 9y'' = -\frac{20}{3} \Rightarrow y'' = -\frac{20}{27}$ (Allow – 0.741)	B1B1 B1 B1B1 B1 M1 A1	 8	 [8]
5	Finds I_1. Integrates by parts. Obtains reduction formula. Attempts to use reduction formula at least once. Obtains I_5, or some intermediate result, correctly. Obtains I_7.	$I_1 = \int_0^1 x e^{-x^2} dx = \left[-\frac{e^{-x^2}}{2}\right]_0^1 = \frac{1}{2} - \frac{1}{2e}$ (AG) $I_{2n+1} = \int_0^1 x^{2n+1} e^{-x^2} dx$ $= \left[-x^{2n}\frac{e^{-x^2}}{2}\right]_0^1 + \int_0^1 2nx^{2n-1}\frac{e^{-x^2}}{2} dx$ $= \left[-\frac{1}{2e}\right] - [0] + nI_{2n-1} = nI_{2n-1} - \frac{1}{2e}$ (AG) $I_3 = \frac{1}{2} - \frac{1}{2e} - \frac{1}{2e} = \frac{1}{2} - \frac{1}{e}$ $I_5 = 2\left(\frac{1}{2} - \frac{1}{e}\right) - \frac{1}{2e} = 1 - \frac{5}{2e}$ $I_7 = 3\left(1 - \frac{5}{2e}\right) - \frac{1}{2e} = 3 - \frac{8}{e}$	M1A1 M1A1 A1 M1 A1 A1	2 3 3	 [8]
6	Reduces to echelon form (N.B. Allow matrix with a row of zeros – not in	$\begin{pmatrix} -2 & 5 & 3 & -1 \\ 0 & 1 & -4 & -2 \\ 6 & -14 & -13 & 1 \\ 1 & 1 & -2 & -11 \end{pmatrix} \sim \begin{pmatrix} -2 & 5 & 3 & -1 \\ 0 & 1 & -4 & -2 \\ 0 & 1 & -4 & -2 \\ 0 & 7 & -1 & -23 \end{pmatrix}$ $\sim \begin{pmatrix} -2 & 5 & 3 & -1 \\ 0 & 1 & -4 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 27 & -9 \end{pmatrix} \sim \begin{pmatrix} -2 & 5 & 3 & -1 \\ 0 & 1 & -4 & -2 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} (\alpha \neq 0)$	M1A1 A1		

	echelon form.) Solves set of equations. Obtains basis. Solves equations in second case. Obtains basis.	$-2x+5y+3z-t=0$ $y-4z-2t=0$ $3z-t=0$ $\Rightarrow K_1 \quad \left\{\begin{pmatrix}25\\10\\1\\3\end{pmatrix}\right\}$ (OE) If $\alpha = 0 \quad -2x+5y+3z-t=0$ $y-4z-2t=0$ $\Rightarrow K_2 \quad \left\{\begin{pmatrix}23\\8\\2\\0\end{pmatrix}, \begin{pmatrix}9\\4\\0\\2\end{pmatrix}\right\}$ (OE) e.g. $\begin{pmatrix}5\\0\\2\\-4\end{pmatrix}$ or $\begin{pmatrix}0\\10\\9\\-23\end{pmatrix}$	M1 A1 M1 A1A1	 8	 [8]
6 ctd	**Other Methods** Working from the start with equations Use of transpose matrices	Sets up **both** sets of equations Solves in the case $\alpha \neq 0$ States K_1. correctly Solves in the case $\alpha = 0$ States K_2. correctly Uses row operations to reduce transpose matrices to echelon form. When $\alpha \neq 0$ $\mathbf{M}^T \sim \begin{pmatrix}-2&0&6&1\\0&2&2&7\\0&0&0&45\\0&0&0&0\end{pmatrix}$ by $\begin{pmatrix}r_1\\5r_1+2r_2\\5r_1+2r_3-4r_4\\50r_1+20r_2+2r_3+6r_4\end{pmatrix}$ $\Rightarrow \quad K_1 \quad \left\{\begin{pmatrix}50\\20\\2\\6\end{pmatrix}\right\}$ or $\left\{\begin{pmatrix}25\\10\\1\\3\end{pmatrix}\right\}$ $\alpha \neq 0$ $\mathbf{M}^T \sim \begin{pmatrix}-2&0&6&0\\0&2&2&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$ by $\begin{pmatrix}r_1\\5r_1+2r_2\\23r_1+8r_2+2r_3\\-9r_1-4r_2-2r_4\end{pmatrix}$	M1A1 M1A1 A1 M1A1 A1 M1A1 M1A1 M1A1		 [8]

