UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

9231 FURTHER MATHEMATICS

9231/23 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question Number	Mark Scheme Details			Part Mark	Total
1	Find tangential acceleration: Find radial acceleration: Combine to give magnitude of acceleration:	$2t = 8$ $(4^2 - 12)^2/2 = 8$ $\sqrt{(8^2 + 8^2)} = 8\sqrt{2}$ *or* $11{\cdot}3$ [ms^{-2}]	B1 B1 M1 A1	4	**[4]**
2	Apply $v^2 = \omega^2 (A^2 - x^2)$ at first point: Apply $v^2 = \omega^2 (A^2 - x^2)$ at second point: Combine to find amplitude A: **A.G.**	$V^2 = \omega^2 (A^2 - 5^2)$ $(9/25)V^2 = \omega^2 (A^2 - 9^2)$ $25(A^2 - 81) = 9(A^2 - 25)$ $16 A^2 = 25 \times 72$, $A = 15\sqrt{2}/2$	B1 B1 M1 A1	4	
	Find ω using $v_{\max} = \omega A$: Find V using one of earlier eqns:	$\omega = (3\sqrt{2})/(15\sqrt{2}/2) = 2/5$ $V^2 = (4/25)(225/2 - 25) = 14$ $V = \sqrt{14}$ *or* $3{\cdot}74$	M1 M1 A1	3	**[7]**
3 **(i)**	Use conservation of energy: Equate radial forces [may imply $R = 0$]: Take $R = 0$ when contact lost: Eliminate v^2 and replace u^2 by $4ag$: Solve for $\cos\theta$:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1 + \cos\theta)$ $[v^2 = 2ag(1 - \cos\theta)]$ $mv^2/a = mg\cos\theta + R$ $mv^2/a = mg\cos\theta$ $[v^2 = ag\cos\theta]$ $4mg - 2mg(1 + \cos\theta) = mg\cos\theta$ $\cos\theta = 2/3$ **A.G.**	B1 M1 A1 M1 A1	5	
(ii)	Find further height h_2 risen: Substitute for v and θ: Find total height risen above centre O:	$h_2 = v^2 \sin^2\theta/2g$ $= (2ag/3)(5/9)/2g = 5a/27$ $a\cos\theta + h_2 = 23a/27$	M1 M1 A1 B1	4	**[9]**
4	Use conservation of momentum: Use Newton's law of restitution: Eliminate v_Q to find e:	$3mv_Q = mu + 3kmu$ $v_Q = e(u - ku)$ $e = (3k + 1)/3(1 - k)$ **A.G.**	M1 A1 M1 A1 M1 A1	6	
	Relate K.E. after and before collision: Replace v_Q by $\frac{1}{3}(1 + 3k)u$ and rearrange: Find root k with $0 < k < 1$: **A.G.** (Simply substituting given k earns M1 A0 A1)	$\frac{1}{2}\,3mv_Q^2 = \frac{2}{3}\,\frac{1}{2}m(u^2 + 3k^2u^2)$ $(1 + 3k)^2 = 2(1 + 3k^2)$ $3k^2 + 6k - 1 = 0$ $k = (-6 + \sqrt{48})/6 = \frac{1}{3}(2\sqrt{3} - 3)$	M1 A1 M1 A1 A1	5	**[11]**
5	Find MI of sphere about diameter: Find MI of sphere about axis through O: Find MI of particle about axis through O: Sum to find MI of system about O:	$I_C = (2/5)\,3M(2a)^2$ $[= 24Ma^2/5]$ $I_C + 3Ma^2$ $[= 39Ma^2/5]$ $M(3a)^2$ $[= 45Ma^2/5]$ $I = 84\,Ma^2/5$ **A.G.**	M1 M1 B1 A1	4	
(i)	State eqn of motion (A.E.F.): Put $\sin\theta \approx \theta$ (implied by using SHM): Find approx. period T from SHM formula: (A.E.F.)	$I\,d^2\theta/dt^2 = -3Mg\,a\sin\theta$ $-Mg\,3a\sin\theta$ $I\,d^2\theta/dt^2 = -6Mga\,\theta$ $[d^2\theta/dt^2 = -(5g/14a)\,\theta]$ $T = 2\pi/\sqrt{(6Mga/(84\,Ma^2/5))}$ $= 2\pi\sqrt{(14a/5g)}$ *or* $10{\cdot}5\sqrt{(a/g)}$	 M1 A1 M1 M1 A1	5	
(ii)	Use appropriate SHM formula: Find time t to $\theta = \frac{1}{2}\alpha$:	$\theta = \alpha\cos\omega t$ $t = (1/\omega)\cos^{-1}\frac{1}{2} = (1/\omega)(\pi/3)$ $= (\pi/3)\sqrt{(14a/5g)}$	M1 M1 A1	3	

