UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2008 question paper

9231 FURTHER MATHEMATICS

9231/01 Paper 1, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$\int_0^h xy^2\,dx = \int_0^h x(rx/h)^2\,dx$	M1
	$= \ldots = r^2h^2/4$ (Q)	A1
	$\bar{x} = Q/V = 3h/4$	M1A1
	OR $(1/3)\rho\pi K^2 h^3 \bar{x} = \int_0^h \rho\pi K^2 x^3\,dx$	M1M1
	$(1/3)h^3\bar{x} = \left[x^4/4\right]_0^h$	A1
	$\bar{x} = 3h/4$ (AG)	A1
2	$u_n = \ln(1+x^{n+1}) - \ln(1+x)$ or for $\ln\{$Product of fractions$\}$	B1
	$\sum_{n=1}^{N} u_n = S_N = \ln\left[\left(1+x^{N+1}\right)/\left(1+x\right)\right]$ (AEF) Cancels $\to$ result	M1A1
	(i) $S_\infty = -\ln(1+x)$ **OR** $\ln\left(\frac{1}{1+x}\right)$	A1
	(ii) $S_\infty = 0$	B1
3	$\mathbf{Ae} = \lambda\mathbf{e}$ and $\mathbf{Be} = \mu\mathbf{e} \Rightarrow \mathbf{Ae} + \mathbf{Be} = \lambda\mathbf{e} + \mu\mathbf{e}$	M1
	$\Rightarrow (\mathbf{A} + \mathbf{B})\mathbf{e} = (\lambda + \mu)\mathbf{e}$	A1
	$\lambda = 4$	B1
	$\mathbf{P} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ -2 & -3 & 1 \end{pmatrix}$	B1
	$\mathbf{D} = \begin{pmatrix} 81 & 0 & 0 \\ 0 & 256 & 0 \\ 0 & 0 & 1296 \end{pmatrix}$	M1A1 (ft)

4 **(i)** $\theta = 2$, $r = 4$ — B1B1

Ignore extra values
Accept as written in MS – co-ords not required

(ii) Graphs: correct location, orientation and concavity required — B1B1

Separate diagrams 1/2, B1 Shapes correct
B1 Intersection correct

(iii) $A_1 = (1/2)\ \int_0^2 (\theta + 2)^2\,\mathrm{d}\theta$ (LNR) — M1

$= \ldots = 28/3$

$A_2 = (1/2)\ \int_0^2 \theta^4\,\mathrm{d}\theta\ = \ldots = 16/5$ (LR) — A1

M1 for 1 correct integral representation plus A1 if both correct

Area $= A_1 - A_2 = 92/15$ (6.13) — A1

S.C. –92/15 M1 A0 A1

Alternative layout: $\frac{1}{2}\int_0^2 (\theta + 2)^2 - \theta^4\ \mathrm{d}\theta$ — M1 (LNR)

$= \frac{1}{2}\left[\frac{\theta^3}{3} + 2\theta^2 + 4\theta - \frac{\theta^5}{5}\right]_0^2$ — A1 (LR)

