MATHEMATICS

Paper 9709/11
Pure Mathematics 1

Key messages

The question paper contains a statement in the rubric on the front cover that 'no marks will be given for unsupported answers from a calculator.' This means that clear working must be shown to justify solutions, particularly in syllabus items such as quadratic equations and trigonometric equations. In the case of quadratic equations, for example, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient for certain marks to be awarded. It is also insufficient to quote only the formula: candidates need to show values substituted into it. When factorising, candidates should ensure that the factors always expand to give the coefficients of the quadratic equation.

Errors in notation were very common, often leading to incorrect use of appropriate methods or, in some cases, use of methods that were irrelevant to the question was seem. Candidates should be reminded that correct use of notation will enable them to demonstrate their understanding of the techniques involved and will also help them to successfully progress with their working.

General comments

Some very good responses were seen but the paper proved very challenging for many candidates. In AS and A Level Mathematics papers the knowledge and use of basic algebraic and trigonometric methods from IGCSE or O Level is expected, as stated in the syllabus.

Comments on specific questions

Question 1

Many candidates successfully found the equation of the curve, including evaluating the constant of integration. A significant number of candidates omitted the constant. Weaker responses used differentiation instead of integration.

Question 2

Many candidates were able to successfully apply the formula for the sum of *n* terms in this question. To find *a* and *d*, it was necessary to substitute the values given into the formula to create two simultaneous equations then to solve them. Candidates should be reminded to think carefully about the most efficient way to solve these equations: complicated substitutions are more prone to error than elimination. A common error was to find the sum of 60 terms instead of the 60$^{\text{th}}$ term. Candidates are reminded to read the question carefully to ensure they are answering what is required.

Question 3

While most candidates used the binomial expansion formula (provided in the List of Formulae MF19), weaker responses involved multiplying out five brackets, which often generated multiple errors.

(a) Several candidates did not include the constant term so were unable to gain one of the marks. Other candidates gave three terms from the term in x^5 downwards, which was not what the question had asked.

(b) Stronger responses expanded $(x + 4)^2$ correctly then successfully identified the three pairs of terms that contributed to the coefficient of x^2. Some candidates obtained only two terms when expanding $(x + 4)^2$. Weaker responses multiplied out the two expansions fully, introducing errors, then did not identify the desired terms within their list of terms.

Question 4

This is a topic that was recently introduced to the syllabus. Most candidates attempted this question but very few candidates obtained fully correct answers. The value for c, the translation parallel to the y-axis, was most often correct while the translation parallel to the x-axis and the stretch parallel to the y-axis were more challenging for candidates. A significant number of candidates showed some working but could not identify numerical values for a, b or c from their working.

Question 5

This question was attempted by almost all candidates though many responses did not use the properties of a geometric progression. A small number of candidates produced working that assumed it was an arithmetic progression, attempting to find the common difference between consecutive terms.

The most successful responses formed an equation for the common ratio from pairs of consecutive terms, solved for k and then used the information that k was negative to obtain $k = –3$. Some candidates subsequently found a correct value for r, the first term and the sum to infinity, though sign errors were fairly common. A small number of candidates used the alternative method, creating an equation for r^2 and solving for r first.

Question 6

Almost all candidates attempted this question, with the majority being awarded the first method mark for equating the curve and the line.

Most responses followed the first method in the mark scheme: forming a 3-term quadratic equation and using the condition for the discriminant to find k. Many algebraic errors were seen in the working, resulting in incorrect final values, but it was still possible to obtain some method marks. A small number of candidates attempted to solve using the alternative (differentiation) method, but few were successful in reaching a value for k via that route.

Question 7

(a) A variety of successful approaches for proving the given identity were seen. Most correct answers used the Pythagorean identity $\sin^2\theta + \cos^2\theta$ then split the expression into two fractions which could be simplified to 1 and $–\tan^2\theta$. Those candidates who were familiar with $\sec\theta$ converted $\frac{1}{\cos^2\theta}$ into $\sec^2\theta$ then $1 + \tan^2\theta$ to obtain the given answer correctly. A common error was to write $\frac{1-2\sin^2\theta}{\cos^2\theta} = 1 - \frac{2\sin^2\theta}{\cos^2\theta}$ then omit the 2 in order to reach the given answer. A few candidates were successful in starting with $1 – \tan^2\theta$, working backwards to obtain the left hand side correctly, and were awarded both marks, although this method is not advised.

In a trigonometric proof, candidates should check carefully that each line of their argument follows logically from the previous line.

(b) The majority of candidates reached the quadratic equation in $\tan^2\theta$: $2\tan^4\theta + \tan^2\theta - 1 = 0$. Many of them were able to solve correctly for $\tan\theta$, though some solved only for $\tan^2\theta$ and did not square root the answer. Candidates should be reminded to check their solution by substituting it into the original equation. Candidates should also be reminded to ensure they include all possible solutions within the range to ensure they are fully answering the question.

Question 8

(a) The most common successful method for answering this question was to state that triangle *PCQ* or *SCR* is equilateral and therefore each angle of the triangle is $\frac{\pi}{3}$. Hence $\angle PCS = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$. Other correct responses used trigonometry in a right-angled triangle formed by dividing triangle *PCQ* in two, for instance. Since the answer was given, it was necessary to produce an argument clearly justifying it.

(b) Many candidates found that the total length of the major arcs was $2 \times 4 \times \frac{2\pi}{3}$. A common mistake was to omit π in the circumference of the semicircles. Candidates needed to ensure that they combine the terms into one exact value and should generally simplify where possible. The question asked for the 'exact perimeter' so those who gave the perimeter as a decimal could not be awarded the final mark.

(c) This question proved to be challenging for most candidates. It required candidates to devise a strategy and apply their knowledge of circles and sectors. In a question of this type, candidates would be helped by writing down an overall strategy first. A fully correct answer was rare, but many attempts at the area of a sector or segment or triangle or semicircle were seen.

Question 9

(a) Many candidates gave a correct response using correct notation. Common errors in incorrect responses included omitting to use a strict inequality and using *x* instead of *y* or f(*x*) to denote the range.

(b) A large number of fully correct answers were seen. Where errors arose, these were generally involving the square root, for example not removing the +/– in the inverse function or writing $\sqrt{(x+4)}$ inappropriately as $\sqrt{x}+2$. Some candidates misunderstood the notation and found f '(*x*) instead of $f^{-1}(x)$.

(c) Most candidates started by forming an equation correctly and many of them rearranged it into a 3-term quadratic equation then solved it. However, many candidates did not realise that only the answer $x = 3$ was valid because of the restriction on the domain of f(*x*), hence they could not be awarded the final mark. Errors were often seen in the following steps: expanding the bracket, multiplying terms by 3 and moving terms across to one side.

(d) Some candidates found gg(*x*) first then substituted $f^{-1}(x)$, while others found f^{-1} initially then composed it with g. It was acceptable to substitute in $x = 12$ at the beginning of the working or later. In either method it was necessary to equate the whole expression to 62 then simplify the result to a 3-term quadratic equation in *a*. The final step was to solve the equation to obtain two values for *a.*

Many candidates misread the notation in this question and used f(*x*) instead of $f^{-1}(x)$, or alternatively used their f '(*x*) found in **part (b)**. Some candidates composed the functions in the wrong order or multiplied two or more functions together. Algebraic errors were fairly common, including the loss of terms when expanding brackets.

Question 10

(a) Many candidates did not realise the need to substitute $y = 0$ into the circle equation in order to find where the tangents cross the *x*-axis. Those who did this easily obtained a quadratic equation and solved it. Most of these candidates obtained the two correct *x*-values. Candidates who attempted to complete the square on *x* and *y* before setting $y = 0$ were more prone to algebraic errors.

(b) Successful responses took one of two approaches to this question. The first approach relied on coordinate geometry. Having found the centre of the circle, they calculated the gradients of the radii ending at the two points found in **part (a)**. They calculated the perpendicular gradients and used those to find the equations of the two tangents. Finally, they equated these lines and solved to find the correct point of intersection (2, 27). The alternative approach used calculus. By differentiating implicitly and substituting in the coordinates of the two points found in **part (a)**, these candidates calculated the gradients of the two tangents. They used these to find the equations of the tangents then the point of intersection as in the first approach.

Numerical and algebraic errors were common. Some candidates had incorrect signs in their coordinates of the centre, while others made sign errors in calculating the gradients or did not find the perpendicular gradients. Not all candidates realised that the two points found in **part (a)** had y-coordinates of 0. Other errors were seen in the equations of the lines or in solving to find the point of intersection. Some of the candidates who attempted implicit differentiation omitted $\frac{dy}{dx}$ for terms in y or forgot to include = 0. Notation errors were also seen, for example implicit differentiation with $\frac{dy}{dx}$ = … at the start of the line.

Question 11

It was apparent from the responses to all part of **Question 11** that some candidates were confused over when to increase and when to decrease the power when using calculus methods.

(a) Many candidates recognised that finding the normal required differentiation to obtain a term with a power of $-\frac{1}{2}$. A common error seen was to omit the coefficient of 3 (not applying the chain rule). Other responses displayed errors in the second term which should have been –1. In some cases, the second term was missing in subsequent working. Having substituted in $x = 4$ to obtain the gradient of the tangent, several candidates did not subsequently find the gradient of the normal, so only the first two marks were available. A significant number of responses showed candidates setting the derivative = 0 as a means of obtaining the gradient. Other candidates did not differentiate at all but substituted $x = 4$ into the equation of the curve.

(b) Most candidates who differentiated correctly or who made only a coefficient error in **part (a)** were able to equate their derivative to zero and solve for the stationary point. Those who had omitted the second term obtained an equation that could not be solved so could proceed no further.

(c) A significant number of candidates found the second derivative correctly and, using the x-value, correctly interpreted their numerical result to identify the maximum. Candidates who made chain rule errors in the first derivative commonly repeated the same error in this. Candidates who instead checked values for the first derivative either side of the point often presented their work unclearly. It is important to obtain numerical values for the first derivative to justify the conclusion about the nature of the stationary point.

(d) Candidates who attempted this part of the question realised the need to integrate the function, substitute in both limits and give their answer in exact form, as specified by the question. Incorrect use of the chain rule was common. A small number of candidates misread the question and attempted to find a volume of revolution. Most candidates showed their substitution of the limits clearly, though a common error was to assume the limit $x = 0$ gave a value of 0 for the expression. Candidates who used a calculator to obtain a numerical answer without showing the substitution could not be awarded the final two marks.

MATHEMATICS

Paper 9709/12
Pure Mathematics 1

Key messages

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Candidates should also familiarise themselves with the terminology used to describe transformations as exemplified in mark scheme and the specimen material.

General comments

The paper was generally well received by candidates and many very good responses were seen. In general, candidates seemed to have sufficient time to finish the paper. Presentation of work was mostly good, although some of the answers appear to be written in pencil which is then superimposed with ink which in some cases makes candidate responses illegible. Centres are strongly advised that candidates should not do this.

Comments on specific questions

Question 1

Part (a) of this question proved to be an accessible start for many candidates with many correct answers seen. Candidates who expanded the answer and compared coefficients were generally more successful than those who attempted to take out a factor of 4 from the question and then re-insert it later. Most candidates did not see the connection between the two parts of the question and in **part (b)** used the discriminant instead. Most realised the need to include *k*, although not all. Those who did were often able to solve the resulting equation correctly to find *k* but only the strongest responses realised that this value of *k* needed to be used to find the value of the root. More efficient methods employed by some candidates involved either using **part (a)** to state *k* or simply stating the root by using $\frac{-b}{2a}$.

Question 2

Transformations is a new topic on the revised syllabus and some candidates were unsure how to proceed and many others did not use the required terminology when describing them in **part (a)**. Translations are best described using a column vector. Stretches should be given with the required scale factor, parallel to, or in the direction of, the relevant axis. Terms such as 'move', 'left', 'right', 'up' or 'down' are not acceptable terminology. In **part (b)** many candidates correctly interpreted the effect of the reflection on the equation of the curve but fewer did so with the stretch. A common response to **part (b)** was $y = -sin\frac{2x}{3} + \frac{5x}{3}$.

Question 3

Candidates should be reminded to present their answer to the accuracy as required by the question, which several candidates were able to correctly answer **parts (a)** and **(b)**, some answers were not given to the 4 decimal places as instructed in the question. Candidates found **part (c)** to be more challenging with many candidates either omitting it or talking about a stationary point or an increasing function. Some realised that the gradients were approaching 2, but very few referred to limits or gradients of chords.

Question 4

Many fully correct solutions were seen for this question on the binomial expansion. Most candidates were able to find the required coefficients and establish a correct equation using the given connection. Some forgot about the possible solution of –1 as well as 1.

Question 5

Nearly all candidates attempted this functions question and understood the required method. In **part (a)** some forgot to square when applying the second *f* function and obtained $2\left(2x^2+3\right)+3$ but could make little progress in **part (b)** because they had an unsolvable equation. This should have indicated to candidates that a mistake had been made. Many fully correct solutions were seen to both parts however in **part (b)** many candidates did not show the required working when solving the quadratic in x^2. Candidates should ensure they show clear attempt of solving quadratics, rather than using their calculator solving functions as these are not condoned. When factorising $4x^4-5x^2+1$, some candidates appeared to have used their calculator to obtain $x^2=1$ or $x^2=\frac{1}{4}$ and then wrote down $(x^2-1)(x^2-\frac{1}{4})$ in attempt to work backwards. Candidates should be reminded that factors are expected to be integers. Candidates should also ensure that when factorising it is important that the brackets do expand to give the coefficients of the original quadratic. Some candidates used the substitution $x=x^2$, but often then forgot to square root. Those who used a letter other than *x* were usually more successful.

Question 6

The use of a diagram for coordinate geometry questions is always recommended. Those who did were generally more successful than those who did not, and they avoided errors such as assuming that *B* lay on the perpendicular bisector. Several different methods were possible which all required the use of the midpoint and the perpendicular gradient to -2. Those who only used one of these two pieces of information were unable to solve the equations with two unknowns or assumed that $(2,0)$ was point *B* rather than the midpoint of *AB*.

Question 7

For this question, candidates who used a diagram tended to be more successful with this question than those who did not. Nearly all candidates attempted **part (a)** and made some progress but relatively few obtained fully correct solutions. Many candidates either showed that the point *A* was on the circle or, more commonly, that the line joining *A* to the centre had a gradient $\frac{3}{2}$ and was therefore perpendicular to line *l* rather than showing both of these. Some candidates took the more difficult approach of solving simultaneously the equation of the circle with line *l* and a few attempted implicit differentiation, however these approaches were rarely successful. Many candidates did not appear to understand what was required in **part (b)** and gave either the equation of the original circle, the equation of a circle centre *A* or the equation of it's perpendicular. Successful responses doubled the difference in coordinates from the original centre to point *A* to find the new centre. The required equation was then usually found although some did not square the radius.