		$\Rightarrow K_2 \quad \left\{ \begin{pmatrix} 23 \\ 8 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 9 \\ 4 \\ 0 \\ 2 \end{pmatrix} \right\}$	M1A1		**[8]**
7	Differentiates twice and substitutes to find value of λ.	$y' = \lambda e^{-x} - \lambda x e^{-x}$, $y'' = -2\lambda e^{-x} + \lambda x e^{-x}$ $y'' + 5y' + 4y = 3\lambda e^{-x} = 6e^{-x} \Rightarrow \lambda = 2$	B1B1 M1A1	4	
	Finds complementary function	$(m+1)(m+4) = 0 \Rightarrow m = -1$ or -4 C.F: $\quad y = Ae^{-x} + Be^{-4x}$	M1 A1		
	and hence general solution.	G.S.: $\quad y = Ae^{-x} + Be^{-4x} + 2xe^{-x}$	A1 B1√		
	Differentiates G.S. Uses initial conditions to find constants, and obtain particular solution.	$\Rightarrow y' = -Ae^{-x} - 4Be^{-4x} + 2e^{-x} - 2xe^{-x}$ $y(0) \Rightarrow A + B = 2$, $\quad y'(0) \Rightarrow 2 - A - 4B = 3$ $\Rightarrow \quad A = 3$ and $B = -1$ $\Rightarrow y = 3e^{-x} - e^{-4x} + 2xe^{-x}$	M1 A1	 6	**[10]**
8	Differentiates and attempts to find $\frac{ds}{dt}$.	$\dot{x} = 3t \quad , \quad \dot{y} = 3t^2 \quad \Rightarrow \quad \frac{ds}{dt} = \sqrt{9t^2 + 9t^4} = 3t\sqrt{1+t^2}$	M1A1		
	Integrates to find arc length.	$s = \int_0^2 3t(1+t^2)^{\frac{1}{2}} dt = \left[(1+t^2)^{\frac{3}{2}} \right]_0^2$ $\Rightarrow s = 5\sqrt{5} - 1 \qquad$ (Allow 10.2)	M1 A1	 4	
	Uses **correct** formulae for x-coordinate.	$\bar{x} = \frac{\int_0^6 xy dx}{\int_0^6 y dx} = \frac{\int_0^2 3\frac{t^2}{2}.t^3.3t dt}{\int_0^2 t^3.3t dt}$	M1 A1		
	Finds value by integration.	$= \frac{\frac{3}{2}\int_0^2 t^6 dt}{\int_0^2 t^4 dt} = \frac{\frac{3}{2}\left[\frac{1}{7}t^7\right]_0^2}{\left[\frac{1}{5}t^5\right]_0^2} \quad = \frac{3}{2} \times \frac{5}{7} \times 4 = \frac{30}{7} \quad$ (Or 4.29)	M1 A1		
	Uses **correct** formulae for y-coordinate.	$\bar{y} = \frac{\frac{1}{2}\int_0^6 y^2 dx}{\int_0^6 y dx} = \frac{\frac{1}{2}\int_0^2 .t^6.3t dt}{\int_0^2 t^3.3t dt}$	M1		
	Finds value by integration.	$= \frac{\frac{1}{2}\int_0^2 t^7 dt}{\int_0^2 t^4 dt} = \frac{\frac{1}{2}\left[\frac{1}{8}t^8\right]_0^2}{\left[\frac{1}{5}t^5\right]_0^2} = \frac{1}{2} \times \frac{5}{8} \times 8 = \frac{5}{2} \quad$ (Or 2.5)	M1 A1	 7	 **[11]**
	Alternative layout:	$\int y dx$ (B1) $\int xy dx$ (B1) $\int \frac{1}{2}y^2 dx$ (B1) (in terms of t.) Then award M1A1 for each of $\bar{x}$ and $\bar{y}$.			
