Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/01**

Paper 1 **For examination from 2017**

MARK SCHEME

Maximum Mark: 75

Specimen

This document consists of **13** printed pages and **1** blank page.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Partial Marks	Guidance
1	$(a+x)^5 = a^5 + {}^5C_1 a^4 x + {}^5C_2 a^3 x^2 + \ldots$ soi	1	**M1**	Ignore subsequent terms
	$\left(-\frac{2}{a} \times (their\ 5a^4) + (their\ 10a^3)\right)(x^2)$	1	**M1**	
	0	1	**A1**	**AG**
		3		
2	$f(x) = x^3 - 7x\ (+c)$	1	**B1**	
	$5 = 27 - 21 + c$	1	**M1**	Sub $x = 3$, $y = 5$. Dep. on c present
	$c = -1 \rightarrow f(x) = x^3 - 7x - 1$	1	**A1**	
		3		
3	$4x^2 + x^2 = 1/2$ soi	1	**B1**	
	Solve as quadratic in x^2	1	**M1**	E.g. $(4x^2 - 1)(2x^2 + 1)$ or $x^2 =$ formula
	$x^2 = 1/4$	1	**A1**	Ignore other solution
	$x = \pm 1/2$	1	**A1**	
		4		

Question	Answer	Marks	Partial Marks	Guidance
4(i)	$4\cos^2\theta + 15\sin\theta = 0$	1	**M1**	Replace $\tan\theta$ by $\frac{\sin\theta}{\cos\theta}$ and multiply by $\sin\theta$ or equivalent
	$4(1-s^2)+15s=0 \rightarrow 4\sin^2\theta - 15\sin\theta - 4 = 0$	2	**M1A1**	Use $c^2 = 1 - s^2$ and rearrange to **AG (**www**)**
		3		
4(ii)	$\sin\theta = -1/4$	1	**B1**	Ignore other solution
	$\theta = 194.5$ or 345.5	2	**B1B1**√	Ft from 1st solution, SC B1 both angles in rads (3.39 and 6.03)
		3		
5(i)	$\frac{dy}{dx} = -\frac{8}{x^2} + 2$ cao	2	**B1B1**	
	$\frac{d^2y}{dx^2} = \frac{16}{x^3}$ cao	1	**B1**	
		3		

Question	Answer	Marks	Partial Marks	Guidance
5(ii)	$-\frac{8}{x^2}+2=0 \rightarrow 2x^2-8=0$	1	**M1**	Set = 0 and rearrange to quadratic form
	$x=\pm 2$	1	**A1**	
	$y=\pm 8$	1	**A1**	If A0A0 scored, SCA1 for just (2, 8)
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}>0$ when $x=2$ hence MINIMUM	1	**B1**√	Ft for "correct" conclusion if $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ incorrect or any valid method inc. a good sketch
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}<0$ when $x=-2$ hence MAXIMUM	1	**B1**√	
		5		
6(i)	$x^2-x+3=3x+a \rightarrow x^2-4x+(3-a)=0$	**1**	**B1**	**AG**
6(ii)	$5+(3-a)=0 \rightarrow a=8$	1	**B1**	Sub $x=-1$ into **(i)**
	$x^2-4x-5=0 \rightarrow x=5$	1	**B1**	**OR B2** for $x=5$ www
		2		

Question	Answer	Marks	Partial Marks	Guidance
6(iii)	$16-4(3-a)=0$ (applying $b^2-4ac=0$)	1	**M1**	**OR** $\mathrm{d}y/\mathrm{d}x=2x-1\rightarrow 2x-1=3$
	$a=-1$	1	**A1**	$x=2$
	$(x-2)^2=0\rightarrow x=2$	1	**A1**	$y=2^2-2+3\rightarrow y=5$
	$y=5$	1	**A1**	$5=6+a\rightarrow a=-1$
		4		
7(i)	$BC^2=r^2+r^2=2r^2\rightarrow BC=r\sqrt{2}$	**1**	**B1**	**AG**

Question	Answer	Marks	Partial Marks	Guidance
7(ii)	Area sector $BCFD = \frac{1}{4}\pi(r\sqrt{2})^2$ soi	1	**M1**	Expect $\frac{1}{2}\pi r^2$
	Area $\Delta BCAD = \frac{1}{2}(2r)r$	1	**M1***	Expect r^2 (could be embedded)
	Area segment $CFDA = \frac{1}{2}\pi r^2 - r^2$.oe	1	**A1**	
	Area semi-circle $CADE = \frac{1}{2}\pi r^2$ Shaded area $\frac{1}{2}\pi r^2 - \left(\frac{1}{2}\pi r^2 - r^2\right)$	1	**B1**	
	or $\pi r^2 - \left(\frac{1}{2}\pi r^2 + \left(\frac{1}{2}\pi r^2 - r^2\right)\right)$	1	**DM1**	Depends on the area ΔBCD
	$= r^2$	1	**A1**	
		6		
8(i)	$x^2 - 4x = 12$	1	**M1**	$4x - x^2 = 12$ scores M1A0
	$x = -2$ or 6	1	**A1**	
	3rd term $= (-2)^2 + 12 = 16$ or $6^2 + 12 = 48$	2	**A1A1**	SC1 for 16, 48 after $x = 2, -6$
		4		

