Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

0332655407

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

1 The coefficient of x^4 in the expansion of $(3 + x)^5$ is equal to the coefficient of x^2 in the expansion of $\left(2x + \frac{a}{x}\right)^6$.

Find the value of the positive constant a. [4]

2 The second and third terms of a geometric progression are 10 and 8 respectively.

Find the sum to infinity. [4]

3 The equation of a curve is such that $\frac{dy}{dx} = 3(4x - 7)^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$. It is given that the curve passes through the point $\left(4, \frac{5}{2}\right)$.

Find the equation of the curve. [4]

4 The first, second and third terms of an arithmetic progression are k, $6k$ and $k + 6$ respectively.

(a) Find the value of the constant k. [2]

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(b) Find the sum of the first 30 terms of the progression. [3]

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5 The equation of a curve is $y = 4x^2 - kx + \frac{1}{2}k^2$ and the equation of a line is $y = x - a$, where k and a are constants.

(a) Given that the curve and the line intersect at the points with x-coordinates 0 and $\frac{3}{4}$, find the values of k and a. [4]

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(b) Given instead that $a = -\frac{7}{2}$, find the values of k for which the line is a tangent to the curve. [5]

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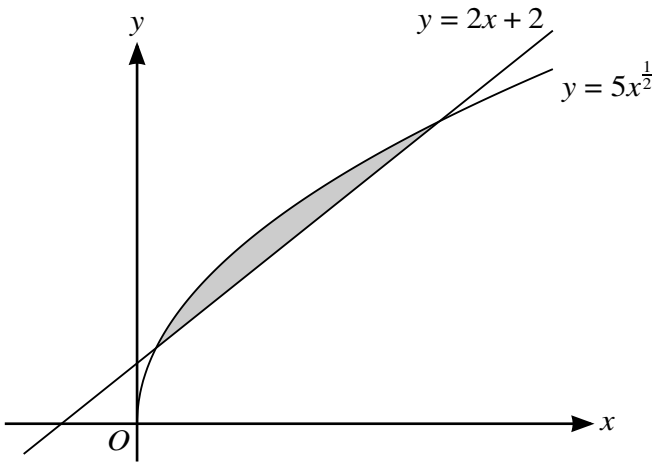

The diagram shows the curve with equation $y = 5x^{\frac{1}{2}}$ and the line with equation $y = 2x + 2$.

Find the exact area of the shaded region which is bounded by the line and the curve. [5]

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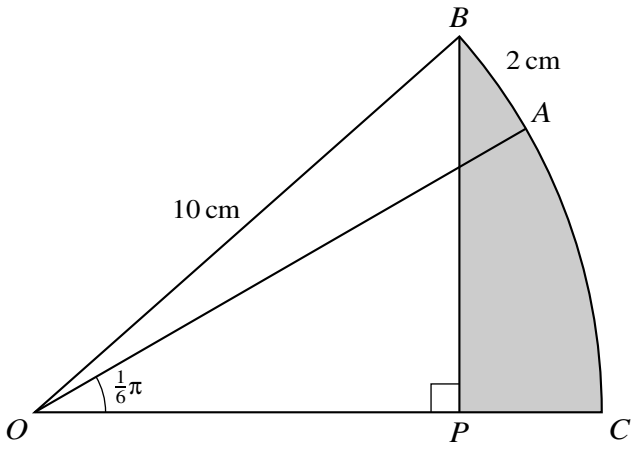

The diagram shows a sector $OBAC$ of a circle with centre O and radius 10 cm. The point P lies on OC and BP is perpendicular to OC. Angle $AOC = \frac{1}{6}\pi$ and the length of the arc AB is 2 cm.

(a) Find the angle BOC. [2]

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(b) Hence find the area of the shaded region BPC giving your answer correct to 3 significant figures. [4]

[Turn over

8 The equation of a circle is $x^2 + y^2 + ax + by - 12 = 0$. The points $A\,(1,\ 1)$ and $B\,(2,\ -6)$ lie on the circle.

(a) Find the values of a and b and hence find the coordinates of the centre of the circle. [4]

(b) Find the equation of the tangent to the circle at the point A, giving your answer in the form $px + qy = k$, where p, q and k are integers. [4]

9 The equation of a curve is $y = 3x + 1 - 4(3x + 1)^{\frac{1}{2}}$ for $x > -\frac{1}{3}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

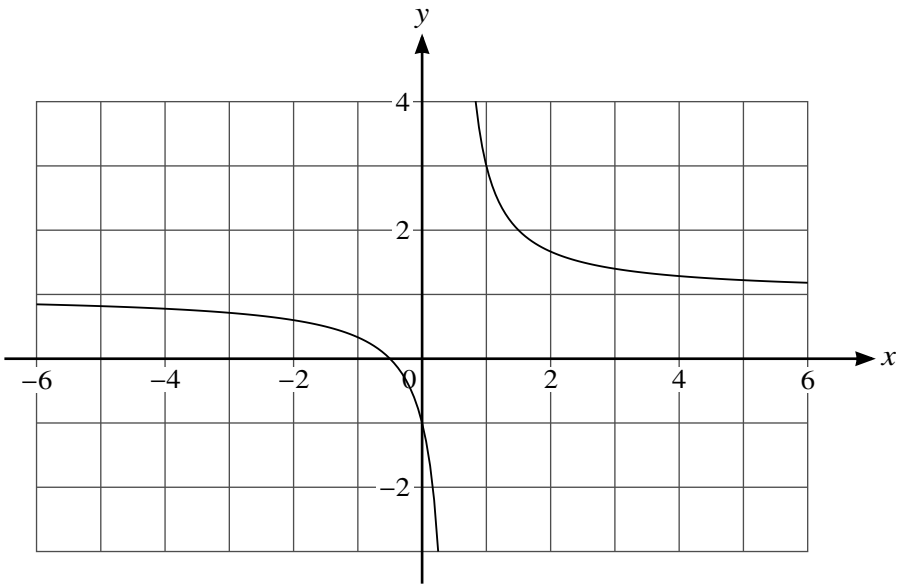
(b) Find the coordinates of the stationary point of the curve and determine its nature. [4]

10 Functions f and g are defined as follows:

$$f(x) = \frac{2x+1}{2x-1} \quad \text{for } x \neq \tfrac{1}{2},$$

$$g(x) = x^2 + 4 \quad \text{for } x \in \mathbb{R}.$$

(a)



The diagram shows part of the graph of $y = f(x)$.

State the domain of f^{-1}. [1]

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(b) Find an expression for $f^{-1}(x)$. [3]

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(c) Find $gf^{-1}(3)$. [2]

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(d) Explain why $g^{-1}(x)$ cannot be found. [1]

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(e) Show that $1 + \frac{2}{2x-1}$ can be expressed as $\frac{2x+1}{2x-1}$. Hence find the area of the triangle enclosed by the tangent to the curve $y = f(x)$ at the point where $x = 1$ and the x- and y-axes. [6]

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11 The function f is given by $f(x) = 4\cos^4 x + \cos^2 x - k$ for $0 \leqslant x \leqslant 2\pi$, where k is a constant.

(a) Given that $k = 3$, find the exact solutions of the equation $f(x) = 0$. [5]

(b) Use the quadratic formula to show that, when $k > 5$, the equation $f(x) = 0$ has no solutions. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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