Question 8

Nearly all candidates attempted **part (a)** but a range of responses were seen. Stronger responses were able to establish the correct relationship between the terms in the arithmetic progression and the geometric progression separately and then combine them to form a quadratic in one variable only. Weaker responses were often unable to establish the correct relationships or treated the two progressions as if they were one. As in **Question 5(b)** some candidates did not show their method of solution for their quadratic equation. Candidates usually found **part (b)** more straightforward and were able to use the correct sum formula to find the required answer.

Question 9

To find the upper limit for the integration candidates could find the *x* coordinate of the point of intersection of the curve $y^2 = x - 2$ and the line $y = 1$ to be 3. However, some square rooted the 3 and others solved $x - 2 = 0$. The formula for finding the volume by integration was well known although a few candidates integrated x^2 instead of y^2 and some only integrated y. Some candidates did not subtract the volume generated by rotating the rectangular area from their answer, although most did. A variety of approaches were used: either integrating 1, subtracting 1 from $x - 2$ then integrating or by recognising that it was the volume of a cylinder that was required, with roughly equal levels of success.

Question 10

Part (a) of this question was very well attempted by most candidates, many of whom presented fully correct solutions. Almost all worked from the left-hand side and attempted to combine the two fractions into one. This was generally successful although it is important to note that all necessary brackets must be shown for this type of proof question. All the required steps in the proof need to be clearly demonstrated.

In **part (b)** most candidates knew to use **part (a)** although some attempted to 'prove' the equation and some equated the expression to 8 rather than 8tan*x.* The equation cos*x* = 0.5 leading to $x = \frac{\pi}{3}$ were very often found but this was usually achieved by cancelling by *tanx* rather than taking it out as a common factor. This meant that the solution *tanx = 0* was almost always lost.

Question 11

Candidates generally effectively dealt with the unknown constant *k* as part of the given differential and were able to differentiate and integrate successfully. In **part (a)** some differentiated the given expression and others equated it to 3.5 rather than 0. Most made a reasonable attempt and were able to solve the subsequent equation successfully. In **part (b)** the vast majority knew that integration was required although some forgot the *+c* and a few used the equation of a straight line. **Part (c)** was generally well answered although some candidates did not multiply the first term by the extra 3 and some other candidates multiplied out the expression before differentiating. In **part (d)** nearly all candidates substituted 2 into their second differential and found its value. It is important to note that if considering the size of $\frac{dy}{dx}$ either side of the stationary point then the values of *x* being considered and the subsequent values of $\frac{dy}{dx}$ need to be stated for the marks to be obtained.

Question 12

Many candidates found this question to be challenging and either omitted it or were unable to make much progress. In **part (a)** some candidates assumed triangle *PAQ* to be equilateral whilst others divided the area or circumference a circle or 2π, by 6, but often did not justify these approaches. Those who explained that 6 of the sectors of size *APQ* or the arc lengths of size *PQ* made up a full circle obtained all available marks. **Part (b)** was done more successfully with some candidates realising that the angle from **part (a)** could be used to find the required 6 arc lengths and the diameter of the given circles for the straight lengths. In **part (c)** a variety of approaches were adopted in stronger responses, which were all roughly equally successful. The most common was to consider the hexagon to be made up of 6 equilateral triangles, with other responses including two trapezia or a rectangle and two triangles. Weaker responses included answers which were not multiples of $\sqrt{3}$ or use of the formula for the area of a sector. Those who had found **part (c)** to be particularly challenging often found **part (d)** to be difficult also. Some good responses were seen though, usually being made up from 6 rectangles, 6 sectors and the answer from **part (c)**. Weaker responses sometimes involved working out the area of a circle and then multiplying that by 7, but often did not progress to find the parts of the area not covered by them.

MATHEMATICS

Paper 9709/13
Pure Mathematics 1

Key messages

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Candidates should be reminded to use the available answer space or the additional answer page at the end of the question paper, rather than using scrap paper for rough working.

General comments

Candidates appeared to have a good understanding of the syllabus content and many very good responses were seen. Candidates are reminded to include all necessary work to ensure their answers are fully justified and to allow available marks to be awarded.

Comments on specific questions

Question 1

Nearly all candidates found this question to be accessible. Most identified the need to integrate, find the constant of integration and present the equation of the curve. Many full correct responses were seen.

Question 2

The requirement of differentiation to find the first derivative was apparent to most candidates. Many error free expressions for the gradient were seen and those who set these expressions to less than zero usually obtained the limit of x and then deduced the required value of a. Candidates were expected to show or state that the gradient was negative in the given range.

Question 3

The most popular route to the solution saw candidates equating the line and curve equations and setting the discriminant of the resulting quadratic equation to zero to find the values of m. Many answers demonstrated clear and logical algebraic processes resulting in correct answers. Use of calculator solving functions was noted in this question and candidates are reminded to show clear method throughout as marks will not be awarded for unsupported answers from a calculator. The alternative method using differentiation to find the gradient of the line in terms of x resulted in a more direct route to the answers, requiring the solution of a simpler quadratic equation and substitution into linear rather than quadratic equations.

Question 4

(a) Many different routes to the given result were seen and providing each step was explained clearly all marks were available. Some candidates did not show a visible method for cancelling terms in the numerator and denominator. The relationship between the three trigonometric ratios was generally well understood.

(b) The need to change of subject of the result given in **part (a)** was apparent to most candidates and usually carried out correctly. A small number did not gather the cos*x* terms before factorising cos*x* from the two terms in which it appeared.

(c) Most candidates realised the need to substitute $k = 4$ into their answer to **part (b)** although some obtained a correct value from the given expression in **part (a)**. Complete solutions were not always seen because many candidates did not correctly identify the second solution or gave additional incorrect solutions in the given range. Several candidates expressed their answers in degrees rather than radians as clearly suggested by the question.

Question 5

(a) Most candidates answered this question correctly. Nearly all candidates used the sector area formula and the area of the sector to find the required angle. Very occasionally this angle was given in degrees instead of radians as requested in the question.

(b) The requirement to use basic trigonometry was seen by many candidates, the arc length formula was generally quoted and used correctly, and many correct answers were seen. It is advised that intermediate calculations should be kept to more than three significant figures to ensure the ensured the final answer is to an acceptable degree of accuracy.

Question 6

(a) Most candidates understood and were able to use a technique to present $f(x)$ and $g(x)$ in completed square form. Finding the connection between $g(x)$ and $f(x + p) + q$ was a notable discriminator. Although the values of p and q were intended to be apparent from a comparison of answers for $f(x)$ and $g(x)$ some algebraic solutions were seen. Occasionally $f(x)$ and $g(x)$ were not shown in completed square form even though this had been requested in the question.

(b) Many candidates stated the transformation to be a translation but did not provide a precise enough description of their translation. Use of column vectors is advised for describing translations. Those candidates who described the translation using a vector were a lot more successful than those who tried to describe the movement without referring to the *x*- and *y*- axes. It was also acceptable to describe the transformation as two single translations.

Question 7

(a) The binomial expansion was well understood by most candidates and many completely correct answers were seen to this part. The occasional sign error and confusing ‘ascending’ with ‘descending’ were the only errors of note.

(b) Finding the appropriate terms in x^2 was completed correctly by many candidates who went on to form and solve a correct quartic equation as a quadratic. Those who factorised correctly or used the quadratic formula or completion of the square often went on to obtain all available marks. Most realised that the negative square roots were valid solutions to their equation.

Question 8

(a) The formation of a composite function was completed successfully by a large majority of candidates. Those who chose to isolate the $(2x + 1)$ term in the resulting equation were often more successful than those who formed a quadratic equation without showing their solution clearly. The strongest solutions featured examination of the two roots and rejection of $x = -\frac{1}{4}$ which was not in the given range of g.

(b) Finding the inverse by change of subject of the expression for fg used in **part (a)** was the favoured method of most candidates. Those who realised the inverse must be negative and used the appropriate square root usually completed the question successfully. Some, otherwise correct, answers were not fully complete because the inverse was left in terms of y rather than x.

Question 9

(a) Most candidates appreciated the need to express 24% in an equivalent numerical form to form an equation linking the second term and sum to infinity. This often led to a correct quadratic equation however it was common that not all supporting working out was shown.

(b) The information in the MF19 booklet was used well by most candidates to formulate equations linking the two given terms and the two given sums. There were several errors noted in attempts to obtain linear equations in *a* and *d* from their fractions, often due to confusion over the form of the first term of the second series. Nonetheless, many correct linear equations were seen and solved correctly to find *a* and *d*.

Question 10

(a) Many different approaches were used to prove *ABC* was a right angle. Showing the gradients of *AB* and *BC* had a product of –1 was the most popular although Pythagoras' theorem, the cosine rule and the scalar product of AB and BC were also seen. Provided the explanation was clear all these methods were able to gain both marks.

(b) Most candidates noticed that if *ABC* was a right angle, *AC* must be a diameter and went on to find its mid-point, *D*, correctly.

(c) The general form of the circle equation given its centre and radius was very familiar to most candidates who used their coordinates of *D* and the length *AD* or *DC* to obtain the correct equation. Longer methods using the three given points rarely ended in a correct equation. Surd form was not acceptable in the final answer and candidates are expected to simplify the *r* term where possible.

(d) Whilst using the mid-point formula or a vector approach provided a quick route to finding *E* some candidates used the lengthier route of solving equation of the circle with the equation of *AD*. Those who chose to solve the equation of the circle with the equation from the length of *AE* had the opportunity to reach the equation of the tangent directly but rarely realised this. Most successful attempts used the perpendicular gradient to *BD* or *BE* with the coordinates of *E* in a straight-line equation. This was the most frequently omitted question part.

Question 11

(a) Most candidates were successful in completing the required differentiation and successfully dealt with terms with fractional indices. Fewer were successful in finding the coordinates of the minimum point due to numerical and algebraic errors in the solution of $\frac{dy}{dx} = 0$. Having found the correct *x*-coordinate a significant number did not continue to find the corresponding *y*-coordinate.

(b) Attempts to find the gradient and *y*-coordinate at the given point were seen in most solutions and subsequently used in a linear equation. Candidates were often successful in substituting into $\frac{dy}{dx}$ and *y*.

(c) Most candidates recognised that integration of *y* was required here, and many successfully integrated the two terms and correctly applyed the limits. The power of $\frac{3}{2}$ was the source of error for some; a common error was to only apply the power to k^2 and not to the coefficient of k^2. However, a majority of candidates reached the correct final answer.

MATHEMATICS

Paper 9709/21
Pure Mathematics 2

Messages

Candidates should ensure that they read each question carefully before they start, to ensure that they have understood the demands of the question. They should then double check their solution to ensure that they have given their final answer in the required form or to the required level of accuracy.

General comments

Some candidates had clearly not prepared well or revised sufficiently for the examination as evidenced by questions that were not attempted. Of those candidates who did attempt all the questions, showing a reasonable understanding of the syllabus, there did not appear to be any issues with time.

Comments on specific questions

Question 1

Most candidates were successful in obtaining the critical values associated with the inequality, either by a pair of correct linear equations or a quadratic equation (ignoring inequality signs). However, fewer candidates then obtained a correct domain.

Question 2

Many candidates were unable to expand out $\sin(\theta + 30°)$ correctly even though the form of this expansion is in the formula booklet provided. Some candidates did not recall that $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$. There were two approaches which could be adopted for answering this question. The more common approach was to expand out $\left(\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta\right) \times \frac{1}{\sin\theta}$ in order to obtain an equation in terms of $\cot\theta$ and hence an equation in $\tan\theta$, which could then be solved. Errors in the expansion of these two terms were common, mainly due to algebraic errors, but many candidates were then able to solve their equation in $\tan\theta$, to obtain at least one solution. The second approach was to multiply each side of the equation by $\sin\theta$ and then simplify the resulting equation to obtain an equation in $\tan\theta$.

Question 3

(a) Few correct responses were seen. Most candidates were able to expand $(\sec x + \cos x)^2$ to obtain $\sec^2 x + \cos^2 x + 2$. Few candidates recognised the need to use the double angle formula to write $\cos^2 x$ as $\frac{1 + \cos 2x}{2}$. In many cases, the power of 2 in the cosine term became a coefficient of the cosine term so that the given form appeared to be obtained. The incorrect response of $\sec^2 x + 2 + \cos 2x$ was the most common solution given by candidates.

(b) Many candidates were able to obtain method marks, even though an incorrect solution had been given for **part (a)**, by integrating $\sec^2 x$ and $\cos 2x$ correctly and then attempting a correct substitution of limits. For those candidates with completely correct solutions, the final accuracy mark was sometimes not awarded as the final answer was not given in exact form, as required by the question. Candidates should ensure that they check the question to ensure that they have given their final answer in the required form.

Question 4

(a) Several candidates demonstrated good knowledge of the basic properties of logarithms and hence a correct value for the parameter was usually obtained. However, many candidates were unable to write $\ln(2t+6)-\ln t$ as $\ln\frac{2t+6}{t}$. Many candidates also attempted to differentiate the expressions for *x* and *y*, with respect to t, without reading the question carefully enough. Many correct derivatives were seen in this question part, with the calculation for finding the value of *t* not always being included.

(b) Few correct responses were seen. Errors included the incorrect differentiation of $\ln(2t+6)$ as $\frac{1}{2t+6}$ rather than the correct $\frac{2}{2t+6}$. Some candidates did not recognise that to find $\frac{dy}{dt}$, differentiation of a product was needed. An exact answer was needed to this question, which many candidates did not show.

Question 5

(a) Use of the quotient rule was widely recognised and applied correctly. However, misuse of brackets and incorrect simplification of $(\ln x)^2$ were common. Many candidates were able to confirm the final result correctly.

(b) Very few correct methods of solution were seen. Many candidates realised that they were looking for a change of sign when considering the value of a function when the two given values were substituted. Most candidates considered the function needed to be $\frac{3x+2}{3\ln x}$ rather than $x-\frac{3x+2}{3\ln x}$ or equivalent. Other methods are acceptable provided a full explanation is given.

(c) Most candidates who attempted this question part were able to obtain a correct result to the required accuracy.

Question 6

(a) Many candidates had difficulty calculating the correct width of the trapezia and hence obtained an incorrect number of incorrect *y* values.

(b) A large number of completely correct answers were seen although an exact answer was required for this question. Provided a correct exact answer was seen using a correct integral, candidates obtained the full marks available even if the exact answer was subsequently evaluated.

(c) Most candidates recognised that a subtraction of the answers obtained in the two previous parts of the question was needed.

(d) Few candidates were able to give a correct answer together with a correct reason. Reasons usually concerned numerical accuracy rather than the shape of the curve and relative trapezia.

Question 7

(a) Most candidates were able to use the factor theorem correctly and obtain a correct answer.

(b) Correct factorisation was usually seen. Most candidates were able to divide by $x - 3$ either algebraically or by synthetic division in order to obtain the correct quadratic factor. Most continued to obtain the correct three linear factors.

(c) Most candidates recognised the connection between the demand of the question and the work done in the previous part, as implied by the use of the word ‘Hence’. Many then proceeded to work, incorrectly, with logarithmic functions rather than recognise a potential quadratic equation.