$= 92/15$ — A1

5 Uses substitution $y = x^3$ — M1

Obtains $y + y^{1/3} - 1 = 0$ — A1

$y = (1 - y)^3$ — A1

$\Rightarrow \ldots \Rightarrow y^3 - 3y^2 + 4y - 1 = 0$ (AG) — A1

$\sum \alpha^6 = \left(\sum \alpha^3\right)^2 - 2\sum \beta^3\gamma^3$ — B1

$= 9 - 8 = 1$ — M1A1

OR put $y = z^{1/2}$ to obtain

$z^3 - z^2 + 10z - 1 = 0$ — M1A1

$\sum \alpha^6 =$ –coefficient of z^2, $= 1$ — A1

6	$y_1 = e^{-t}/(4 - 2t)$	(Tolerate sign error)	M1
	y_2 = any correct form in t	Omission of $\frac{\mathrm{dt}}{\mathrm{dx}}$ M0	M1A1
	$y_2 = (t - 1)\mathrm{e}^{-t}/4(2 - t)^3$ (AG)		A1
	Mean value = (4/7) $\int_0^{7/4} y_2\,\mathrm{d}x$		M1
	(Limits may be given as $t = 0$ to $t = 1/2$)		
	$\int_0^{7/4} y_2\,\mathrm{d}x = [y_1]_0^{7/4} = (1/3)e^{-1/2} - 1/4$ (AEF)		M1(LNR) A1(LR)
	$= (1/21)\,(4e^{-1/2} - 3)$ (AG)		A1
7	Verifies H_1 to be true		B1
	$H_k : \sum_{r=1}^{k} (3r^5 + r^3) = (1/2)k^3(k + 1)^3$		B1
	$H_k \Rightarrow \sum_{r=1}^{k+1} (3r^5 + r^3) = (1/2)k^3(k + 1)^3 + 3(k + 1)^5 + (k + 1)^3$		M1
	$= \ldots = (1/2)(k + 1)^3(k + 2)^3$		A1
	Thus $H_k \Rightarrow H_{k+1}$ and concludes		A1
	$3\sum_{r=1}^{n} r^5 + (1/4)n^2(n + 1)^2 = (1/2)n^3(n + 1)^3$		M1
	$\Rightarrow \ldots \Rightarrow \sum_{r=1}^{n} r^5 = (1/12)n^2(n + 1)^2(2n^2 + 2n - 1)$		M1A1

8 **(i)** $\int_0^{\pi/2} t^n s dt = \left[-t^n c\right]_0^{\pi/2} + n\int_0^{\pi/2} t^{n-1} c dt$ M1A1(LR)

$= \left[nt^{n-1}s\right]_0^{\pi/2} - n(n-1)I_{n-2}$ M1A1

$\Rightarrow I_n = n(\pi/2)^{n-1} - n(n-1)I_{n-2}$ (AG) A1

(ii) L = length of arc = $\int_0^{\pi/2}\sqrt{\dot{x}^2+\dot{y}^2}\,dt$ with integrand expressed in terms of t M1

$\sqrt{\dot{x}^2+\dot{y}^2} = 2t^4 s$ B1

$L = \int 2t^4 s\, dt = 2I_4$ A1

$I_2 = \pi - 2$ A1

$L = \pi^3 - 24\pi + 48$ (AEF), e.g., $L = 8(\pi/2)^3 - 24(\pi-2)$ Accept 3.61 A1

9 One asymptote is $x = -1$ (Allow $x \to -1$, $y \to x - 3$) B1

$y = x - 3 + (\lambda+3)/(x+1)$ M1

$\Rightarrow y = x - 3$ A1

$\lambda = 1$ B1

Axes plus both asymptotes drawn B1 ft

DEF: Here Z denotes 'correct shape, orientation and approximately correct location'

RH branch with Z B1 ft

LH branch with Z B1

$\lambda = -4$: RH branch with Z B1

LH branch with Z B1

Intersections with x-axis $(1+\sqrt{5}, 0)$, $(1-\sqrt{5}, 0)$ B1

10 $\sum_{n=1}^{N} z^{2n-1} = \left(z - z^{2N+1}\right)/\left(1 - z^2\right)$ (Sum of G.P.) B1

$S_N = \Re\left[\left(z - z^{2N+1}\right)/\left(1 - z^2\right)\right]$ (Real Part) M1

$S_N = \Re\left[\left(z - z^{2N+1}\right)\left(1 - \bar{z}^2\right)/\left(2 - 2\cos 2\theta\right)\right]$ (AEF with a real denominator) M1

$= ... = \left(\cos(2N-1)\theta - \cos(2N+1)\theta\right)/\left(2 - 2\cos 2\theta\right)$ (AEF in θ and N, only.) M1A1

Multiplying out numerator

$= ... = \sin 2N\theta / 2\sin\theta$ (AG) A1

$-\sum_{n=1}^{N}(2n-1)\sin(2n-1)\theta = N\cos 2N\theta\csc\theta - (1/2)\sin 2N\theta\csc\theta\cot\theta$ (AEF) M1A1(LHS) A1(RHS)

Put $\theta = \pi/N$ to obtain required result (AG) A1

11 $y_1 = x^{\alpha}w_1 + \alpha x^{\alpha-1}w,\ y_2 = x^{\alpha}w_2 + 2\alpha x^{\alpha-1}w_1 + \alpha(\alpha-1)x^{\alpha-2}w$ B1B1
(Must see general results with α)