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6	Integrate to find $F(x)$ for $1 \le x \le 3$:	$F(x) = \frac{1}{2}(x - 1)$	B1		
	State $F(x)$ for other intervals of x:	$0\ (x < 1),\ 1\ (x > 3)$	B1	2	
(i)	Relate dist. fn. $G(y)$ of Y to X: (working may be omitted)	$G(y) = P(Y < y) = P(X^3 < y)$ $= P(X < y^{1/3}) = F(y^{1/3})$ $= \frac{1}{2}(y^{1/3} - 1)$	M1 A1		
	Differentiate to find $g(y)$:	$g(y) = y^{-2/3}/6\ \ (1 \le y \le 27)$ [$= 0$ otherwise]	B1	3	
(ii)	Find expected value of Y (or X^3):	$E(Y) = \int_1^{27} y\,(y^{-2/3}/6)\,dy$ $= [y^{4/3}/8]_1^{27}$ *or* $[x^4/8]_1^3$ $= (81 - 1)/8 = 10$	B1		
	Find variance of Y:	$E(Y^2) = \int_1^{27} y^2\,(y^{-2/3}/6)\,dy$ $= [y^{7/3}/14]_1^{27}$ *or* $[x^7/14]_1^3$ $= (2187 - 1)/14 = 1093/7$ $\text{Var}(Y) = E(Y^2) - 10^2$ $= 393/7$ *or* $56{\cdot}1[4]$	M1 A1	3	**[8]**
7	State or find value of λ:	$\lambda = 1/2000$ *or* $0{\cdot}0005$	B1		
	Find $p = P(T \ge 1000)$:	$1 - \int_0^{1000} \lambda\, e^{-\lambda t}\,dt = 1 + [e^{-\lambda t}]_0^{1000}$ $= e^{-0{\cdot}5} = 0{\cdot}607$	M1 A1	3	
	Find $P(N = 1)$ where N of the 6 bulbs have $T < 1000$:	$P(N = 1) = 6\,p^5\,(1-p)$ [$= 0{\cdot}194$]	B1		
	Hence find $P(N \le 1)$:	$P(N \le 1) = P(N = 1) + p^6 = 0{\cdot}244$	M1 A1	3	
	Formulate inequality for new λ:	$0{\cdot}001 > \int_0^4 \lambda\, e^{-\lambda t}\,dt$	M1		
		$= [-e^{-\lambda t}]_0^4 = 1 - e^{-4\lambda}$	A1		
		$-4\lambda > \ln 0{\cdot}999$	A1		
	Find minimum mean from $1/\lambda$:	$1/\lambda > -4/\ln 0{\cdot}999$, min is 4000	M1 A1	5	**[11]**
8 (i)	Find value of k by integrating $f(x)$:	$[\frac{1}{3}kx^3]_0^6 = 1,\ k = 3/6^3 = 1/72$	B1		
	State and evaluate expression for a: **A.G.**	$a = 216\,[\frac{1}{3}kx^3]_2^3 = 3^3 - 2^3 = 19$	B1		
	Find b and c: (**MR**: $f(x)$ as distn. of table: max 3/4)	$b = 216\,[\frac{1}{3}kx^3]_3^4 = 37$, $c = 216\,[\frac{1}{3}kx^3]_4^5$ *or* $216 - 155 = 61$	M1 A1	4	
(ii)	State (at least) null hypothesis:	H_0: $f(x)$ fits data (A.E.F.)	B1		
	Combine first 2 cells since exp. value < 5:	O: 4 . . . E: 8 . . .	B1		
	Calculate χ^2 (to 2 dp):	$\chi^2 = 6{\cdot}69[4]$	M1 *A1		
	Compare consistent tabular value (to 2 dp):	$\chi^2_{4,\,0.9} = 7{\cdot}779$			
	[or if 3 or 0 cells combined:	$\chi^2_{3,\,0.9} = 6{\cdot}251$, $\chi^2_{5,\,0.9} = 9{\cdot}236$]	*B1		
	Valid method for reaching conclusion:	Accept H_0 if $\chi^2 <$ tabular value	M1		
	Conclusion (A.E.F., dep *A1, *B1):	$6{\cdot}69 < 7{\cdot}78$ so $f(x)$ does fit	A1	7	**[11]**