MATHEMATICS

Paper 9709/22
Pure Mathematics 2

Key messages

It is essential that a question is read carefully so that its demands are understood. Candidates should also check that their answer is given in the required form and, if not exact, to the required level of accuracy.

General comments

Some candidates did not appear to have a full understanding of the syllabus requirements as many questions were not attempted. However, the candidates who attempted all the questions demonstrated a reasonable understanding of the syllabus.

Comments on specific questions

Question 1

(a) Most candidates were able to use one relevant logarithm property correctly, with many obtaining a correct solution. Some candidates made the error of writing $\ln(2+x)-\ln x$ as $\frac{\ln(2+x)}{\ln x}$, rather than the correct $\ln\frac{2+x}{x}$.

(b) It is important that candidates took notice of the word 'Hence' in this question part. Too many candidates attempted to start the question again not realising that x in the equation in **part (a)** had been replaced by $\cot y$ in **part (b)**. Of those candidates who wrote down $\cot y = \frac{1}{4}$, some had difficulty in solving this equation. Candidates could be guided by the mark allocation. In this case the question was 2 marks which suggests less work is needed than for **part (a)** which was 3 marks.

Question 2

Very few correct solutions were seen. Candidates should ensure that they read the question carefully to ensure they take the necessary steps for solving. It appeared that many candidates did not read the question carefully as the first step taken was to square the terms in the expression $|3a-1|+|7b-1|$. No attempt was made to find the values of *a* and b first.

Of the candidates who did consider the equation $5|x| = 5-2x$, many did not deal with the modulus involving one term only. Many candidates squared the $|x|$ term, but not its coefficient of 5 or $(5-2x)$. As a result, correct values for *a* and *b* were rarely seen. Candidates who did obtain values for a and b, whether correct or not, often did not take account of the fact that $a < b$ and made incorrect substitutions. After substitutions into $|3a-1|+|7b-1|$ were made, many candidates then attempted to square the numbers obtained. It is important that candidates are aware of how to deal with varied modulus function questions, as the initial response of most candidates in answering this question was to square any modulus function irrespective of other terms involved.

Question 3

Most candidates were able expand at least one of the trigonometric functions correctly. There were sometimes sign errors in the expansion of $\cos(2\theta + 60°)$. Candidates are reminded to familiarise themselves with and utilise the MF19 formula book as many helpful formulae are provided, which included the formulae for these expansions, however these went unnoticed by many candidates. Common errors tended to be in the simplification of the resulting terms in order to obtain an equation in terms of $\tan 2\theta$. Occasionally, the multiple angle was omitted. The candidates that obtained a correct equation in terms of $\tan 2\theta$ very often only gave one solution, not considering that there was another solution in the given range.

Question 4

(a) Many correct responses were seen, showing a good understanding of integration involving exponential functions. Some candidates did not gain the final accuracy mark if they had not given an exact answer in their solution, as requested by the question. This highlights the need to ensure that final answers are given in the required form.

(b) Few completely correct responses were seen as some candidates did not make use of the double angle formula and write $\sin^2 2x$ as $\frac{1-\cos 4x}{2}$. Most candidates realised that $\tan^2 x$ needed to be written as $\sec^2 x - 1$ so that integration could take place.

Question 5

(a) Most candidates made a reasonable attempt at algebraic long division in order to obtain the quotient. It was essential that the divisor was $x^2 - 4x + 4$. The process of algebraic long division was the most successful when the expression was written as $x^4 + 0x^3 + 0x^2 - 32x + 55$. Methods involving the formation of an identity were less common but equally successful.

(b) Most candidates had expressions containing $(x-2)^2$ and their quotient from **part (a)**. Many candidates assumed that this quotient could be factorised and went on to produce an incorrect expression involving 4 linear factors.

(c) Most candidates realised that the *x* term from **part (b)** needed to be replaced by e^{-3y}. Fewer deduced that there was only one possible solution of $e^{-3y} = 2$, with some candidates considering either solutions from incorrect responses to **part (b)** as well. For this question it was necessary to give an exact answer, with some candidates giving their answers in decimal form.

Question 6

(a) Few correct solutions were seen. Common errors occurred with the differentiation of the term $(\ln x)^2$ in the given equation. It was essential that candidates made use of the chain rule, but many made incorrect use of the power rule for logarithms. Errors also occurred when candidates attempted to find the coordinates of the points *A* and *B*, with the term $(\ln x)^2$ again causing problems. Many candidates divided through by $\ln x$ and were only able to find the solution $x = e^2$.

(b) Few correct solutions were seen as most candidates had obtained an incorrect expression for $\frac{dy}{dx}$ in **part (a)**, although most equated their $\frac{dy}{dx}$ to zero and attempted to solve their incorrect equation.

Question 7

(a) A mark allocation of 1 mark and not much answer space should have alerted candidates that not too much work needed to be done. Many candidates did not obtain this mark as they did make use of $y = 6t\sin 2t$, replacing *y* by 3 and *t* by *p*, and then rearranging the resulting equation.

(b) Very few correct methods of solution were seen. Many candidates realised that they were looking for a change of sign when considering the value of a function when the two given values were substituted. Most candidates considered the function needed to be $\frac{1}{2\sin 2p}$ rather than $p - \frac{1}{2\sin 2p}$ or equivalent. Other methods are acceptable provided a full explanation is given.

(c) Most candidates who attempted this question part were able to obtain a correct result to the required accuracy. Errors commonly occurred when candidates had their calculator in degree mode rather than radian mode. The fact that there was no apparent convergence to a value should alert candidates to check their solutions and their calculator mode.

(d) Most candidates were able to find $\frac{\mathrm{d}x}{\mathrm{d}t}$ correctly and recognise that to obtain $\frac{\mathrm{d}y}{\mathrm{d}t}$, differentiation of a product was needed. Some errors occurred when the coefficient of the derivative of $\sin 2t$ was incorrect. Most candidates obtained $\frac{\mathrm{d}y}{\mathrm{d}x}$ using their $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ correctly, but few correct answers were seen due to arithmetic slips in the substitution of a correct answer to **part (c)**.

MATHEMATICS

Paper 9709/23
Pure Mathematics 2

Key messages

It is essential that a question is read carefully so that its demands are understood. Candidates should also check that their answer is given in the required form and, if not exact, to the required level of accuracy.

General comments

Some candidates did not appear to have a full understanding of the syllabus requirements as many questions were not attempted. However, the candidates who attempted all the questions demonstrated a reasonable understanding of the syllabus.

Comments on specific questions

Question 1

(a) Most candidates were able to use one relevant logarithm property correctly, with many obtaining a correct solution. Some candidates made the error of writing $\ln(2+x)-\ln x$ as $\frac{\ln(2+x)}{\ln x}$, rather than the correct $\ln\frac{2+x}{x}$.

(b) It is important that candidates took notice of the word 'Hence' in this question part. Too many candidates attempted to start the question again not realising that x in the equation in **part (a)** had been replaced by $\cot y$ in **part (b)**. Of those candidates who wrote down $\cot y = \frac{1}{4}$, some had difficulty in solving this equation. Candidates could be guided by the mark allocation. In this case the question was 2 marks which suggests less work is needed than for **part (a)** which was 3 marks.

Question 2

Very few correct solutions were seen. Candidates should ensure that they read the question carefully to ensure they take the necessary steps for solving. It appeared that many candidates did not read the question carefully as the first step taken was to square the terms in the expression $|3a-1|+|7b-1|$. No attempt was made to find the values of *a* and b first.

Of the candidates who did consider the equation $5|x| = 5-2x$, many did not deal with the modulus involving one term only. Many candidates squared the $|x|$ term, but not its coefficient of 5 or $(5-2x)$. As a result, correct values for *a* and *b* were rarely seen. Candidates who did obtain values for a and b, whether correct or not, often did not take account of the fact that $a < b$ and made incorrect substitutions. After substitutions into $|3a-1|+|7b-1|$ were made, many candidates then attempted to square the numbers obtained. It is important that candidates are aware of how to deal with varied modulus function questions, as the initial response of most candidates in answering this question was to square any modulus function irrespective of other terms involved.

Question 3

Most candidates were able expand at least one of the trigonometric functions correctly. There were sometimes sign errors in the expansion of $\cos(2\theta + 60^\circ)$. Candidates are reminded to familiarise themselves with and utilise the MF19 formula book as many helpful formulae are provided, which included the formulae for these expansions, however these went unnoticed by many candidates. Common errors tended to be in the simplification of the resulting terms in order to obtain an equation in terms of $\tan 2\theta$. Occasionally, the multiple angle was omitted. The candidates that obtained a correct equation in terms of $\tan 2\theta$ very often only gave one solution, not considering that there was another solution in the given range.

Question 4

(a) Many correct responses were seen, showing a good understanding of integration involving exponential functions. Some candidates did not gain the final accuracy mark if they had not given an exact answer in their solution, as requested by the question. This highlights the need to ensure that final answers are given in the required form.

(b) Few completely correct responses were seen as some candidates did not make use of the double angle formula and write $\sin^2 2x$ as $\frac{1-\cos 4x}{2}$. Most candidates realised that $\tan^2 x$ needed to be written as $\sec^2 x - 1$ so that integration could take place.

Question 5

(a) Most candidates made a reasonable attempt at algebraic long division in order to obtain the quotient. It was essential that the divisor was $x^2 - 4x + 4$. The process of algebraic long division was the most successful when the expression was written as $x^4 + 0x^3 + 0x^2 - 32x + 55$. Methods involving the formation of an identity were less common but equally successful.

(b) Most candidates had expressions containing $(x-2)^2$ and their quotient from **part (a)**. Many candidates assumed that this quotient could be factorised and went on to produce an incorrect expression involving 4 linear factors.

(c) Most candidates realised that the *x* term from **part (b)** needed to be replaced by e^{-3y}. Fewer deduced that there was only one possible solution of $e^{-3y} = 2$, with some candidates considering either solutions from incorrect responses to **part (b)** as well. For this question it was necessary to give an exact answer, with some candidates giving their answers in decimal form.

Question 6

(a) Few correct solutions were seen. Common errors occurred with the differentiation of the term $(\ln x)^2$ in the given equation. It was essential that candidates made use of the chain rule, but many made incorrect use of the power rule for logarithms. Errors also occurred when candidates attempted to find the coordinates of the points *A* and *B*, with the term $(\ln x)^2$ again causing problems. Many candidates divided through by $\ln x$ and were only able to find the solution $x = e^2$.

(b) Few correct solutions were seen as most candidates had obtained an incorrect expression for $\frac{dy}{dx}$ in **part (a)**, although most equated their $\frac{dy}{dx}$ to zero and attempted to solve their incorrect equation.

Question 7

(a) A mark allocation of 1 mark and not much answer space should have alerted candidates that not too much work needed to be done. Many candidates did not obtain this mark as they did make use of $y = 6t\sin 2t$, replacing *y* by 3 and *t* by *p*, and then rearranging the resulting equation.

(b) Very few correct methods of solution were seen. Many candidates realised that they were looking for a change of sign when considering the value of a function when the two given values were substituted. Most candidates considered the function needed to be $\frac{1}{2\sin 2p}$ rather than $p - \frac{1}{2\sin 2p}$ or equivalent. Other methods are acceptable provided a full explanation is given.

(c) Most candidates who attempted this question part were able to obtain a correct result to the required accuracy. Errors commonly occurred when candidates had their calculator in degree mode rather than radian mode. The fact that there was no apparent convergence to a value should alert candidates to check their solutions and their calculator mode.

(d) Most candidates were able to find $\frac{\mathrm{d}x}{\mathrm{d}t}$ correctly and recognise that to obtain $\frac{\mathrm{d}y}{\mathrm{d}t}$, differentiation of a product was needed. Some errors occurred when the coefficient of the derivative of $\sin 2t$ was incorrect. Most candidates obtained $\frac{\mathrm{d}y}{\mathrm{d}x}$ using their $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ correctly, but few correct answers were seen due to arithmetic slips in the substitution of a correct answer to **part (c)**.

MATHEMATICS

Paper 9709/31
Pure Mathematics 3

General comments

There were a wide range of responses noted throughout the question paper. Some candidates demonstrated a strong understanding of the topics covered and good problem-solving skills. In many scripts, candidates offered no responses to part or all of several questions, this was particularly noted in the later questions. It was noted that some candidates struggled with basic algebraic procedures.

The questions candidates found to be most accessible were **Question 1** (modular inequality), **Question 6(a)** (differentiation of a parametric function), 7(c) (applying an iterative process), **Question 8(a)** (angle between two vectors), **Question 9(a)** (differentiation of a product), **Question 9(b)** (integration by parts), and **Question 10** (differential equation). Several candidates offered a solution to **Question 2** (quadratic equation in e^x), but very few had a correct approach. The question that commonly had no response was **Question 5** (complex numbers).

Key messages for candidates

- Candidates must ensure that they show clear working throughout the question paper.
- Candidates should ensure they have sufficient subject knowledge as they should be prepared to answer questions on all topics on the syllabus.
- Candidates are reminded to write clearly and do not overwrite one solution with another as this can be very difficult to read.
- Candidates should check their algebra and arithmetic carefully, and ensure they use brackets correctly.
- If a question asks to obtain a given answer then take particular care to show full working.
- If a question asks for an exact answer then decimal working is not appropriate.
- Double checking answers to ensure it fits with the context of the question and what the question is asking is advised.

Comments on specific questions

Question 1

Most candidates showed an understanding of how to start on the solution to this question, and many obtained the correct answer. For those candidates who attempted to remove the modulus signs by squaring both sides of the inequality, the most common error was not squaring the 2. For candidates attempting to form two linear inequalities, the most common errors were sign errors and slips in the algebra. Candidates who obtained the correct critical values usually went on to obtain the correct answer.

Question 2

Several candidates multiplied both sides of the equation by $2+e^x$, but very few recognised the result as a quadratic equation in e^x. The most common incorrect approach was to try to express the logarithm of a sum of terms as the sum of the logarithms of the individual terms.

The best responses gave a clear explanation of why the negative root of the quadratic did not give a root. The conclusion ‘math error’ with no further comment was not sufficient.

Question 3

(a) There were several fully correct responses to this question, with candidates able to quote and use the basic trigonometric formulae. There were some sign errors in quoting the formulae, but the most common error was to use an incorrect formula such as $\cos(x-30°)=\cos x-\cos 30°$. A few candidates were not able to use an expression in $\sin x$ and $\cos x$ to find a value for $\tan x$.

(b) The majority of candidates recognised the link between the two parts of the question, and many obtained the solution $x=-6.2°$. Some candidates stopped at that point and did not go on to state a solution in the required range. Some candidates stated just one of the possible solutions. Those candidates who worked directly from the equation given in the question, rather than link it to **part (a)** generally did not make any progress.