Question	Answer	Marks	Partial Marks	Guidance
8(ii)	$r^2 = \frac{x^2}{4x}\left(=\frac{x}{4}\right)$ soi	1	**M1**	
	$\frac{4x}{1-\frac{x}{4}} = 8$	1	**M1**	Accept use of unsimplified $\frac{x^2}{4x}$ or $\frac{4x}{x^2}$ or $\frac{4}{x}$
	$x = \frac{4}{3}$ or $r = \frac{1}{3}$	1	**A1**	
	3^{rd} term $= \frac{16}{27}$ (or 0.593)	1	**A1**	
		4		
	ALTERNATIVE METHOD $\frac{4x}{1-r} = 8 \rightarrow r = 1 - \frac{1}{2}x$ **or** $\frac{4x}{1-r} = 8 \rightarrow x = 2(1-r)$	1	**(M1)**	
	$x^2 = 4x\left(1-\frac{1}{2}x\right)$ $\quad r = \frac{2(1-r)}{4}$	1	**(M1)**	
	$x = \frac{4}{3}$ $\quad r = \frac{1}{3}$	1	**(A1)**	
9(i)	$-(1)(x-3)^2 + 4$	**3**	**B1B1B1**	
9(ii)	Smallest (m) is 3	**1**	**B1**√	Accept $m \geqslant 3$, $m = 3$. **Not** $x \geqslant 3$. Ft *their b*

Question	Answer	Marks	Partial Marks	Guidance
9(iii)	$(x-3)^2 = 4-y$	1	**M1**	Or x/y transposed. Ft *their a*, *b*, *c*
	Correct order of operations	1	**M1**	
	$f^{-1}(x) = 3+\sqrt{4-x}$ cao	1	**A1**	Accept $y =$ if clear
	Domain is $x \leqslant 0$	1	**B1**	
		4		
10(i)	PM = $2\mathbf{i} - 10\mathbf{k} + \frac{1}{2}(6\mathbf{j} + 8\mathbf{k})$ oe	1	**M1**	Any valid method
	PM = $2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$	1	**A1**	
	$\div\sqrt{4+9+36}$	1	**M1**	
	Unit vector = $\frac{1}{7}(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$	1	**A1**	
		4		

Question	Answer	Marks	Partial Marks	Guidance
10(ii)	AT = $6\mathbf{j} + 8\mathbf{k}$, PT = $a\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ soi (or TA and TP)	1	**B1**	Allow 1 vector reversed at this stage.
	$(\cos ATP) = \frac{(6\mathbf{j}+8\mathbf{k}).(a\mathbf{i}+6\mathbf{j}-2\mathbf{k})}{\sqrt{36+64}\sqrt{a^2+36+4}}$	1	**M1**	(**AM** or **MT** could be used for **AT**)
	$= \frac{36-16}{\sqrt{36+64}\sqrt{a^2+36+4}}$ $\frac{20}{10\sqrt{a^2+40}}$	1	**A1**√	Ft from their **AT** and **PT**
	$\frac{2}{\sqrt{a^2+40}} = \frac{2}{7}$ oe and attempt to solve	1	**M1**	
	$a = 3$	1	**A1**	Withheld if only 1 vector reversed
		5		
	ALTERNATIVE METHOD Alt (Cosine Rule) Vectors (AT, PT etc.)	1	**(B1)**	
	$\cos ATP = \frac{a^2+36+4+36+64-(100+a^2)}{2\sqrt{(a^2+40)}\sqrt{100}}$ then as above	2	**(M1A1)**	

Question	Answer	Marks	Partial Marks	Guidance
11(i)	$\frac{dy}{dx} = \left[\frac{1}{2}(1+4x)^{-1/2}\right] \times [4]$	2	**B1B1**	
	At $x = 6, \frac{dy}{dx} = \frac{2}{5}$	1	**B1**	
	Gradient of normal at $P = -\frac{1}{2}$	1	**B1**√	**OR** eqn of norm $y - 5 = \textit{their} -\frac{5}{2}(x-6)$ When $y = 0, x = 8$ hence result
	Gradient of $PQ = -\frac{5}{2}$ hence PQ is a normal, or $m_1 m_2 = -1$	1	**B1**	
		5		

Question	Answer	Marks	Partial Marks	Guidance
11(ii)	Vol for curve $=(\pi)\int(1+4x)$ and attempt to integrate y^2	1	**M1***	
	$=(\pi)\left[x+2x^2\right]$ ignore '+ c'	1	**A1**	
	$=(\pi)\left[6+72-0\right]$	1	**DM1**	Apply limits $0\rightarrow 6$ (allow reversed if corrected later)
	$=78(\pi)$	1	**A1**	
	Vol for line $=\frac{1}{3}\times(\pi)\times 5^2\times 2$	1	**M1**	**OR** $(\pi)\left[\frac{\left(-\frac{5}{2}x+20\right)^3}{3\times-\frac{5}{2}}\right]_6^8$
	$=\frac{50}{3}(\pi)$	1	**A1**	
	Total Vol $=78\pi+50\pi/3=94\frac{2}{3}\pi$ (or $284\pi/3$)	1	**A1**	
		7		

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