Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2023 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **17** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

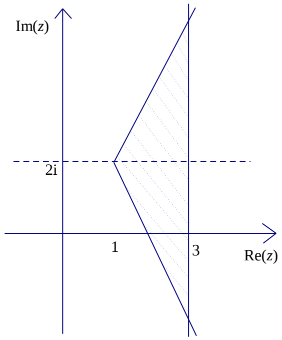
FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a quotient or express x as $\ln e^x$	**M1**	$x = \ln[(2y - 3)/(y + 4)]$ or $\ln e^x = \ln(2y - 3) - \ln(y + 4)]$.
	Remove logarithms and obtain a correct equation e.g. $e^x = \dfrac{2y-3}{y+4}$	**A1**	
	Obtain answer $y = \dfrac{3+4e^x}{2-e^x}$	**A1**	OE ISW
		3	

Question	Answer	Marks	Guidance
2(a)	Show correct half-lines from 1 + 2i, symmetrical about $y = 2i$ (drawn between $\dfrac{\pi}{4}$ and $\dfrac{5\pi}{12}$).	**B1**	
	Show the line $x = 3$ extending in both quadrants.	**B1**	
	Shade the correct region. Allow dashes on axes as scale. FT If only error is one of following: FULL lines or $x \neq 3$ or one sign error in 1 + 2i or angle outside tolerance or scale missing on one axis.	**B1 FT**	
			SC No scale on either axis allow **B1 FT** for otherwise correct figure in correct position.
		3	

Question	Answer	Marks	Guidance
2(b)	Carry out a complete method for finding the least value of arg z	**M1**	e.g. $-\tan^{-1}\frac{\left(2\sqrt{3}-2\right)}{3}$ or $\tan^{-1}\frac{\left(-2\sqrt{3}+2\right)}{3}$.
	Obtain answer –0.454 3dp	**A1**	**SC B1** 0.454 .
		2	

Question	Answer	Marks	Guidance
3	Commence division and reach partial quotient $2x^2 + (a \pm 2)x$	**M1**	$2x^2 + (a + 2)x + a$ need $2x^2 + (a \pm 2)x$ $(x^2 - x + 1)\ 2x^4 + ax^3 + 0x^2 + bx - 1$ $2x^4 - 2x^3 + 2x^2$ $(a + 2)x^3 - 2x^2 + bx$ $(a + 2)x^3 - (a + 2)x^2 + (a + 2)x$ $ax^2 + (b - (a + 2))x - 1$ $ax^2 - ax + a$ $(b - 2)x - (1 + a)$ $3x + 2$ Working backwards from remainder: $2x^2 + (\ldots.)x \pm 3$ **M1** $2x^2 - x - 3$ **A1**
	Obtain correct quotient $2x^2 + (a + 2)x + a$	**A1**	Allow sign error e.g. in $b - 2$.
	Set *their* linear remainder equal to part of "$3x + 2$" and solve for a or for b	**M1**	Remainder = $3x + 2 = (b - 2)x - 1 - a$. Allow for just equating x term or constant term.
	Obtain answer $a = -3$	**A1**	
	Obtain answer $b = 5$	**A1**	

Question	Answer	Marks	Guidance
3	**Alternative method for Question 3**		
	State $2x^4+ax^3+0x^2+bx-1=\left(x^2-x+1\right)\left(2x^2+Ax+B\right)+3x+2$ and form and solve equation(s) to obtain A or B	**M1**	e.g. 0 = B – A + 2 and –1 = B + 2.
	Obtain A = –1, B = –3	**A1**	
	Form and solve equations for a or for b	**M1**	e.g. a = A – 2 or b = –B + A + 3.
	Obtain answer a = –3	**A1**	
	Obtain answer b = 5	**A1**	
	Alternative method for Question 3		
	Use remainder theorem with $x=\dfrac{1\pm\sqrt{-3}}{2}$ or $x=\dfrac{1\pm i\sqrt{3}}{2}$	**M1**	Allow for correct use of a reasonable attempt at either root in exact or decimal form in the remainder theorem $x^2=\dfrac{-1+\sqrt{-3}}{2}$ $x^3=-1$ $x^4=\dfrac{-1-\sqrt{-3}}{2}$.
	Obtain $-a+\dfrac{b}{2}\pm\dfrac{b\sqrt{-3}}{2}\mp\sqrt{-3}-2=\dfrac{7}{2}\pm\dfrac{3\sqrt{-3}}{2}$ or $-a+\dfrac{b}{2}\pm\dfrac{bi\sqrt{3}}{2}\mp i\sqrt{3}-2=\dfrac{7}{2}\pm\dfrac{3i\sqrt{3}}{2}$	**A1**	Expand brackets and obtain exact equation for either root. Accept exact equivalent.
	Solve simultaneous equations, or single equation, for a or for b	**M1**	
	Obtain answer a = –3 from exact working	**A1**	
	Obtain answer b = 5 from exact working	**A1**	
		5	