Question 4

(a) Those candidates who used $1-\cos 2\theta=2\sin^2\theta$ and $1+\cos 2\theta=2\cos^2\theta$ proved the identity very quickly. Candidates who omitted and stated equations such as $1-\cos 2\theta=1-1-2\sin^2\theta$ frequently made sign errors at the next step, making them unable to gain the method mark and the accuracy mark by implying the use of an incorrect formula. Candidates are reminded to ensure their work is clearly presented throughout and include essential brackets in places required and used.

(b) Many candidates found this definite integration question challenging. Those who did not use the result from **part (a)** often made little progress. Only a minority of the candidates who used $\int\tan^2\theta\,\mathrm{d}\theta$ processed this correctly.

Question 5

A high proportion of candidates offered no response to any part of this question.

(a) Candidates were expected to the use of the quadratic formula to find the roots of the quadratic equation in *z*. Candidates who did this often made slips in simplifying $\sqrt{(-2p\mathrm{i})^2+4q}$, common errors being $\sqrt{4p^2+4q}$ and $\sqrt{-4p+4q}$.

(b) Very few candidates recognised that for the points to lie on the imaginary axis the discriminant of the quadratic needed to be negative.

(c) Some candidates stated the conditions for the triangle to be equilateral, but very few were able to apply these correctly to their values of *z*.

Question 6

(a) The majority of candidates started by trying to find $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$. The expression for $\frac{\mathrm{d}y}{\mathrm{d}t}$ was often correct, but the 3 was frequently missing from the numerator in $\frac{\mathrm{d}x}{\mathrm{d}t}$. Most candidates knew how to use these two derivatives to obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. Some candidates understood that the range of values for *t* was limited due to the use of $\ln(2+3t)$, and they were able to use this to explain why the gradient of the curve is always positive.

(b) The candidates who started by using the fact that $x=0$ on the *y* axis were often able to use this to find the values for *t*, *y* and the gradient of the curve and form the equation of the tangent. Some candidates did not seem to know where to start in solving the problem.

Question 7

(a) The stronger responses worked from the fact that *M* is described as the maximum point and started by differentiating the function. The application of the quotient rule was often correct. There were some slips in the algebra after equating the derivative to zero, and some long routes were followed, but several candidates did reach the given answer correctly.

(b) The formula given in **part (a)** lends itself to considering the solution of $\mathrm{f}(x) = x$ and several candidates were successful in following this route. Some candidates preferred to rearrange the formula to consider $\mathrm{f}(x) = 0.$ There was evidence of confusion between the two options with several candidates using the former pattern and then expecting a sign change. Several candidates scored no marks because they were working in degrees, or because the values they stated were not recognised and they had not said how they had been obtained.

(c) The process of using an iterative formula was well understood, and those candidates who were working in radians usually reached a correct conclusion.

Question 8

(a) Some candidates did not use the correct vectors to find the required angle, but the direction vector of the line was often identified correctly. The process of using the scalar product was understood by those candidates who attempted to use it.

(b) Very few candidates made any progress with this question. Some got as far as an expression for $\overrightarrow{AP}$ or $\overrightarrow{BP}$, but it was unusual to see an expression equating the lengths of these two vectors.

Question 9

(a) Several candidates applied the product rule correctly to obtain a correct expression for the gradient of the curve. Most went on to try to find the *x* coordinate at the stationary point, but some made errors in dealing with the indices. Using their value for *x* to obtain the value for *y* proved to be more difficult, but there were several fully correct solutions.

(b) Most of the candidates who attempted this question recognised the need to use integration by parts. This was often completed correctly, but there were some errors in the coefficients. The limits were usually used correctly. Some candidates did not explain why $6\ln 8 = 18\ln 2;$ an explanation was expected as they were working towards a given answer.

Question 10

Although the step to separate the variables was not always seen, the instruction to use partial fractions led many candidates to work on splitting $\dfrac{1}{x^2(1+2x)}.$ A correct form for the partial fractions was often used, and several candidates earned the marks for correct completion of the partial fractions. Some candidates gave incorrect coefficients with no indication of how these had been obtained, so the only mark available to them was for using the correct form for the partial fractions.

In the integration, several candidates used the correct forms for $\int \frac{1}{x}\,\mathrm{d}x$ and $\int \frac{1}{1+2x}\,\mathrm{d}x$, but they had difficulty with $\int \frac{1}{x^2}\,\mathrm{d}x.$ Those candidates who completed the integration correctly usually went on to use $x = 1$ when $t = 0$ to obtain a correct expression for *t* in terms of *x*.

MATHEMATICS

Paper 9709/32
Pure Mathematics 3

General comments

The candidate responses to this paper covered the full range of marks available, with some excellent responses noted. Many candidates offered responses to all eleven questions. Overall candidates did well in more routine questions such as **Question 1** (modular inequality), **Question 5** (square root of a complex number), **Question 9(a)** (partial fractions) and **Question 10(d)** (application of an iterative formula). The questions that proved to be most challenging were **Question 4** (use of integration by parts to integrate $\tan^{-1}\left(\frac{x}{2}\right)$), **Question 8** (stationary points of a trigonometric function), **Question 10(a)** (area of a trapezium) and **Question 11(b)** (lengths of vectors).

Several candidates appeared to miss certain aspects of the questions. **Questions 2**, **4** and **5** were quite often omitted or solutions were attempted that indicated that candidates had little understanding of the topic.

There were many instances of responses that were difficult to read because the candidate had written over their original response, candidates are reminded to ensure their working is presented clearly throughout the question paper, paying particular attention to the clarity of signs and numerals.

Key messages for candidates

- Candidates must ensure that they show clear working throughout the question paper.
- Candidates should ensure they have sufficient subject knowledge as they should be prepared to answer questions on all topics on the syllabus.
- Candidates are reminded to write clearly and do not overwrite one solution with another as this can be very difficult to read.
- Candidates should check their algebra and arithmetic carefully, and ensure they use brackets correctly.
- If a question asks to obtain a given answer then take particular care to show full working.
- If a question asks for an exact answer then decimal working is not appropriate.
- Double checking answers to ensure it fits with the context of the question and what the question is asking is advised.

Comments on specific questions

Question 1

The majority of candidates started by squaring both sides of the inequality to remove the modulus signs. The most common error at this point was to forget to square the 3. There were several errors in the algebra and the arithmetic, but many candidates obtained the correct quadratic and the correct critical values. The most common incorrect answers were the impossible answer '$x < -4$ and $x > -\frac{2}{5}$' and the incorrect statement $-\frac{2}{5} < x < -4$. Those candidates who used a pair of linear equations or inequalities were often successful. There were some slips in the arithmetic and also several candidates thinking that $|2x-1| = 2x+1$.

Question 2

The standard of the diagrams offered was variable. Candidates are not expected to use accurate constructions, but at the same time, many diagrams would have benefitted from the use of a ruler or other straight edge. Most candidates drew a circle of radius 1, usually with the centre in the correct place. Several circles strayed into the first and third quadrants: this could be avoided if candidates drew the circle first then drew the axes. The half-line proved to be more challenging, with several candidates thinking that the line needed to pass through the centre of the circle. Candidates should be aware that it is much easier to draw the loci correctly if they use the same scales on both axes.

Question 3

(a) Many candidates dealt with the logs correctly, to earn the first mark. The question asks for a result involving $\ln x$, so working with logarithms to base 3 was not helpful unless accompanied by the correct process for the change of base of logarithms. The second mark was more challenging, with some candidates offering no explanation of the straight line. Others quoted $y = mx + c$ without making sure that their equation was in a comparable form. The gradient was often stated incorrectly: $-\frac{\ln x}{\ln 3}$ was a common incorrect answer. The question asked for an exact value, but several candidates offered decimal approximations.

(b) Many candidates showed confusion between whether it was $X\left(=\ln x\right)$ or x that was being set to 0. A large number used the impossible $\ln(0)$ in their working. The majority of seemingly correct answers were fortuitous, coming from equating $\ln(0)$ to 0. Candidates who realised that they just needed to use the y-intercept were in the minority.

Question 4

This was a challenging question for many, with candidates not knowing how to use the given formulae or mistaking $\tan^{-1}\frac{x}{2}$ for $\cot\frac{x}{2}$. Some candidates seemed unaware of the use of ‘1’ to create parts ready for integration, with attempts involving $u = \tan^{-1}$ and $\frac{dv}{dx} = \frac{1}{2}x$ being used. Those candidates who did realise what was required often made errors with the coefficients when differentiating. Of the candidates who completed the first stage of integration by parts correctly, about half recognised that the integral now involved a logarithm. When substituting limits, candidates need to be aware that the use of degrees is not appropriate when integrating. Several candidates equated $2\tan^{-1}1$ to 90.

Question 5

It appeared that many candidates were not familiar with the procedure for finding the square roots of a complex number. Some simply offered no response, or there were weak attempts involving multiplication by the complex conjugate or involving the squaring of u.

Most successful responses squared $a + bi$ and compared real and imaginary coefficients. A small number incorrectly used $i^2 = 1$. Many candidates made sign errors when comparing coefficients. Of those who earned the first 4 marks, not all then went on to match up the pairs of values required for a correct final answer.

A small minority of candidates attempted the alternative route of using the modulus and argument. They often obtained a correct value for the modulus of the square root but did not have an exact method for dealing with the argument.

Some candidates were successful in solving the simultaneous equations $a^2 - b^2 = 10$ and $a^2 + b^2 = 14$.

Question 6

(a) The level of success with this part of the question depended on the route adopted. A small number of candidates used the incorrect statement $\operatorname{cosec}2\theta = \frac{1}{\cos 2\theta}$. Those candidates who started by stating $\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}$ often provided clear and concise proofs. Brackets were not always used when required; for example, the incorrect expression $\frac{1-1-2\sin^2\theta}{2\sin\theta\cos\theta}$ was common, and often led to an incorrect conclusion. Some of the candidates with this first step did not appreciate the advantage gained by the two denominators being the same. Their next step was $\frac{\sin 2\theta - \sin 2\theta\cos 2\theta}{\sin^2 2\theta}$, leading to more complicated work and offering the potential for errors. Many candidates had a first step of $\frac{1}{\sin 2\theta}-\frac{1}{\tan 2\theta}$ leading to $\frac{1}{2\sin\theta\cos\theta}-\frac{1-\tan^2\theta}{2\tan\theta}$. Eventual success did occur for some of these candidates but only after lengthy working.

(b) Many candidates recognised the integral and provided detailed steps showing that $-\ln\frac{1}{2}+\ln\frac{1}{\sqrt{2}}$ is equal to $\frac{1}{2}\ln 2$. Most errors were due to sign errors in the integration, or to the use of incorrect values for the exact trigonometric ratios. Some candidates used decimal approximations and consequently lost accuracy marks.

Those candidates who did not recognise the integral as a standard form often used substitution, which was usually successful if they substituted $u=\cos\theta$.

Question 7

Many candidates with a correct first step of $\frac{dy}{dx}=\frac{ky}{(x+1)^{\frac{1}{2}}}$ followed by sensible separation of the variables, were able to work through successfully without much difficulty. Those candidates who moved *k* to the left-hand side of the equation before integrating were not quite as successful because they often made errors in the coefficient for that integral. Some candidates assumed $k=1$. They earned some credit for making progress with the differential equation $\frac{dy}{dx}=\frac{y}{(x+1)^{\frac{1}{2}}}$. Other candidates thought that the two given points, $(0, 1)$ and $(3, e)$ needed to be used at the outset in some way. Some candidates stated that $k=\frac{e-1}{3-0}$. Candidates reaching the correct penultimate step of $\ln y=\sqrt{x+1}-1$ did not always go on to express *y* in terms of *x*.

Question 8

Most candidates were able to show the correct use of the product rule. Some candidates chose to use the quotient rule on $\frac{\tan^2 x}{e^{5x}}$. The main difficulties were due to sign errors in the formula used and incorrect attempts to differentiate $\tan^2 x$. Rather than differentiate $\tan^2 x$ as a function of a function, several candidates worked all the way through differentiating $\frac{\sin^2 x}{\cos^2 x}$ using the quotient rule. Many candidates retained the exponential terms long after they needed to, and they were slow to factorise in general, leading to many lines of cumbersome work. Candidates were often too quick to divide through by $\tan x$ and often overlooked the solution $x=0$. Those candidates with a correct derivative who used the correct relationship between $\sec^2 x$ and $\tan^2 x$ usually went on to earn most of the remaining marks. Those who expressed the derivative in terms of $\sin x$ and $\cos x$ often got as far as $5\sin x\cos x=2$ but did not continue. There were only a few instances of extra solutions being offered, usually −0.464 and −1.107. A few candidates used inappropriate substitutions such as $\tan x = x$ when trying to solve their equations and in turn confused themselves.

Question 9

(a) Almost all candidates stated the correct partial fractions structure and used a correct method for finding the coefficients. Several candidates made errors due to the omission of brackets, with $Bx+C(2+x)$ used in place of $(Bx+C)(2+x)$.

(b) Candidates need to be careful in transcribing results from one part of a question to another. Many candidates demonstrated a good understanding of how to expand the terms, but accuracy marks were often not able to be awarded due to the use of $-(2x+1)$ in place of $(-2x+1)$.

Question 10

(a) Some candidates offered no attempt at this part of the question. Candidates who found *DC* correctly generally went on to earn full marks provided they kept a clear head in setting up their equation. There were several attempts to work back from the given answer, but these were not successful.

(b) The majority of candidates were able to calculate the values of a relevant expression using radians. Candidates working with $\mathrm{f}(x)=0,$ who obtained answers +/– 0.016, were more likely to be able to complete the argument correctly. Some candidates working with $\mathrm{f}(x)=x,$ who obtained answers 0.484 and 0.716, gave incorrect statements because they were looking for positive and negative signs, while they were supposed to compare their values with 0.5 and 0.7 and recognise the change of direction of inequality signs.

(c) There were many fully correct solutions to this problem. Some candidates appeared confused about what was expected and retained the subscripts throughout their working. Some candidates completed the iteration in **parts (c)**, with several realising their error and repeating the iteration in **part (d)**.

(d) Those candidates who worked in radians frequently gave a fully correct solution. For those candidates who did not use the information from **part (b)**, the common starting points were 1, $\frac{\pi}{4}$ and $\frac{\pi}{2}$. Some candidates who used a starting point of 0.5 or 0.7 often stopped the iterative process prematurely. In contrast, some candidates continued until successive iterations gave the same result to 4 or more decimal places before drawing their conclusion.

Question 11

(a) Some candidates did not appear familiar with the notation used in the question. Most could find the lengths of *OA* and *OB* for the scalar product work, but several did not realise what was required for the first mark. Some candidates with correct work in the scalar product, including a negative value for the cosine of the angle, gave the final answer as an acute angle.

(b) Very few candidates made any progress with this part of the question. Very few candidates were aware of how to find the position vector of *M*, by far the most common assumption was $\overrightarrow{OM}=\frac{1}{2}\overrightarrow{AB}$. Some candidates did use their $\overrightarrow{OM}$ to form an expression for $\overrightarrow{OP}$, but few went on to find $\overrightarrow{AP}$ or to write down a correct expression connecting the lengths. A small number of candidates had some success using the alternative methods involving cosine rule or right-angled triangles.