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9	Calculate sample mean:	$\bar{d}$ = 2623/5 = 524·6	M1		
	Estimate population variance using 1st sample: (allow biased here: 11·04 *or* $3{\cdot}323^2$)	$s_1^{\ 2} = (1376081 - 2623^2/5)/4$ [= 13·8 *or* $3{\cdot}715^2$]	M1		
	Find confidence interval (allow z in place of t) e.g.: (inconsistent use of 4 or 5 loses M1)	$524{\cdot}6 \pm t\,\sqrt{(13{\cdot}8/5)}$	M1		
	Use of correct tabular value:	$t_{4,\,0.975} = 2{\cdot}776$ (2 d.p.)	A1		
	Evaluate C.I. correct to 3 s.f. (needs correct s, t):	$524{\cdot}6 \pm 4{\cdot}6[1]$ *or* [520·0, 529·2]	A1	5	
	State hypotheses:	H_0: $\mu_b = \mu_a$, H_1: $\mu_b \neq \mu_a$	B1		
	Estimate population variance using 2nd sample: (allow biased here: 11·44 *or* $3{\cdot}382^2$)	$s_2^{\ 2} = (2720780 - 5216^2/10)/9$ [= 572/45 *or* 12·711 *or* $3{\cdot}565^2$]	M1		
	Estimate population variance for combined sample:	$s^2 = (4{\times}13{\cdot}8 + 9{\times}12{\cdot}71)/13$ = 848/65 *or* 13·05	M1 A1		
	Calculate value of t (to 2 dp):	$t = (524{\cdot}6 - 521{\cdot}6)/(\,s\sqrt{(5^{-1} + 10^{-1})})$ = 1·52	M1 *A1		
	Compare with correct tabular t value:	$t_{13,\,0.95} = 1{\cdot}77[1]$	*B1		
	Correct conclusion (AEF, dep *A1, *B1):	No difference in means	B1	8	**[13]**
10a	Take moments about P for system [i.e. rod]:	$F_A\,h = Wa \cos\theta$	M1 A1		
	Take moments about B for rod:	$F_A\,2a \sin\theta + Wa\cos\theta$ $= R_A\,2a\cos\theta$	M1 A1		
	Eliminate F_A to give R_A :	$R_A = \tfrac{1}{2}W + (Wa\sin\theta)/h$	M1		
	Find inequality for μ:	$\mu \geq F_A/R_A$ $\mu \geq 2a\cos\theta/(h + 2a\sin\theta)$ **A.G.**	M1 A1	7	
(i)	Use $kW = T$ to express in terms of F_A or R_A :	$kW = F_A/\sin\theta$ *or* $(W - R_A)/\cos\theta$	M1		
	Substitute for F_A or R_A :	$k = (a/h)\cot\theta$ *or* $(\tfrac{1}{2} - (a/h)\sin\theta)/\cos\theta$	M1 A1		
	Substitute for h and θ :	$k = \sqrt{5}/6$ *or* 0·373	A1	4	
(ii)	Find horizontal component N_P :	$N_P = T\sin\theta$ *or* $kW\sin\theta$	M1		
	Substitute for k and θ :	$(\sqrt{5}/6)(2/3)W = \sqrt{5}W/9$ *or* $0{\cdot}248W$	M1 A1	3	**[14]**

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10b (i)	Use regression line or 1st normal eqn, e.g.:	$\Sigma y/5 = 2{\cdot}5\ \Sigma x/5 - 1{\cdot}5$	B1		
	Use data to substitute for Σx and Σy:	$11 + p + q = 2{\cdot}5 \times 15 - 5 \times 1{\cdot}5$			
		$p + q = 37{\cdot}5 - 7{\cdot}5 - 11 = 19$ **A.G.**	M1 A1	3	
(ii)	Use formula for b or 2nd normal eqn: $2{\cdot}5 = (32+2p+6q - 15\times30/5)/(61 - 15^2/5)$				
		or $32+2p+6q = 2{\cdot}5\times61 - 1{\cdot}5\times15$	M2 A1		
	(A.E.F.)	$p + 3q = 49$ (*or* $3q - p = 41$)	A1		
	Solve any two simultaneous eqns for p, q:	$p = 4$, $q = 15$	M1 A1	6	
(iii)	Find correlation coefficient r:				
	$r = (32+2p+6q - 15\times30/5)/\sqrt{\{(61 - 15^2/5)\ (49 + p^2 + q^2 - (11 + p + q)^2/5)\}}$		M1		
	$= (130 - 15 \times 30/5)/\sqrt{\{(61 - 15^2/5)\ (290 - 30^2/5)\}}$		A1		
	or $2{\cdot}5\sqrt{\{(61 - 15^2/5)/\ (49 + p^2 + q^2 - (11 + p + q)^2/5)\}}$		(M1)		
	$= 2{\cdot}5\ \sqrt{\{(61 - 15^2/5)/\ (290 - 30^2/5)\}}$		(A1)		
	$= 40/\sqrt{(16\times110)}$ *or* $2{\cdot}5\sqrt{(16/110)} = 0{\cdot}953$		A1	3	
(iv)	(**a**) State eqn of actual regression line:	$y = 0{\cdot}25x - 1{\cdot}5$	B1		
	(**b**) State new value of r or say unchanged:	Same value as found in (iii)	B1	2	**[14]**