Obtain any DE of the form $P(x,\alpha)w_2 + Q(x,\alpha)w_1 + R(x,\alpha)w = f(x)$ M1

Sets $\alpha = -2$ and obtains required $x - w$ DE (AG) M1A1 oew

Complementary function $= Ae^{-x} + Be^{-x/2}$ M1A1

Puts $P\sin 2x + Q\cos 2x$ to obtain 2 linear equations in P and Q M1A1

Solves to obtain $P = 0,\ Q = -1$ A1

$y = x^{-2}\left[Ae^{-x} + Be^{-x/2} - \cos 2x\right]$ A1

Allow $x^2 y = \left[Ae^{-x} + Be^{-x/2} - \cos 2x\right]$

12 EITHER

(i) $\overrightarrow{AB} \times \overrightarrow{CD} = (\lambda - 2)\mathbf{i} + (4\lambda - 12)\mathbf{j} - 4\mathbf{k}$ M1A1

$(-5\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}).\left[(\lambda - 2)\mathbf{i} + (4\lambda - 12)\mathbf{j} - 4\mathbf{k}\right] = 3\lambda - 30$ M1A1

$\left|3\lambda - 30\right| / \sqrt{(\lambda - 2)^2 + (4\lambda - 12)^2 + 16} = 3$ M1A1

$\lambda^2 - 5\lambda + 4 = 0$ (AG) A1

(ii) $\lambda - 1, 4$ B1

$\lambda = 1 : \mathbf{n}_1 = 5\mathbf{i} + 13\mathbf{j} - 7\mathbf{k}$ M1A1 cao

$\lambda = 4 : \mathbf{n}_2 = 8\mathbf{i} + 25\mathbf{j} - 7\mathbf{k}$ M1A1 cao

Acute angle between planes $= \cos^{-1}\left|\mathbf{n}_1.\mathbf{n}_2\right| / \left|\mathbf{n}_1\right|\left|\mathbf{n}_2\right| = 12.1°$ M1A1 cao

[For lines 2 and 3: $\dfrac{10 - 5\lambda - 24 + 8\lambda - 16}{\sqrt{(2-\lambda)^2 + (12 - 4\lambda)^2 + 4^2}} = 3$ M1A1

$(\lambda - 10)^2 = 164 - 100\lambda + 17\lambda^2$ M1A1]

12 OR

(i) Transform given matrix to an echelon form, e.g.,

$$\begin{pmatrix} 1 & 2 & -1 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

M1A1

$dim(V) = 3$ OR BY EQUIVALENT METHOD If written down with no working 1/3 B1

(ii) $\alpha_1 \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ 3 \\ 0 \\ 3 \end{pmatrix} + \alpha_3 \begin{pmatrix} -1 \\ -1 \\ 3 \\ -4 \end{pmatrix} = \mathbf{0} \Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0,$ shown $\Rightarrow$ linear independence M2A2

(iii) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 3 \\ -4 \end{pmatrix} \right\}$ B1

(iv) W not a vector space since W does not contain the zero vector, or equivalent or not closed wrt addition — B1

(v) Reduces $\begin{pmatrix} 1 & 2 & -1 & x \\ 1 & 3 & -1 & y \\ 1 & 0 & 3 & z \\ 0 & 3 & -4 & t \end{pmatrix} \to \begin{pmatrix} 1 & 2 & -1 & x \\ 0 & 1 & 0 & y-x \\ 0 & 0 & 4 & -3x+2y+z \\ 0 & 0 & 0 & -y+z+t \end{pmatrix}$ — M1A1

$\Rightarrow \mathbf{b} \equiv \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \in V$ iff $y - z - t = 0$ or equivalent method — M1A1

so that $b \in W \Rightarrow y - z - t \neq 0$ — A1

Alternative: suppose $\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 3 \\ 0 \\ 3 \end{pmatrix} + \gamma \begin{pmatrix} -1 \\ -1 \\ 3 \\ -4 \end{pmatrix}$ (i.e. in V)

$$\left.\begin{array}{l} x = \ldots \\ y = \ldots \\ z = \ldots \\ t = \ldots \end{array}\right. \Rightarrow y - z - t = \ldots = 0$$

Hence $\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \in W \Rightarrow y - z - t \neq 0$