MATHEMATICS

Paper 9709/33
Pure Mathematics 3

Key messages

Candidates need to be aware of what is required for sketching graphs, details of this are given in **Question 6(a)**. Candidates should also ensure that include sufficient detail and clear method is justifying a statement or proving a given result, as explained in **Question 7(a)**. An understanding of vector equations of a line is expected, therefore candidates should ensure they are familiar with this, as was required for **Question 9(b)**.

Candidates are reminded that calculators cannot be used in complex number questions since they need to show working to justify their result. This follows the rubric on the front of the question paper and was important for **Question 10(a)** and **(b)**.

General comments

The standard of work on this paper was high, with a considerable number of candidates performing well on many of the questions. It was pleasing to see that candidates were generally aware of the need to show sufficient working in their solutions, especially in **Question 2** and **Question 10(a)** and **(b)**.

Candidates should be reminded to set out their work clearly to ensure it is legible.

Comments on specific questions

Question 1

Most candidates produced fully correct answers. Some errors were seen, for example $3x$ was replaced by x, terms were written in descending order rather than ascending order, or the correct answer was multiplied throughout by 3 to remove fractions.

Question 2

Many candidates took ln of both sides of the given equation, applied a number of log laws incorrectly then solved their equation, which could not gain any credit. More successful responses first converted the given equation into a quadratic equation for 4^x, or substituted another variable, for example $u = 4^x$.

Question 3

(a) Many candidates did not simplify their answer, for example leaving the exponential terms within a bracket, hence could not be awarded the final accuracy mark. A common error involved differentiating the t term within the equation for x as 0, not 1. Other candidates made errors in applying the product rule when differentiating y. Multiplying out the expression for y before differentiating was less efficient because it created two terms on which the product rule needed to be used.

(b) If the answer in **part (a)** had already been factorised then very little work was required. Most candidates used their factors or solved a quadratic equation. Some could not be awarded the second mark, for example because they also gave an answer for $t = -2$, which should have been rejected. Some candidates gave a decimal answer instead of an answer in exact form.

Question 4

(a) This question was well answered by the majority of candidates. Only a few mixed up their coefficients of the partial fractions.

(b) This part of the question proved testing for a few candidates, as both integrals required ln terms with use of the chain rule in reverse. Errors were seen in the coefficients of either or both terms and in applying log laws to manipulate the answer into the correct form.

Question 5

(a) Most candidates were able to apply the double angle formula to both $\tan 4\theta$ and $\tan 2\theta$ so could be awarded the first two method marks. Few candidates spotted that the solution became much easier to handle if they divided by the $\tan\theta$ term throughout as soon as they reached an equation solely in terms of $\tan\theta$. Many candidates squared the expansion of $\tan 2\theta$ incorrectly when expressed in terms of $\tan\theta$ or made errors later in rearranging their equation into the correct form.

(b) Most candidates found the angle in the first quadrant and were awarded the first two marks. However, a large number of candidates did not obtain a second value by finding an angle in the required domain from the negative value of $\tan\theta$. Some answers were seen without supporting working so, if incorrect, could not be awarded the method mark. Candidates are reminded to include all necessary working to support their answers.

Question 6

(a) Many candidates' attempts to sketch both graphs showed they did not quite understand the properties of each type of graph and its transformations. For the exponential graph $y = 1 + e^{-x}$, few candidates included the asymptote at $y = 1$ and some graphs approached the x-axis. It was important to mark the y-intercept of 2 clearly on their graph. For $y = \cot\left(\frac{x}{2}\right)$, it was common to see the graph terminating before reaching the point $(\pi, 0)$ or a small distance above it. However, this point needed to be clearly specified to demonstrate understanding of the stretch to the cot graph. The curvature of this graph was important, as was its behaviour for small values of x.

In this type of question, candidates need to either mark the point of intersection of their graphs or to state that there is a point of intersection. Either of these would show that they know the point corresponds to the root.

(b) This question was generally answered well, with many candidates gaining full marks. It was important to ensure that their calculator was in radian mode and to check arithmetical calculations carefully. Candidates chose to use a variety of functions, for example $y = \cot\left(\frac{x}{2}\right) - 1 - e^{-x}$ and $y = 0$; $y = \cot\left(\frac{x}{2}\right)$ and $y = 1 + e^{-x}$; $y = 2\tan^{-1}\left(\frac{1}{(1 + e^{-x})}\right)$ and $y = x$; $y = \ln\left(\cot\left(\left(\frac{x}{2}\right) - 1\right)\right)$ and $y = x$.

(c) Almost all candidates produced fully correct answers and were awarded all three marks.

Question 7

(a) (i) Very few candidates were able to justify the statement, with most simply writing the given expression. It was important to produce a short argument that was rigorous and correct. Here are three possible ways of answering this question:
Using a geometrical method: take angle α as the angle between the tangent and the horizontal and state angle α as MPN or show on the diagram. Then $\tan a = \frac{dy}{dx}$ and $\tan a = \frac{MN}{y}$ leading to the given answer. An alternative similar argument refers to similar triangles instead of angles.

Using a calculus method: state $\frac{dy}{dx}$ = gradient of tangent = $-\frac{1}{(\text{gradient of normal})}$. Use gradient of the normal = $-\frac{y}{MN}$ leading to the given answer.
Or, using a coordinate geometry method:
gradient of normal = $\frac{(0-y)}{(x_N - x_M)} = -\frac{y}{MN}$ then use $\frac{dy}{dx} = -\frac{1}{(\text{gradient of normal})}$, leading to the given answer.

(ii) The proof of this expression required these clear statements: area of *PMN* = $\left(\frac{1}{2}\right)$. *PM.MN* = tan*x*, combined with *PM* = *y* and *MN* = *y*. $\frac{dy}{dx}$ leading to the given answer.

(b) Many clear and strong solutions that were fully correct were seen. A sign error when integrating tan*x* was fairly common, as was integrating tan*x* to $\sec^2 x$.

Question 8

(a) Many excellent solutions were seen to this question part. A few candidates made errors with the quotient rule, for instance an incorrect sign in the numerator, terms reversed in the numerator or a missing denominator. The question required exact coordinates for *x* and *y* so candidates needed to know how to solve $\ln x = \frac{1}{4}$ without using their calculator.

(b) Many candidates demonstrated excellent use of integration by parts although the conclusion proved challenging for most candidates. Some candidates chose a numerical value for the constant *a* while others considered the function as *a* tended to infinity. Sign errors and coefficient errors were also seen on occasion.

Question 9

(a) Most candidates offered fully correct solutions for this question. Besides a few arithmetical slips there were some candidates who made an error in the direction of one of their vectors, for example using ***DC*** instead of ***CD***.

(b) Many candidates did not give their answer in the correct form as a vector equation as required by the question. The right-hand side should have been equated to **r**, the position vector of a point on the line *AB.*

(c) There were many ways of solving this problem and any correct solution could be awarded all available marks. There were some candidates who did not find both the perpendicular distance and the area of the trapezium. Here is a selection of the main approaches taken by candidates:

1. Finding the vector from a general point on one line to either of the given points on the other line then establishing when this is perpendicular to ***AB*** or ***CD***. This was done in a variety of ways: by finding the scalar product of this vector with ***AB*** or ***CD***, or by minimising the length of the vector via calculus or by completing the square of the quadratic expression.
2. Using the scalar product to project the vector between one of the given points on *AB* and one of the given points on *CD* onto one of those two parallel lines. Then it was straightforward to find the required distance by Pythagoras.
3. Using the vector product to find a perpendicular vector followed by its magnitude.
4. Use of the cosine rule to find one of the angles in the trapezium.
5. Scalar product rule to find one of the angles in the trapezium.
6. Showing lengths *AD* and *BC* are equal so the trapezium is symmetric.
7. Establishing the length of sides of the two triangles and using Heron's formula to find their areas. The combined area could then be used in formula for the area of the trapezium to find the perpendicular distance.
8. Finding vectors from the origin to a general point on each line. Then scalar products of these vectors with direction vector of *AB* identified points on the lines which were in the plane

perpendicular to the two parallel lines and contained the origin. Subtraction of these vectors led to the required perpendicular distance.

Question 10

(a) Most candidates gave fully correct responses to this question, however some candidates attempted to factorise the given expression into quadratic factors in order to show this result, which was not necessary. All that was required was the substitution of *z* into the quartic equation then careful expansion of both z^2 and z^4. Each of these needed to consist of three or more terms to demonstrate that the expansion had not been performed on the calculator.

(b) Some candidates gave the three correct roots with no working; it is expected that candidates show their clear working, as stated on the front of the question paper that 'no marks will be given for unsupported answers from a calculator.' As a result, these candidates were only able to gain one mark as one of the roots could be derived from their answer to **part (a)**. Most successful answers started by using factors, derived from the root given in **(a)** and its conjugate, to produce a quadratic factor. The other quadratic factor was then obtained by long division or by inspection. Slight variations of this approach using product and sum of roots were often seen. Several candidates did not find the complex roots from their second quadratic. Sign errors were also fairly common throughout candidates' solutions.

MATHEMATICS

Paper 9709/41
Mechanics

Key messages

- Non-exact numerical answers are required correct to 3 significant figures as stated on the question paper and cases where this was most frequently not adhered to were seen in **Question 1**, and **Question 6**. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation or a work/energy equation. Such a diagram would be particularly useful in **Question 1, Question 2, Question 6 and Question 7**.
- In questions such as **Question 5** in this paper, where acceleration is given as a function of time, it is important to identify that calculus must be used and that it is not possible to apply the equations of constant acceleration.

General comments

The paper was well attempted by many candidates although a wide range of marks was seen.

The examination allowed candidates at all levels to show their knowledge of the subject, whilst differentiating well between even the stronger candidates. **Questions 1 and 4(a)** were found to be the most accessible questions whilst **Questions 5 and 7** proved to be the most challenging for candidates.

Comments on specific questions

Question 1

This question was answered well by most candidates. There were two main alternative approaches. The first involved finding the total force that needs to be applied by the winch, namely the sum of the weight component along the plane plus the resistance force. The work done was then found by multiplying this force by 5 m, the distance moved by the force. An alternative approach was to use the work-energy principle. The total work done by the winch equalled the sum of the gain in potential energy of the system plus the work done against the resistance. Common errors seen were using the wrong component when resolving the weight and not including the resistance force within the calculations.

Question 2

(a) Most candidates attempted answering this question. It was necessary to apply Newton's second law to each particle or to the system in order to set up two simultaneous equations involving the unknown values of the mass, m and the acceleration, a. Solving these equations gave the required value of m. Some candidates made errors in solving these equations and others incorrectly assumed that the acceleration was g.

(b) The value of acceleration was usually found in solving **part (a)** although some candidates found the value of a in this part. This value of a must be used here in order to find v from the equation $v^2 = u^2 + 2as$ with $u = 0$, $s = 0.9$ and a being the value found earlier. Most candidates attempted this question however some used the value of acceleration as g rather than the actual acceleration of the particle.

Question 3

(a) Most candidates attempted to answer this question. It was necessary to use the principle of conservation of linear momentum for the collision between particle *P* and particle *Q*. Many candidates wrote down the correct form of equation, however some used the wrong sign when dealing with the fact that particle *P* rebounds with a speed of $1\,\text{ms}^{-1}$ which led to an incorrect answer. Candidates should be reminded that momentum is a vector quantity; this is often the reason for this sign error.

(b) When *Q* collided with *R* it was again necessary to apply the principle of conservation of linear momentum which involved three terms, stating that the momentum of *Q* before impact is equal to the sum of the momentum of *Q* after impact and the momentum of *R* after impact. The speed of *Q* which was found in **part (a)** was needed to find the momentum of *Q* before impact. Since there were no further collisions between *P* and *Q*, the greatest speed of Q after impact must be $1\,\text{ms}^{-1}$ which was the given rebound speed of *P*. Care must be taken with the sign of this term and a diagram showing the speeds and directions would make this easier for candidates to visualise. If this value of $1\,\text{ms}^{-1}$ is used with the correct sign, then the required value of *V* could be found. Some common errors made were to assume that the speed of *Q* after collision with *R* was either zero or that *Q* and *R* coalesced after the collision.

Question 4

(a) This question was well answered by most candidates. It is necessary to find the distances travelled by the two cyclists. In the case of Isabella, the first stage of her motion involved constant acceleration and the distance travelled in metres can be found as $\frac{1}{2}\times a\times 5^2$. Her speed after 5 seconds was $5a$ and she travelled for 10 seconds at this speed; travelling a further $5a\times 10$. In the final 5 seconds she travelled the same distance as in the first 5 seconds due to the symmetry of the motion. Maria travelled 27.5 m and reached a speed of $5\,\text{ms}^{-1}$ and so in the next 10 seconds she travelled 5×10 m and in the final 5 seconds her deceleration was $1\,\text{ms}^{-2}$, so she travelled a distance $\frac{1}{2}\times 1\times 5^2$. By adding each of these stages for each cyclist it led to the total distance travelled by each of them. An alternative approach was to sketch the motion for each cyclist on a velocity-time graph which enabled the distance travelled to be found by calculating the areas under each graph. Many well-presented and strong solutions were seen for this question with the main errors seen being numerical.

(b) Candidates generally found this question part to be challenging. It was necessary to express the distance travelled by Isabella in terms of the acceleration *a*. This could have been done either by adding the three stages together or by using a single expression for area from the velocity-time graph. Once this had been done, the expression is then equated to 90 which is the distance travelled by Maria and the resulting equation can be solved for the required acceleration. Common errors were to use an incorrect approach for finding the distance travelled by Isabella.

Question 5

(a) In this question the acceleration was given as a function of time and hence constant acceleration formulae cannot be used. In order to find *v*, it was necessary to integrate the given expression for *a* with respect to *t*. Once this had been done the expression found was set equal to zero since *B* is the point at which the particle is at instantaneous rest. The equation $v = 0$ needed to be solved for *t* to achieve the required answer. Some candidates did not identify that point B referred to in the question is where $v = 0$ and a number of candidates solved $a = 0$ instead.

(b) In this part of the question it was necessary to determine when $a = 0$ and this was achieved by setting the given expression to zero and solving for *t*. The question asked for the distance travelled and so this involved finding the displacement by integrating the expression found for *v* in **part (a)** with respect to *t*. This expression needed to be evaluated between limits of $t = 0$ and $t = 9$ which is the time at which the acceleration of the particle is again zero. Candidates commonly made errors during their integration with several using differentiation instead of integration. Some candidates also did not find the correct value of *t* when the acceleration is zero.

Question 6

(a) In this question it was given that the force in the *x*-direction is zero and so it was necessary to resolve all three forces parallel to the *x*-direction and set this expression to zero. The expression was an equation involving $\cos\alpha$ and could be solved to give the required value of α. Almost all candidates found this value correctly. Once the value of α has been determined, the resultant force was found by resolving the three forces in the *y*-direction which gives the required resultant. A common error seen was to use the wrong components when resolving forces in either direction. Overall, most candidates performed well on this question.