	Eliminating t.	Area (B1) $\int xy dx$ (B1) $\int \frac{1}{2}y^2 dx$ (B1) Then award M1A1 for each of $\bar{x}$ and $\bar{y}$.			

9	Proves initial result.	**Ae** = λ**e** **BMe** = **MAM**$^{-1}$**Me** = (**MAIe**) = **MAe** = **M**λ**e** = λ**Me** (CWO) (**Me** $\neq$ **0** since **M** non-singular $\Rightarrow \lambda$ is an eigenvalue.)	B1 M1 A1	3	
	States eigenvalues.	Eigenvalues are: –1 , 1 , 2	B1		
	Finds one eigenvector.	$\lambda = -1 \quad \begin{vmatrix} i & j & k \\ 0 & 2 & 1 \\ 0 & 2 & 4 \end{vmatrix} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\lambda = 1 \quad \begin{vmatrix} i & j & k \\ -2 & 2 & 1 \\ 0 & 2 & 4 \end{vmatrix} = \begin{pmatrix} 8 \\ 8 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\lambda = 2 \quad \begin{vmatrix} i & j & k \\ -3 & 2 & 1 \\ 0 & -1 & 4 \end{vmatrix} = \begin{pmatrix} 9 \\ 12 \\ 3 \end{pmatrix} \sim \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$	M1A1		
	All correct.		A1	4	
	States eigenvalues of **B**.	Eigenvalues of **B** are –1 , 1 , 2	B1		
	Finds eigenvectors of **B.** (N.B. Same as **A**'s is M0)	Corresponding eigenvectors are: $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix}$	M1A1 A1	4	**[11]**
10	Uses identity.	$2\sin\theta\cos\left(\theta - \frac{1}{4}\pi\right) = \sin\left(2\theta - \frac{1}{4}\pi\right) + \sin\left(\frac{1}{4}\pi\right)$	M1		
	Uses sine-cosine link. Obtains result.	$= \cos\left(\frac{1}{2}\pi - 2\theta + \frac{1}{4}\pi\right) + \frac{1}{\sqrt{2}}$ $= \cos\left(2\theta - \frac{3}{4}\pi\right) + \frac{1}{\sqrt{2}}$ (AG)	M1 A1	3	
	Sketches graph.	Closed loop through origin, in correct position.	B1		
	Obtains line of symmetry.	For line of symmetry $2\theta - \frac{3}{4}\pi = 0 \Rightarrow \theta = \frac{3}{8}\pi$.	B1B1	3	
	Uses area of sector formula. Rearranges.	$A = \frac{1}{2}\int_0^{\frac{3}{4}\pi}\left\{\cos^2\left(2\theta - \frac{3}{4}\pi\right) + \sqrt{2}\cos\left(2\theta - \frac{3}{4}\pi\right) + \frac{1}{2}\right\} d\theta$ $= \frac{1}{2}\int_0^{\frac{3}{4}\pi}\left\{\frac{1}{2}\cos\left[4\theta - \frac{3}{2}\pi\right] + \sqrt{2}\cos\left[2\theta - \frac{3}{4}\pi\right] + 1\right\} d\theta$	M1 A1		
	Integrates correctly.	$= \left[\frac{1}{16}\sin\left(4\theta - \frac{3}{2}\pi\right) + \frac{1}{2\sqrt{2}}\sin\left(2\theta - \frac{3}{4}\pi\right) + \frac{\theta}{2}\right]_0^{\frac{3}{4}\pi}$	dM1A1		
	Substitutes limits.	$= \left[-\frac{1}{16} + \frac{1}{4} + \frac{3}{8}\pi\right] - \left[\frac{1}{16} - \frac{1}{4}\right]$	dM1		
	Obtains given answer.	$= \frac{3}{8}(\pi + 1)$ (AG)	A1	6	**[12]**
		N.B Method marks are dependent in final part. If $\frac{1}{2}$ factor missing throughout – award M's (Max 3) If $2 \times \frac{1}{2}\int_0^{\frac{3}{8}} r^2 d\theta$, penultimate line is $= \left[\frac{3}{8}\pi\right] - \left[\frac{1}{8} - \frac{1}{2}\right]$			

11 E	Finds **PQ**.	**PQ** = $\begin{pmatrix} -3+\mu-3\lambda \\ -6\mu-2\lambda \\ 12-2\mu+\lambda \end{pmatrix}$	M1A1		
	Finds direction of common perpendicular.	$\begin{vmatrix} i & j & k \\ 3 & 2 & -1 \\ 1 & -6 & -2 \end{vmatrix} = \begin{pmatrix} -10 \\ 5 \\ -20 \end{pmatrix} \sim \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$	M1		
	(Or uses scalar product of **PQ** with the direction vector of each line.)				