Question	Answer	Marks	Guidance
4	Substitute $z = x + \mathrm{i}y$ and $z^* = x - \mathrm{i}y$ to obtain a correct equation, horizontal or with $(1-2\mathrm{i})/(1-2\mathrm{i})$ seen, in x and y	**B1**	$5(x + \mathrm{i}y) - (x + \mathrm{i}y)(x - \mathrm{i}y)(1 + 2\mathrm{i}) + (30 + 10\mathrm{i})(1 + 2\mathrm{i}) = 0$ $5(x + \mathrm{i}y)(1 - 2\mathrm{i})/[(1 + 2\mathrm{i})(1 - 2\mathrm{i})] - (x + \mathrm{i}y)(x - \mathrm{i}y) + (30 + 10\mathrm{i}) = 0$ $x - 2\mathrm{i}x + \mathrm{i}y + 2y - x^2 - y^2 + 30 + 10\mathrm{i} = 0$.
	Use $\mathrm{i}^2 = -1$ at least once and equate real and imaginary parts to zero	***M1**	OE For their horizontal equation.
	Obtain two correct equations e.g. $x + 2y - x^2 - y^2 + 30 = 0$ and $-2x + y + 10 = 0$	**A1**	$5x - (x^2 + y^2) + 10 = 0$ $5y - 2(x^2 + y^2) + 70 = 0$ $5y - 10x + 50 = 0$ $x + 2y - (x^2 + y^2) + 30 = 0$ Allow $-2\mathrm{i}x + \mathrm{i}y + 10\mathrm{i} = 0$.
	Solve quadratic equation for x or for y	**DM1**	$x^2 - 9x + 18 = (x - 3)(x - 6) = 0$ $y^2 + 2y - 8 = (y + 4)(y - 2) = 0$ **DM0** If x or y imaginary.
	Obtain answers $3 - 4\mathrm{i}$ and $6 + 2\mathrm{i}$	**A1**	
		5	

Question	Answer	Marks	Guidance
5(a)	Obtain $\frac{dx}{dt} = e^{2t} + 2te^{2t}$	**B1**	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$	**M1**	$\frac{dy}{dx} = \frac{2t+1}{e^{2t}(1+2t)}$.
	Obtain the given answer $\frac{dy}{dx} = e^{-2t}$	**A1**	AG Need to see $e^{2t}(1 + 2t)$ in denominator.
		3	
5(b)	Obtain $x = -e^{-2}$ or $-\frac{1}{e^2}$ and $y = 3$ at $t = -1$	**B1**	
	Obtain gradient of normal = $-e^{-2}$ or $-\frac{1}{e^2}$	**B1**	
	$x = 0$ substituted into equation of normal or use of gradients to give $y = 3 - \frac{1}{e^4}$ with no errors	**B1**	Equation of normal $y - 3 = -e^{-2}\left(x - -e^{-2}\right)$. AG **SC** Decimals **B0 B1 B0** − 0.135 .
		3	

Question	Answer	Marks	Guidance
6(a)	State $R = 13$	**B1**	Allow if $\sqrt{(12^2 + (-5)^2)}$ seen.
	Use correct trig formulae to find $\alpha = \tan^{-1}(\pm 5/12) = \cos^{-1}(\pm 12/13) = \sin^{-1}(\pm 5/13)$	**M1**	$\cos(\alpha) = 12$ and $\sin(\alpha) = 5$ M0 However, $\sin(\alpha)/\cos(\alpha) = 5/12$ or $-5/12$ with no error seen, or $\tan(\alpha) = 5/12$ or $-5/12$ quoted then allow.
	Obtain $\alpha = 0.395$	**A1**	CWO If negative sign seen when finding R then A0 here. If degrees 22.6 A0 MR. Only penalise degrees once in **(a)** and **(b)**. Note $\alpha = 0.39479\ldots$
		3	
6(b)	$\cos^{-1}\left(\frac{6}{R}\right)$	**B1FT**	SOI 1.0910… FT *their* incorrect R.
	Use correct method to find a value of $2x$ in the interval	**M1**	$2x = \cos^{-1}\left(\frac{6}{R}\right) + \alpha$ or $2\pi - \cos^{-1}\left(\frac{6}{R}\right) + \alpha$. Allow if $\cos(2x + 0.395)$ seen
	Obtain answer, e.g. $x = 0.743$ or 0.742	**A1**	42.5 or 42.6 degrees.
	Obtain second answer, e.g. $x = 2.79$ and no others in the interval	**A1**	159.8, 159.9 or 160.0 degrees all possible depending whether using 3 dp or 4 dp.
		4	