(b) In this part of the question a new value of α was given. It was necessary to resolve forces in two perpendicular directions, and here it was best to use the *x* and *y* directions. Using this method produced the components *X* and *Y* of the system of forces in the *x* and *y* directions respectively. Once these values are found, use of Pythagoras gave the magnitude of the resultant force and trigonometry can be used to determine the direction of this resultant. Most candidates made a good attempt to find *X* and *Y* but often did not continue past this point. Some candidates made a good attempt to find the magnitude, but several did not express the direction sufficiently well. Candidates should ensure that the direction must be uniquely described either in words or by means of a vector diagram.

Question 7

(a) (i) This question could have been approached in two different ways. One method was to use energy principles for motion on the slope. An alternative approach was to apply Newton's second law for the motion. If energy principles were used then the question can be solved simply by using the fact that the potential energy lost by the child is equal to the gain in kinetic energy and this gave the required speed. If Newton's second law is used then resolving forces along the slide gave the acceleration down the slide and once this was found, use of the constant acceleration formulae led to the required speed. Most candidates who attempted this question followed one or other of these methods and some good solutions were seen. An error seen was to wrongly use *g* as the acceleration down the plane rather than a component.

(ii) There were a number of different ways that candidates could have answered this question. In all alternative methods the value for the speed found in **(a)(i)** can be used. The total kinetic energy at the bottom of the slide is all lost to friction as the child comes to rest, and so since it is given that 250 J are lost to friction for every metre travelled; the energy method gives $\frac{1}{2}\times 35\times 5^2 = 250d$ where *d* is the distance travelled along the horizontal section. This equation can readily be solved for *d*. Whilst many different approaches were seen, a common error seen was to use $\frac{250}{d}$ rather than $250d$ when using this equation.

(b) Again, there are several approaches that can be taken to solve this question, similar to those in **Question 7(a)**. The method most commonly used was to apply energy principles for the motion from start to finish. The initial potential energy is found. All of this could then be used against friction on the slope and the friction on the horizontal section. If *F* is the frictional force on the slope then the energy equation takes the form $35\times g\times 2.5\sin 30 = F\times 2.5 + 250\times 1.05$. *F* can be expressed as $F = \mu\times 35g\cos 30$, where μ is the coefficient of friction between the child and the sloping section of the slide. Combining these expressions gives the required value for μ. Whilst many other approaches were seen, it was also noted that candidates found this to be one of the most challenging questions on the paper.

MATHEMATICS

Paper 9709/42
Mechanics

Key messages

- Non-exact numerical answers are required correct to 3 significant figures as stated on the question paper and cases where this was most frequently not adhered to were seen in **Question 2, Question 3, Question 4, Question 5 and Question 6**. Candidates are advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation or a work/energy equation. Such a diagram would be particularly useful in **Question 3, Question 4** and **Question 5**.
- In questions such as **Question 7** in this paper, where acceleration is given as a function of time, it is important for candidates to identify that calculus must be used and that it is not possible to apply the equations of constant acceleration.

General comments

The paper was generally very well answered by many candidates, with a wide range of marks seen.

The examination allowed candidates at all levels to show their knowledge of the subject, whilst differentiating well between even the stronger candidates. **Questions 1** and **2** were found to be the most accessible questions whilst **Question 6** and **Question 7** proved to be the most challenging for candidates.

In questions such as **Question 2** and **Question 5(b)** where the sine of an angle is given, it is not necessary to evaluate the angle and in fact by doing so may lead to approximations which could affect the accuracy.

Comments on specific questions

Question 1

Many candidates answered this question well, although a significant number made errors within their solution. When finding the potential energy some candidates used the 15 m given in the question as the height change rather than using the correct form, 15 sin 10. Some did not include the effect of the initial speed of the particle. Other candidates included the effects of all necessary terms, a potential energy term and two kinetic energy terms but used incorrect signs when combining them. Another common error was to include potential energy and work done by the weight which is essentially using the potential energy twice. The question specifically asked for the use of an energy method, but some candidates used constant acceleration formulae and hence were not able to gain all marks because of this. It is important that candidates read the question carefully and pay close attention to the instruction.

Question 2

In this question the exact values of $\sin\alpha$ and $\sin\theta$ are given so candidates do are not expected to evaluate the angles to answer it effectively. For those candidates who did evaluate the angles this led to a slight loss of accuracy. This question was generally well answered by most candidates. The most common approach to this question was to resolve forces in the direction of the 30 N force and in the direction perpendicular to the 30 N force leading to force components X and Y respectively. Almost all candidates were successful in doing this. It was then necessary to find the magnitude of the resultant and the most straightforward way of doing this is to use Pythagoras to combine the two perpendicular forces X and Y. Once the magnitude had been found the angle at which the force acts could be determined using trigonometry. Although many candidates found both the magnitude and the angle, they did not use the value of the angle to uniquely describe the direction of the resultant. There are many ways that a candidate can express this direction which can be explained either in words or by use of a suitably annotated diagram.

Question 3

This question was well answered by many candidates. Whilst an energy approach could be taken to solve this question, most candidates resolved forces and attempted to find the acceleration of the particle. By using this method, it was necessary to resolve forces perpendicular to the rod first of all in order to find the normal reaction. This reaction can be expressed as $R = 0.3g - 8\sin 10$. A common error made by candidates included replacing the minus sign with a plus sign. Another error seen was to wrongly assume that $R = 0.3g$. Once this reaction had been found the equation of motion along the rod could be expressed by using Newton's second law as $8\cos 10 - F = 0.3a$, where F is the friction force which can be expressed as $F = 0.8R$. This enabled the acceleration *a* to be found. Candidates could then make use of the constant acceleration equations such as $s = ut + \frac{1}{2}at^2$ using the value of *a* found, $s = 0.6$ and $u = 0$ in order to find the required value of *t*.

Question 4

This question asked for the greatest value of P. As the particle is stationary, the two extreme values of P are when the particle is on the point of slipping down the plane and when it is on the point of slipping up the plane. The latter of which will give rise to the largest value of P. Almost all candidates adopted the same approach to this question, by resolving forces along and perpendicular to the plane. Each of these equations involved three terms. Resolving perpendicular to the plane gives an expression for the normal reaction R in the form $R = 12g\cos 25 - P\sin 8$. Some candidates made sign errors when setting up this equation and others did not include the term which involves P. When resolving parallel to the plane the equation takes the form $P\cos 8 = 12g\sin 25 + F$ where F is the friction term. In this equation, errors seen included sign errors, mixing sine and cosine and the omission of some terms. The relationship $F = 0.3R$, where 0.3 is the coefficient of friction between the particle and the plane, was then needed. Once this is done the two equations formed a pair of simultaneous equations in R and P. Most candidates made a good attempt at this problem and either substituted directly for R or used a standard method for solving simultaneous equations.

Question 5

(a) (i) This question asked for the power developed by the car. As it travels at constant speed there is no net force acting on the system and hence the driving force is equal to the total resistance. Some candidates incorrectly assumed that only the resistance on the car was required but the engine has to provide power to drive the car and caravan system and so the driving force was (440 + 280) N. Candidates then needed to make use of the definition of power as $P = Fv$ with $F = 720$ and $v = 30$.

(ii) In this question it is first necessary to find the new driving force following the reduction in power. Once this was done Newton's second law could be applied either to the car, the caravan or the system. Two of these equations are needed to find the deceleration and the tension in the tow-bar. Using the system equation gives the deceleration directly. A common error that was seen was to take the new power as 8 kW rather than to reduce it by 8 kW. Most candidates made a reasonable attempt at this question, often finding the deceleration correctly but not finding the required tension.

(b) (i) In this part of the question the car and caravan travel up an incline at constant speed and hence the net force acting on the system must be zero. This means that the driving force consists of the total resistance plus the component of the weight acting along the slope. Common errors seen when finding the driving force were to use the component of mass rather than weight or to ignore the effect of the weight. Once this driving force, F, is found, use of the formula $P = Fv$ with $P = 28000$ used gives the required constant speed.

(ii) In this question the distance travelled in one minute at constant speed is required and this distance d is given by $d = v \times 60$ m, where *v* is the constant speed found in **5(b)(i)**. The required potential energy of the caravan is $800g \times d \times 0.06$. A common error was to find the potential energy of the car or of the system rather than for the caravan. Another error was to use d rather than $d \times 0.06$ as the height change in one minute.

Question 6

Candidates found this question to be particularly challenging. There are several stages involved in the solution. The particles are projected at the same time and it is necessary to determine three key facts. These are the time after which the collision occurs, the position of the particles when they collide and the velocities of the particles when the collision occurs. Knowing the velocities of the two particles as they collide, the principle of conservation of linear momentum can be used to determine the velocity of the combined particle after collision. In addition, there are two cases to consider with the mass of one particle being twice that of the other. Solving these equations for the two cases gives the two different possible velocities for the combined particle after collision. Knowing the position at which the collision occurs enables each of these velocities to be used to find the time taken to return to the ground level in each case. The final answer required is the difference between these two times. Some excellent solutions were seen but many candidates did not progress to answer the question fully. When finding the time at which collision occurs, the distance travelled by *A* is $30t - \frac{1}{2}gt^2$ and the distance travelled by *B* is $\frac{1}{2}gt^2$ and the sum of these is equal to 15. A common error often seen in setting up this equation was to include sign errors in the $\frac{1}{2}gt^2$ term which lead to an incorrect time of collision. Many candidates used the initial velocities of 0 and 30 when applying the conservation of linear momentum.

Question 7

(a) In this question it was first necessary to find the velocity in each of the three stages. This was achieved by integrating each of the given expressions for acceleration with respect to t. Each integral also includes an arbitrary constant which must be found. Many candidates assumed that all these constants were zero. In fact, the constant in stage 1 is zero because $v = 0$ at $t = 0$. By equating the velocities in stage 1 and stage 2 at $t = 2$ the constant of integration in stage 2 can be found. Using the given condition that $v = 0$ at $t = 16$ it can be shown that the constant of integration in stage 3 is zero. On examination of the three expressions for velocity it is clear that the velocity can only reach a value of 55 in stage 3. Setting this expression for velocity equal to 55 leads to a quadratic equation which can be solved for the two possible values of t. Many candidates who did not find the correct arbitrary constant in stage 2 found extra incorrect values of t at which $v = 55$. Most candidates made some progress on this question but unless the correct arbitrary constant was found in stage 2 then it was not possible to obtain full marks.

(b) This question asked for the graph to be completed. The straight-line representing stage 2, $v = 14t - 8$, is shown on the graph. In the section $0 \leqslant t \leqslant 2$ the velocity is a quadratic graph passing through the origin and the bottom point of the given line. In the section $4 \leqslant t \leqslant 16$ the velocity is also a quadratic with a maximum value at $t = 8$ and this curve passes through the end point of the line at $t = 4$ and also through the point (16,0). Although some excellent graphs were seen, many candidates incorrectly drew straight lines for both of the missing stages.

(c) To find the time during which the particle is decelerating it is necessary to locate the maximum value of v which is where $a = 0$ and this occurs at $t = 8$. In order to find the distance travelled while the particle is decelerating, the expression for velocity in stage 3 must be integrated between limits of $t = 8$ and $t = 16$. Many candidates knew that it was necessary to integrate the velocity in stage 3 but a significant number chose to integrate with a lower limit of $t = 4$. Another error which was frequently seen was where candidates had assumed that the graph in part **(b)** consisted of 3 or more straight lines and hence found the area bounded by triangles and trapezia.

MATHEMATICS

Paper 9709/43
Mechanics

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper rather than correct to two significant figures as sometimes seen e.g. **Question 1** and **Question 6(c)**.
- When a question provides a value for the sine, cosine or tangent of an angle it is not necessary to find the angle itself since the given value can be used to obtain exactly any other trigonometric values needed. This was the case in **Question 3** and **Question 7(a)**.
- A complete force diagram can be helpful when applying Newton's Second Law to ensure that all forces are represented correctly in the equation of motion e.g. **Question 5(b)** and **Question 7(b)** and **(c)**.

General comments

This paper was generally well attempted with many responses of a very high standard. **Question 2(a)**, **Question 3** and **Question 4(a)** were found to be the most accessible questions. **Question 5(b)** and **Question 7(c)** were the most challenging questions.

Comments on specific questions

Question 1

The solution to this problem required the use of the conservation of momentum law. Whilst this was frequently applied correctly, the momentum before the collision was quite often calculated as $(0.4 \times 2.5 + 0.5 \times 1.5) = 1.75$ instead of $(0.4 \times 2.5 - 0.5 \times 1.5) = 0.25$. The condition that 'the speed of Q is twice the speed of P' was sometimes misinterpreted and misapplied by candidates.

To obtain a second solution candidates needed to recognise that after the collision the particles either travelled in the same direction with momentum $(0.4v + 0.5(2v))$ or in opposite directions $(-0.4v + 0.5(2v))$. Some solutions concluded with a negative velocity rather than the speed of particle P. Final answers were occasionally seen as $0.18\,\text{ms}^{-1}$ and $0.42\,\text{ms}^{-1}$ rather than the three significant figure values expected. A few candidates considered conservation of energy rather than conservation of momentum.

Question 2

This question was found to be generally straightforward with fully correct solutions frequently seen in both parts.

(a) The majority of candidates applied Newton's Second Law and solved the resulting equation accurately. Occasionally the resistance force was omitted, thus $\frac{150}{4} = 0.25m$ leading to $m = 150\,\text{kg}$. In other solutions the resistance appeared to be in the same direction as the driving force, thus $\frac{150}{4} + 20 = 0.25m$ leading to $m = 230\,\text{kg}$ following the sign error.

(b) **Part (b)** included a change from acceleration to a constant speed of $3\,\text{ms}^{-1}$ and a change from the horizontal road to a hill. The errors seen included an additional acceleration term (ma) when resolving; an unchanged driving force ($\frac{150}{4}$ instead of $\frac{150}{3}$); and $mg\cos\theta$ or $m\sin\theta$ instead of $mg\sin\theta$ used for the component of weight down the hill.

Question 3

This question was well attempted by nearly all candidates who frequently resolved the forces and solved the resulting simultaneous equations accurately. Final answers of $F = 16.6$ and $\theta = 65.8$ instead of $F = 16.5$ and $\theta = 65.7$ were seen following premature approximations such as $F\sin\theta = 15.1$ or $\tan\theta = 2.22$. Other errors included sign errors or the use of sine instead of cosine or cosine instead of sine when resolving or when substituting for $\sin\alpha$, $\sin\beta$, $\cos\alpha$ or $\cos\beta$. Since values for $\sin\alpha^{\circ}$ and $\sin\beta^{\circ}$ were given in the question it was possible to calculate $\cos\alpha^{\circ}$ and $\cos\beta^{\circ}$ exactly without finding and using approximations for the angles α and β.