	Obtains two correct equations. e.g.	$-3+\mu-3\lambda = 12\mu+4\lambda$ $-24\mu-8\lambda = -12+2\mu-\lambda$	A1 A1		
	Solves.	$\mu = 1$, $\lambda = -2$	M1A1		
	Finds **p** and **q**.	$p = \begin{pmatrix} 4+3\lambda \\ 7+2\lambda \\ -1-\lambda \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ $q = \begin{pmatrix} 1+\mu \\ 7-6\mu \\ 11-2\mu \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix}$	A1	8	
	Finds common perpendicular.	**AB** × **PQ** = $\begin{vmatrix} i & j & k \\ -1 & 0 & 4 \\ 2 & -1 & 4 \end{vmatrix} = \begin{pmatrix} 4 \\ 12 \\ 1 \end{pmatrix}$	M1A1		
	Finds **PA** (or **QA** or **PB** or **QB**)	**PA** $\begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix}$ **QA** $\begin{pmatrix} 2 \\ 6 \\ -10 \end{pmatrix}$ **PB** $\begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$ **QB** $\begin{pmatrix} -1 \\ 6 \\ 2 \end{pmatrix}$	B1		
	Uses triple scalar product to find shortest distance.	$\dfrac{\begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 12 \\ 1 \end{pmatrix}}{\sqrt{16+144+1}} = \dfrac{24+48-2}{\sqrt{161}} = \dfrac{70}{\sqrt{161}} = 5.52$	M1A1 A1	6	**[14]**
	Alternative for last 4 marks:	Plane through e.g. P in direction **PA :** $4x + 12y - 29 = 0$. Award M1A1. Then use of distance of point from line formula to get $\dfrac{70}{\sqrt{161}}$. Award M1A1.			

11 O	Draws Argand diagram.	Shows position of 3 cube roots on Argand diagram.	B1	1	
	E.g. Uses De M's theorem.	$1 = \cos 2k\pi + i\sin 2k\pi$; $k = 0, 1, 2.$ $\Rightarrow \omega = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$ and $\omega^2 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$	M1A1		
	Gives a + i*b* form.	$\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, $\omega^2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$	A1	3	
		S.C. Award B1 for cube roots without/incorrect working. Look out for algebraic forms from $(z-1)(z^2+z+1)=0$ then squaring one to get the other, which scores M1A1 A1.			
	Expands determinant.	$(6-\omega^3) - 3\omega(9\omega^2 - 2\omega^2) + 2\omega^2(3\omega - 4\omega)$	M1A1		
	Uses $\omega^3 = 1$ etc.	$= 5 - 21 - 2 = -18$	B1	3	
	Uses $a + ib$ forms correctly.	$z = 4\sqrt{3}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) - 4\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$	M1A1		
	Simplifies.	$= -2\sqrt{3} - 6i - 2\sqrt{3} + 2i$ $= -4(\sqrt{3} + i)$	A1		
	Rearranges	$= -8\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$			
	Reverts to *r*, θ form.	$= -8\left(\cos\frac{1}{6}\pi + i\sin\frac{1}{6}\pi\right) = 8\left(\cos\frac{7}{6}\pi + i\sin\frac{7}{6}\pi\right)$	M1A1	5	
	One cube root.	Cube roots are: $2\left(\cos\frac{7}{18}\pi + i\sin\frac{7}{18}\pi\right)$,	B1		
	Other two.	$2\left(\cos\frac{19}{18}\pi + i\sin\frac{19}{18}\pi\right)$, $2\left(\cos\frac{31}{18}\pi + i\sin\frac{31}{18}\pi\right)$	B1	2	**[14]**