Question	Answer	Marks	Guidance
7(a)	State or imply area of major sector = $\frac{1}{2}r^2(2\pi - x)$	**B1**	OE
	State or imply area of shaded segment = $\frac{1}{2}r^2 x - \frac{1}{2}r^2 \sin x$	**B1**	OE $r^2 \sin(x/2) \cos(x/2)$ B0 until changed to $(1/2)r^2 \sin x$.
	State $\frac{1}{2}r^2(2\pi - x) = 3\left(\frac{1}{2}r^2 x - \frac{1}{2}r^2 \sin x\right)$	**M1**	OE Area of major sector = 3 times (area of minor sector – area of triangle). Allow $r^2 \sin(x/2) \cos(x/2)$.
	Obtain the given answer $x = \frac{3}{4}\sin x + \frac{1}{2}\pi$ after full and correct working	**A1**	AG Allow rectified slip if before penultimate line.
		4	
7(b)	Calculate the values of a relevant expression or pair of expressions at x = 2 and x = 2.5	**M1**	x= 2 x = 2.5 $(3/4) \sin x + (1/2)\pi$ 2.2(5277) 2.0(197) 2 < 2.2 or 2.3 2.5 > 2.0 $x - (3/4) \sin x - (1/2)\pi$ − 0.2(5277) < 0 + 0.4(803) > 0 or change of sign Attempt both values and one correct for M1.
	Complete the argument correctly with correct calculated values	**A1**	Degrees award 0/2
		2	

Question	Answer	Marks	Guidance
7(c)	Use the iterative formula correctly at least twice	M1	
	Obtain final answer 2.18	A1	
	Show sufficient iterations to 4 d.p. to justify 2.18 to 2 d.p. or show there is a sign change in the interval (2.175, 2.185)	A1	2 / 2.2528 / 2.1530 / 2.1972 / 2.1784 / 2.1866 / 2.1830 / 2.1846 2.25 / 2.1543(5) / 2.1967 / 2.1786 / 2.1865 / 2.1831 / 2.1845 2.5 / 2.0196(5) / 2.2465 / 2.1560 / 2.1960 / 2.1789 / 2.1863 / 2.1831 / 2.1845 Degrees award 0/3
		3	

Question	Answer	Marks	Guidance
8(a)	Use the product rule correctly	*M1	x^3 d/dx(ln$x$) + d/dx(x^3) ln$x$.
	Obtain the correct derivative in any form	A1	e.g. $\frac{x^3}{x} + 3x^2 \ln x$.
	Equate derivative to zero and solve exactly for x	DM1	Reaching $x = e^a$.
	Obtain answer $\left(\frac{1}{\sqrt[3]{e}}, -\frac{1}{3e}\right)$ or exact equivalent	A1	ISW
		4	

Question	Answer	Marks	Guidance
8(b)	Integrate by parts and reach $ax^4 \ln x + b\int(x^4 / x)\mathrm{d}x$	*M1	
	Obtain $\frac{x^4}{4}\ln x - \frac{1}{4}\int(x^4 / x)\mathrm{d}x$	A1	OE
	Complete integration and obtain $\frac{x^4}{4}\ln x - \frac{x^4}{16}$	A1	OE
	Use limits of $x = \frac{1}{2}$ and $x = 1$ in the correct order, having integrated twice	DM1	Correct substitution [(1/4)ln1 or 0 − 1/16] – [(1/64)ln(1/2) – $(1/16)^2$] or minus this value CWO. Allow omission of (1/4)ln1 or 0.
	Obtain answer $\frac{15}{256} - \frac{1}{64}\ln 2$ or exact equivalent final answer	A1	
		5	