Question 4

(a) The great majority of candidates applied $s = ut + \frac{1}{2}at^2$ appropriately to obtain the given result $u = 22$. A few mistakenly assumed a greatest height of 24 m with velocity $v = 0$ and applied $v^2 = u^2 + 2as$ to obtain $u = \sqrt{480} = 22$ approximately.

(b) This part was found to be more challenging and produced a variety of attempted methods. The question involved the use of constant acceleration formulae usually in two stages to find h. It was essential to realise that 3.6 seconds was the time spent above h m with 1.8 seconds rising and 1.8 seconds falling. It was common to see $s = ut - \frac{1}{2}gt^2$ applied with $u = 22$ and $t = 3.6$ leading mistakenly to s = 14.4, the displacement from the ground after 3.6 s rather than h. Many candidates were able to find the greatest height reached (24.2 m) or the time taken to reach it (2.2 s), but could not always progress further to find h. One possible method was to find the time taken $\left(2.2 - \frac{3.6}{2}\right)$ s to reach h m and then apply $s = ut - \frac{1}{2}gt^2$. Alternatively, the distance above h m could be found using $s = 0\,t + \frac{1}{2}gt^2$ with $t = 1.8$ s. Those who used this method sometimes forgot to subtract the distance above h m from the greatest height to find h.

Question 5

(a) The most common method of solution was to form and solve a work/energy equation for the whole system. In forming the expected equation PE loss + WD by the engine = KE gain, the common errors were to omit one of the three terms or to combine the terms with an incorrect sign or both. For example, $1900\text{g} \times s \times \sin 5 = \frac{1}{2} \times 1900 \times (30^2 - 20^2) + 150\,000$, $s = 377$ (sign error) or $150\,000 = 1900g \times s \times \sin 5$, $s = 90.6$ (no kinetic energy and a sign error). Occasionally there was a repeated $1900g \sin 5 \times s$ if it was not realised that the work done by the component of weight is the potential energy loss. Those who formed a work/energy equation in terms of h (the change in height) needed to apply $\frac{h}{\sin 5}$ to find the distance required. Those who attempted work/energy equations for either the car or the trailer separately needed to realise that both equations were necessary since this method involved two unknowns: the tension in the tow-bar as well as the length of the hill.

(b) The most straightforward solutions used $v^2 = u^2 + 2as$ to find the acceleration down the hill and then applied $F = ma$ to the trailer. Those who involved either the car or the whole system in the solution frequently either overlooked the driving force or assumed it to be 150000÷200 with the work done by the engine unchanged from **part (a)**. Thus it was common to see the solution of erroneous simultaneous equations such as $1400g\sin 5 - T = 1400a$ and $T + 500g\sin 5 - 100 = 500a$ or $750 + 1400g\sin 5 - T = 1400a$ and $T + 500g\sin 5 - 100 = 500a$.

Some candidates attempted a longer solution using a work/energy equation for the system in order to find the actual driving force before attempting simultaneous equations. Those who attempted this method either for the system or for the car alone, sometimes confused the tension and the driving force without realising that both were acting on the car. A clear and complete force diagram may have been helpful in assessing the forces acting on the car, the trailer and the system as a whole.

Question 6

(a) Candidates were expected to solve the equation $3t^2 - 5t + 2 = 0$ and to find the minimum velocity. Whilst the solution of the quadratic equation was straightforward, several candidates misread the question, finding only the minimum velocity. The most common method to find this minimum value was to use differentiation which usually led to a correct minimum velocity although some solutions stated the time for the minimum velocity rather than the velocity itself. A few stated the speed $+0.125\,\text{ms}^{-1}$ instead of the velocity.

(b) The best sketches showed the quadratic shape and identified the key values using the information found in **part (a)**. Those who did not make a connection with **part (a)** or realise the features of a quadratic function often plotted points for $t = 0$, 1, 2 and 3 and then joined these either with a curve or with straight lines. A variety of other incorrect sketches included straight lines, points of inflection or more than one turning point.

(c) Integration was frequently used with the correct limits to find the distance. Whilst the integration was nearly always correct, the limits sometimes included extra time intervals such as from $t = 0$ to $t = 1$ and from $t = 1.5$ to $t = 3$. The final answer was sometimes left as –0.0417 showing displacement rather than distance, and sometimes shown as 0.042 given correct to two instead of three significant figures A few candidates who had drawn straight line graphs in **part (b)** attempted to find, for example, the area of a triangle. A few others attempted mistakenly to use constant acceleration formulae, but the majority recognised that integration was needed.

Question 7

(a) This was a 'show that' question with an exact answer given and requiring clear and exact working. Since $\sin\theta$ was given as an exact value, it was possible to find $\cos\theta$ without calculating and using an approximate value (16.3°) for the angle θ. The solution involved resolving parallel and perpendicular to the plane and applying $F = \mu R$. Both F (the frictional force) and R (the normal reaction) combined a component of $0.3g$ N with a component of the 4 N force. A common error was to omit one component when resolving, e.g. $R = 3\cos\theta$. The clearest solutions showed the substitutions for $\sin\theta$ and $\cos\theta$ using exact values either before or after the application of $F = \mu R$.

(b) The application of Newton's Second Law was usually recognised to be the method of solution. The equation of motion for particle P should have contained the component of weight and the frictional force acting down the plane, and the 4 N force acting up the plane. One common error was to assume that the frictional force was unchanged by the new direction of the 4 N force. Thus $4 - 3\sin\theta - 3 = 0.3a$, $a = 0.533$ was seen repeatedly, with $\mu R = 3$ instead of $\mu R = 0.75 \times 3\cos\theta$. Other solutions omitted either the weight component $4 - 0.75 \times 3\cos\theta = 0.3a$, $a = 6.13$ or omitted the frictional force $4 - 3\sin\theta = 0.3a$, $a = 10.5$.

(c) This was found to be a more challenging question which was not always attempted. The problem involved two stages of motion, before and after the removal of the 4 N force. Having calculated the initial acceleration of the particle in **part (b)**, the 'suvat' formulae could be used to find the distance moved up the plane whilst accelerating, and the speed of particle P when the 4 N force was removed. The main errors arose when calculating the acceleration after the removal of the 4 N force. This required another application of $F = ma$. The acceleration was sometimes stated erroneously as $a = -g\sin\theta$ or $a = -g$ or calculated with the component of weight and the frictional force acting in opposite directions, or with the frictional force still including the 4 N force. A final application of $v^2 = u^2 + 2as$ enabled the distance whilst decelerating to be calculated. In some cases, the particle appeared to have an acceleration to $v = 0$ rather than a deceleration to $v = 0$.

MATHEMATICS

Paper 9709/51
Probability and Statistics 1

Key messages

Candidates should ensure they communicate clearly, showing the necessary steps required to support their answer, particularly when they are required to explain a given numerical value.

Where a diagram is required it should be clearly and accurately drawn and appropriately labelled.

Candidates should state only non-exact answers correct to 3 significant figures and exact answers should be stated exactly, for example as a fraction or surd. To justify a final answer correct to 3 significant figures candidates should use values correct to at least 4 significant figures throughout their working. There is no requirement for fractions to be converted to decimals, this can often lead to a lack of accuracy.

General comments

Most candidates used the answer space effectively. Where there is more than one attempt at a question it should be made clear which alternative is to be marked, particularly when the Additional Page is used. Candidates are reminded to cross out any work not to be marked, and not to offer multiple alternative solutions to a question.

The use of diagrams and sketches were often seen in good solutions to organise information and support explanations. Candidates are recommended to include these as they can help to visualise a scenario.

A range of responses were noted throughout the question paper; some strong solutions were seen in **Question 4**; however, many found the context for the probabilities in **Question 7** challenging.

Comments on specific questions

Question 1

Some candidates found this question to be particularly challenging. This situation which requires the multiplication of combinations and then the addition of scenarios is quite familiar for most candidates. Here many candidates appreciated that there were 3 appropriate scenarios and attempted to add them.

Those who identified the correct scenarios and realised that the marbles were of different sizes and hence distinguishable were often able to complete the question successfully. Some added ${}^4C_4$ and ${}^8C_1$ for example to get the number of combinations of 4 blue marbles and 1 red marble rather than finding their product. Those who did not appreciate that the marbles were of different sizes attempted to list possible options.

Question 2

Good responses performed the standardisation process competently with few applying a continuity correction or using the variance. The most efficient solutions used the symmetry of the situation to find the probability, these were often characterised by the inclusion of a helpfully shaded diagram. Candidates must ensure they maintain accuracy when using the normal tables without rounding or truncating the values obtained. Most candidates realised that they had to multiply by 500 to find the expected number of rods with better answers being given as an integer with no reference to abbreviation or approximation.

A significant number of candidates misunderstood the context and attempted to use 0.5 as the boundary for the probability and were unable to proceed as this gave them too large a value for z, whilst others seemed to have no understanding of the normal distribution and found it difficult to access any marks.

A small minority of candidates provided no evidence of standardisation to support their probability of 0.7888. Candidates are reminded to include all necessary working and that 'no marks will be given for unsupported answers from a calculator' as stated on the front of the question paper.

Question 3

Almost all candidates identified the need to use factorials to find arrangements.

(a) Many solutions including 8! were seen with better responses dividing by 3! to deal with the repetition of the Es.

(b) Stronger solutions were often accompanied by a layout indicating that there was only one way to use the letters L, E, D in that order, meaning that there were now 6 items to arrange. Weaker responses used all the arrangements of the letters L, E, D and included a factor of 3! in their total number of arrangements or omitted the denominator of 2! to deal with the repetition of the remaining Es.

(c) A substantial number of incomplete solutions were seen to this part of the question with candidates often not providing a probability. Those who arranged the 6 letters R, E, L, E, S, E then multiplied by the number of ways of placing A and D in nonadjacent places were often successful; better responses correctly using a denominator of 3! to deal with the repetition of the Es. This approach was least often seen however those that chose it were quite successful. The method most often employed was to subtract the arrangements with A and D together from the total number of arrangements. This method was usually done well. Some candidates needed to be aware of necessity of including the 2 different orders of A and D. Another effective approach, less often seen, was to find the probability of the arrangements with A and D together and subtract from 1, with the understanding that the letters A and D could appear in either order.

Question 4

In some solutions it was not always clear when candidates omitted the 0 at the beginning of decimals then used a dot to signify multiplication.

(a) Many good solutions were provided. The best responses showed the labels in full or used recognisable abbreviations and applied clear notation to each set of branches. Candidates should be aware of the need of accurate communication in their diagrams. The context of the question was challenging for some as the path to taking the practical test depended on a successful attempt at the first or second written test. Some included extra branches such as a second attempt at the written test after the first was successful and an attempt at the practical test following a failure at the written test. Weaker responses dealt with the two tests in separate diagrams and were often unable to continue.

(b) This was well done by almost all who produced a correct tree diagram. A few successfully found the probabilities of the 2 routes to being offered a place at college but then multiplied them together.

(c) This part was generally less well attempted. Many candidates did not seem to know what was required to calculate a conditional probability and simply gave the probability of gaining a place at college following a pass at the first written test. Some of those who calculated conditional probabilities either gave an incorrectly rounded value of 0.869 or prematurely approximated. The best solutions either gave their answer as a fraction or rounded correctly.

Question 5

(a) Where candidates calculated frequency density, they were often able to access most of the remaining marks. Many others simply plotted frequency and time, and a few cumulative frequency curves were also seen. Good solutions stated the frequency densities before drawing the graph; candidates would be well advised to adopt this approach to assist with plotting accuracy. The data allowed simple scales to be used on each axis which the majority of graphs demonstrated. Most classes had the correct bounds with only a very few candidates using a continuity correction of 9.5 or 10.5 etc. Weaker responses miscalculated the scales and attempted to extend the given answer frame or made all the classes of equal width. Several scripts were seen where candidates had tried to correct their graphs by overwriting or squeezing a second graph into the space; they would be well advised to seek another grid. The use of a ruler is essential to ensure that candidates can draw graphs to the appropriate degree of accuracy. Candidates should be reminded that axes should be carefully labelled, including appropriate units for the variable.

(b) There were many solutions demonstrating the correct approach to finding the mean. Midpoints for each class were often within the correct range and most divided by the total frequency of 200. Weaker responses summed the midpoints and divided by 5. Many good responses included an expanded data table as evidence supporting their method. Candidates should be reminded of the requirement to show their working. An unsimplified expression containing midpoints would be appropriate but must be present.

(c) Candidates found this part most challenging. Some good, concise solutions were seen but a significant number made the error of using the upper bound of 59 or 59.5. Weaker responses simply subtracted the positions of the quartiles; 150 – 50 or 75% – 25%.

Question 6

A large number of candidates identified that the question related to the binomial distribution, but some candidates were unable to make any progress at all. Candidates are reminded to work more accurately in the method than the required level of accuracy in the answer.

(a) Most of the successful solutions made a clear statement of 1 – the probabilities of X being 10, 11 or 12. Some candidates used unsuitable boundaries demonstrating a misunderstanding of the conditions whilst others simply provided one binomial term or omitted the probability of X being 10. Weaker responses sometimes omitted the binomial coefficient for each term. Those who simply summed the probabilities of the outcomes that met the success criteria were sometimes successful but often evaluated incorrectly.

(b) Good solutions included clear unsimplified calculations for the mean and variance before their substitution into the standardisation formula. A large number realised that a continuity correction was required as the data was discrete. A helpful diagram often supported the correct choice of area in stronger responses. There was some evidence that the z value had been rounded too soon or truncated resulting in a loss of accuracy. Candidates should be reminded that they must provide evidence of standardisation to support their given probability of 0.9433 to gain credit.

(c) The strongest responses provided an efficient justification but the vast majority of candidates found this challenging. Whilst many knew they had to compare a value to 5 either npq or np was often used.

Question 7

(a) Good solutions demonstrated an understanding that the situation was one where the tins could not be replaced once opened with the best candidates displaying an awareness of the requirement to justify their calculation since the answer was given. Weaker responses selected the tins from 7 each time.

(b) A table was present in most candidates' responses. Stronger responses appropriately identified the correct outcomes. Weaker responses included 0, 6 or 7 with some omitting 4 and 5. The weakest solutions tabulated the number of tins for each vegetable. The best solutions included calculations to justify the entries in their tables; many candidates recognised that the probability that 1 tin was needed was $\frac{3}{7}$. Those who used decimals instead of fractions were far less likely to obtain a sum to 1 for the last mark and candidates would be better advised to work in fractions.

(c) Many candidates did not attempt this part of the question. Good solutions were often characterised by the inclusion of a table of xp and x^2p to inform their answer. Where the variance formula was used it was usually correct; when errors were seen it was often with using the mean rather than the mean squared. Candidates should be reminded of the need to support their working; on occasion the mean was simply stated without any supporting working.

MATHEMATICS

Paper 9709/52
Probability and Statistics 1

Key messages

Candidates should ensure they communicate clearly, showing the necessary steps required to support their answer, particularly when they are required to explain a given numerical value. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution. When errors are corrected, candidates would be well advised to cross through and replace the term and are strongly recommended not to write over their work.