Question	Answer	Marks	Guidance
9	Separate variables correctly and obtain e^{-3y} and $\sin^2 2x$ on the opposite sides	**B1**	
	Obtain term $-\frac{1}{3}e^{-3y}$	**B1**	
	Use correct double angle formula for $\sin^2 2x$ = (1/2)[1 – cos 4x]	**M1**	
	Obtain terms $\frac{1}{2}\left[x-\frac{1}{4}\sin 4x\right]$ oe	**A1**	
	Use x = 0, y = 0 to evaluate a constant or as limits in a solution containing terms of the form ax and $b\sin 4x$ and $ce^{\pm 3y}$	**M1**	
	Obtain correct answer in any form e.g. $-\frac{1}{3}e^{-3y}=\frac{1}{2}\left[x-\frac{1}{4}\sin 4x\right]-\frac{1}{3}$	**A1**	
	Substitute x = $\frac{1}{2}$ and obtain y = 0.175 or $-\frac{1}{3}\ln\left(\frac{1}{4}+\frac{3}{8}\sin 2\right)$	**A1**	OE ISW
		7	

Question	Answer	Marks	Guidance
10(a)	Carry out correct process for evaluating the scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OB}$	**M1**	$\pm(3, -1, 2).(1, 2, -3) = \pm(3-2-6) = [-5]$.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and obtain $\cos^{-1}\{\pm(3-2-6)/[\sqrt{(3^2+(-1)^2+2^2)}\ \sqrt{(1^2+2^2+(-3)^2)}]\}$	**A1**	
	Obtain answer 110.9° or 1.94^c	**A1**	
		3	
10(b)	Use a correct method to form an equation for line through *AB*	**M1**	
	Obtain $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \mu_1(2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$	**A1**	OE e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \mu_2(-2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k})$. Need **r** or (x, y, z).
		2	
10(c)	Obtain a correct equation for line through *CD* e.g. $[\mathbf{r} =]\ \mathbf{i} - 2\mathbf{j} + 5\mathbf{k} + \lambda_1(-4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$	**B1**	OE e.g. $[\mathbf{r} =]\ 5\mathbf{i} - 6\mathbf{j} + 11\mathbf{k} + \lambda_2(-4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$. **r** can be omitted or another symbol used.
	Equate two pairs of components of general points on *their l* and *their CD* and solve for λ or for μ	**M1**	
	Obtain e.g. $\lambda_1 = -2$ or $\mu_1 = 3$ or $\lambda_2 = -1$ or $\mu_2 = -4$	**A1**	
	Obtain position vector $9\mathbf{i} - 10\mathbf{j} + 17\mathbf{k}$	**A1**	Condone (9, –10, 17) but not $(9\mathbf{i}, -10\mathbf{j}, 17\mathbf{k})$.
		4	

Question	Answer	Marks	Guidance
11(a)	State or imply the form $\dfrac{Ax+B}{4+x^2}+\dfrac{C}{1+x}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	$(Ax + B)(1 + x) + C(4 + x^2) = 5x^2 + x + 11$.
	Obtain one of $A = 2$, $B = -1$ and $C = 3$	**A1**	If error present in above still allow A1 for C.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	If $A = 0$ then max M1 A1 (for C).
		5	
11(b)	Integrate and obtain terms $\left(\dfrac{A}{2}\right)\ln\left(4+x^2\right)+\dfrac{B}{2}\tan^{-1}\left(\dfrac{x}{2}\right)+C\ln\left(1+x\right)$	**B1FT + B1FT + B1FT**	The FT is on A, B and C. Integral of $\dfrac{Ax+B}{4+x^2}$ = $\dfrac{B}{2}\tan^{-1}\left(\dfrac{x}{2}\right)$ or $(A/2)\ln(4 + x^2)$ only. B0FT unless clearly from single term.
	Substitute limits 0 and 2 correctly in an integral of the form $a\ln\left(4+x^2\right)+b\tan^{-1}\left(\dfrac{x}{2}\right)+c\ln\left(1+x\right)$, where $abc\neq 0$	**M1**	$a\ln\left(4+4\right)+b\tan^{-1}\left(\dfrac{2}{2}\right)+c\ln\left(1+2\right)$ $-$ [$a\ln 4+b\tan^{-1}0+c\ln\left(1\right)$]. Allow one sign or substitution error. Allow omission of $b\tan^{-1}0+c\ln\left(1\right)$.
	Obtain answer $\ln 54-\dfrac{\pi}{8}$ after full and correct working	**A1**	AG – work to combine or simplify at least 2 of ln terms is required CWO. A0 if any non exact value(s) seen.
		5	