Candidates should state only non-exact answers correct to 3 significant figures and exact answers should be stated exactly, for example as a fraction or surd. To justify a final answer correct to 3 significant figures candidates should use values correct to at least 4 significant figures throughout their working. There is no requirement for fractions to be converted to decimals, this can often lead to a lack of accuracy.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. A significant number of clear back-to-back stem-and-leaf diagrams were noted and the inclusion of appropriate units in the key.

Sufficient time seems to have been available for candidates to complete all the work they were required to, although some candidates did not complete the very last question. Candidates should ensure they are familiar with all aspects of the syllabus for this component, one area of particular difficulty was the geometric distribution. Many good solutions were seen for **Questions 3**, **4** and **7**. The context in **Questions 1** and **6** was found to be challenging for many.

Comments on specific questions

Question 1

Many candidates successfully identified that the geometric distribution was the most appropriate approach for the question. A significant minority attempted to use the binomial distribution without success. Several candidates interpreted the context of ‘an ordinary fair die’ as a ‘die with 5 sides’.

(a) Good solutions simply stated that in a geometric approximation, mean $X = \dfrac{1}{\text{probability } X}$, and evaluated with their probability substituted. Weaker solutions calculated the expected value from throwing a 5. A significant minority simply stated their probability of throwing a 5.

(b) The most successful solutions used the simple approach of summing the probabilities of the outcomes that met the stated criteria. A number of candidates did not evaluate their correct expression with sufficient accuracy, arriving at a total of 0.299 because of premature approximation. Some candidates attempted to use the formula for the sum of a geometric series, but in this context, it was not very efficient to do so. The majority of candidates chose to use the more efficient method of P($X \leqslant 7$) – P($X \leqslant 3$) but struggled to interpret the required conditions accurately. A very common error was P($X \leqslant 7$) – P($X > 3$), although many other incorrect conditions were applied. Candidates should be encouraged to review how to interpret limiting conditions in preparation for the assessment.

(c) Candidates were often more successful in their interpretation of the success criteria here. Good solutions stated the calculation for the efficient method identified in **part (b)** clearly and evaluated accurately. A few candidates simply summed the probabilities of all the possible outcomes, although some did use their answer from **part (b)** within their expression. As in the previous part, many candidates had difficulty interpreting the success criteria and P($X \leqslant 10$) or P($X = 9$) were the most common errors.

Question 2

Many fully correct solutions were seen. The best stated clearly the probability of success as a decimal and found the appropriate z-value. The normal standardisation formula was stated with the values substituted and an equation formed. Many showed a clear algebraic process to find σ, rounding their final answer to 3 significant figures. A common error was to state σ to 3 decimal places, which illustrated the misconception about leading zeros in decimal values. Weaker solutions often equated to the probability or gave no evidence as to how the numerator was calculated. Candidates are well advised to clearly indicate how values are substituted into formulas within the syllabus content.

Some candidates found that a simple sketch of the normal distribution assisted in the interpretation of the context.

Question 3

Many solutions contained a tree diagram, which often clarified the approach that was being taken by the candidate. A very small number of candidates misinterpreted the initial probability information and assumed that probabilities stated for being late for each of the transport method were conditional probabilities.

(a) The best solutions formed the unsimplified 3-term equation for not being late and provided a clear solution. The alternative approach of forming the unsimplified 3-term equation for being late and then finding the complement was often less successful. Weaker solutions often had arithmetical inaccuracies or did not form the initial equation successfully.

Several solutions were difficult to understand as there was inconsistency in the use of the unknown, with x often being used to mean 'being late' and 'not being late' in the same algebraic solution. It is good practice to state the meaning of algebraic unknowns.

(b) This question had a fairly standard conditional probability context, which was recognised by many. The best solutions realised that the complement of **part (a)** could be used to calculate the denominator and stated an unsimplified conditional probability. However, the majority of successful solutions simply calculated the denominator from first principles. Weaker responses often simply calculated the probability that Alexa travelled by train and was late, without making an attempt at a conditional probability. It was noted that many candidates stated their final answer to 2 significant figures, possibly to be maintain the same accuracy as some of the probabilities stated, rather than the 3 significant figures expected. A few candidates stated their probability as an accurate fraction; it is good practice not to convert to a less accurate form unnecessarily.

Question 4

(a) The best solutions generated the outcome space which was used to complete the required probability distribution table. Common errors were often linked to outcomes involving the –2 value on the spinner or stating outcomes that could not be achieved from the values on the spinners, such as 4. Several candidates did not ensure that the sum of the probabilities was 1, and that all probability values were between 0 and 1. A few candidates simply stated the frequency of each outcome.

(b) Most candidates were able to use the correct mean and variance formulae and provided unsimplified expressions as supporting evidence for their calculations. This enabled candidates who made errors in **part (a)** to gain credit. Weaker solutions frequently omitted the subtraction of $E(X)^2$ in the variance formula. Candidates were expected to identify the specific value that was being calculated and not just state the two values obtained.

Question 5

Most candidates identified the binomial distribution was the focus of the question. Each part was a fairly standard application of the skill involved, but many did not interpret the different success requirements accurately.

(a) Almost all candidates attempted this part. The best solutions identified that the P(not late) = P(on time) + P(early) initially. This value was then used to find the required probability. A significant number of candidates used an incorrect probability, often assuming that the P(not late) = 1 – P(early). A common misconception was that $(x + y)^3 = x^3 + y^3$ and used this approach in the solution. A small number of candidates simply calculated 3 × P(not late), and accepted a final probability value greater than 1, which should have been a clear indication of an error.

(b) The majority of candidates recognised that this was a standard application of the binomial distribution and provided at least one binomial term in the correct form. The most common approach was 1 – P(arrives early less than 3 times). Again, many candidates did not identify the required outcomes accurately and would benefit reviewing the interpretation of success criteria in preparation for the assessment.

(c) Many candidates recognised that the normal distribution was the appropriate approximation to use. Good solutions provided evidence of how the mean and variance were calculated, substituted accurately into the normal standardisation formula using the appropriate continuity correction. Then having evaluated the *z*-value, used a suitable method to determine the probability, often with a simple sketch of the normal distribution to highlight the expected probability area. The stronger responses also included evidence that the conditions required for the use of the normal approximation were met. Several candidates did not use any continuity correction, presumably not recognising that the number of flights was discrete data.

A small number of candidates did not follow the instruction in the question to use an approximation and used a calculator function to evaluate a binomial distribution.

Question 6

Although most candidates were able to identify that combinations should be used in this question, many found the context very challenging. A very small number of candidates assumed that the repeated letters were identifiable in some way, whereas in this context the repeated items cannot be distinguished.

(a) A large majority of candidates were successful here. Good solutions included a clear statement of the unsimplified expression before any evaluations were undertaken. Weaker solutions either did not eliminate the effect of the repeated letters, or simply evaluated their expression inaccurately.

(b) Most candidates found this question very challenging and few fully correct solutions were seen. The use of a simple diagram to interpret the required condition was often seen in good solutions. The most successful approach was to calculate the total number of arrangements with the Rs at each end and subtract the number of arrangements with the Rs at each end and all the Os together. A number of solutions did not eliminate the effect of the repeated Os in the total and more often assume that the Os could be individually identified when they were together. Many solutions had double the anticipated number of arrangements which would suggest that the Rs could be individually identified.

The less successful approach was to calculate the number of arrangements where the Os were all separated and add the number of arrangements where there was a single O and a group of 2 Os separated. Many candidates who used this approach only calculated one of these options or assumed that the Os could be individually identified.

(c) This question was found challenging by many candidates. Of several different approaches that could be used, the most successful was to identify the 5 possible scenarios of O and R that fulfilled

the conditions and then calculate the number of selections that were possible for each. A common error was to ignore that O and R were repeated within TOMORROW and not multiply by the appropriate number of ways these letters could be selected. A significant number of candidates either included ORRR, which was impossible, or omitted OORR in the list of scenarios.
As a probability was required, the total number of selections possible was calculated separately before the probability was stated. This step was omitted by almost all candidates. It is good practice to suggest that candidates read the question again after completing their solution to ensure that they have fulfilled all the requests in the instructions, and that their result seems sensible.

Stronger responses often used a probability approach, identifying the 5 possible scenarios of O and R that fulfilled the conditions and then calculating the probabilities of making the selections without replacement. Again, a common error was to ignore that the letters could be selected in different orders, and so the probabilities needed to be multiplied by the appropriate factor.

Question 7

(a) Almost all candidates attempted this part, appropriately considering the general advantages rather than simply comparing the data provided. The best comments highlighted the difference in the amount of data that was presented and included reference to the original data still being available in the stem-and-leaf diagram, often stating other mathematical processes that could be undertaken which were not possible from a box-and-whisker plot.

Weaker solutions often focused on ease or speed of construction and ease of reading data from the diagram. Candidates should be aware that 'mathematical' links are expected in responses to gain credit.

(b) Many clear, accurate stem-and-leaf diagrams were seen. The best ensured that the leaves had appropriate titles, the data was ordered and aligned vertically accurately, the stem included all the terms and the single key used a clear example values from the data with the team and units stated, Weaker solutions often had poor vertical alignment, included punctuation between the values or values out of order or omitted.

A small number of candidates used a single digit stem, which was not appropriate to give a visual presentation of the spread of the data. A similar number of candidates did not follow the instructions in the question to place the Amazons on the left of the diagram.

Candidates should be aware that errors need to be corrected without affecting the vertical alignment. It is good mathematical practice to use pencil to draw diagrams, as errors can then be easily be erased.

(c) Most candidates showed an understanding of how to calculate the interquartile range. Good solutions stated and identified clearly both quartiles before performing the calculation. A common error was to find one of the quartiles inaccurately by not using the 'median' of the data above or below the median value.

(d) Many candidates found the correct height of the unknown Amazon player. Good solutions provided clear explanation of the calculations being performed – identifying what was being found in a structured manner. Weaker solutions did not present the logic of their reasoning but an unordered collection of calculations which they used to find the missing value.

A common misconception was that the missing value could be identified by simply comparing the means of the two data sets.

MATHEMATICS

Paper 9709/53
Probability and Statistics 1

Key messages

Candidates are reminded that they must show their working and that unsupported answers will not be rewarded. Solutions relying entirely on calculator technology are not acceptable; all stages in the response must be evident. Candidates should also be aware that, if they make more than one attempt at a question, only one of their responses will be marked. Candidates are reminded to cross out any work not to be marked, and not to offer multiple alternative solutions to a question.

General comments

In both **Question 2** and **Question 5(a)**, candidates were required to form a pair of simultaneous equations to be solved. Most candidates showed their working clearly, either by using an algebraic method (substitution or elimination) or by putting the equations in the same linear form, preferably with the two unknowns and their coefficients on one side of the equal sign and the constant on the other before using their calculator. Candidates are reminded that 'no marks will be given for unsupported answers from a calculator', as stated on the front of the question paper.

In **Questions 3** and **7(a)** where candidates were required to extract information from a given table or summary, the individual numbers must be shown if used to ensure all available marks can be awarded. Often a total was given without showing how it was derived.

Premature approximation was also commonly seen, particularly in **Questions 5** and **7(b)**. Candidates need to be aware that if their final answer is to be correct to 3 significant figures, all input numbers need to be correct to at least 4 significant figures.

Comments on specific questions

Question 1

(a) This question was well answered with most candidates giving the correct answer. A few candidates applied 'less than' to the number of candidates as well as the height and put a < sign in front of the 60 or gave the incorrect answer of 59.

(b) Stronger responses found 65% of 160 = 104 and then to find the height that corresponded to a cumulative frequency of 104 by drawing a horizontal line on the graph at 104 and dropping down to 136 cm on the horizontal axis. As the question states 'use the graph' we expect to see some indication on the graph of its use, usually a horizontal and vertical line drawn on the graph. Most candidates were able to interpret the horizontal scale correctly. The most common error was to misunderstand the idea of a 65^{th} percentile and to read across from 65 on the vertical axis and give the final answer as 104 cm.

(c) This question was well answered with most candidates knowing to find the positions of the lower and upper quartiles and then use the graph to find the corresponding values in cm and subtract the value of the lower quartile from that of the upper quartile. The most frequent errors were to give the difference in the positions (120 – 40) as the value of the interquartile range or to state the interquartile range as being from 76 to 150 and not subtracting.

Question 2

Most strong responses formed two equations connecting the with two unknowns. One equation was formed by summing the probabilities and equating them to 1 and the other from the given information that $P(X \geqslant 0) = 3P(X < 0)$. They then went on to solve the two equations simultaneously, for which clear method was needed. If a calculator was used, the equations needed to be prepared in the same format, ideally with the unknowns on one side of the = sign and the constant on the other, with this being evidenced in the answer space.

There were some intuitive alternative methods seen which bypassed solving a pair of simultaneous equations. One was to use the ratio 1:3 so that $P(X < 0) = 0.25$ and therefore $2p = 0.25$, quickly giving $p = 0.125$. Another was to use the fact that $P(X \geqslant 0) = 1 – P(X < 0) = 1 – 2p$ and then equate this to $6p$ as $3P(X < 0) = 6p$.

Question 3

(a) Most candidates knew to add the totals of the x and y values and then to divide by 17, the total number of players in both teams. $\frac{(1051+1991)}{17} = 178.9$ was needed to be seen and writing $\frac{(\sum x + \sum y)}{17} = 178.9$ without substituted values shown was not sufficient. The most common error was to find the means of the two teams separately and then either to add them or to divide the sum of the two means by 2. Few candidates rounded prematurely or rounded incorrectly with some truncating the answer to 178.8.

(b) Most candidates applied the correct variance or standard deviation formula but, as in **part (a)**, a significant number did not show which numbers they were substituting in. The sums of squares (193700 and 366400) added and divided by 17 as well as the value of the mean that was to be squared and subtracted were all needed to be seen. Most knew to use either the exact value of the mean or a value to at least 4 significant figures. However, premature approximation was seen in some responses.

Question 4

(a) This question was generally well answered. Most candidates recognised that there were three ways of obtaining a total of 4 from three die, although weaker responses often did not obtain the required probability of $\left(\frac{1}{6}\right)^3$ and others obtained the correct probability but did not go on to multiply by 3.

(b) Recognition of a geometric distribution was good here with only a few attempting a solution using binomial notation and most candidates obtained the correct answer. Incorrect probabilities were sometimes carried over from the previous part while others used the answer from **part (a)** as P(18).

Some candidates prematurely rounded their final answer and candidates are reminded to ensure they provide an answer to at least 3 significant figures.

Question 5

(a) Most candidates used the tables the correct way around with very few producing probabilities rather than z-values and most knew to give the z-values to at least 3 decimal places before they standardised and produced two equations in μ and σ. As in **Question 2**, we insisted on seeing evidence of how the simultaneous equations were solved. Those who chose to use calculators rather than the preferred methods of elimination or substitution needed to prepare their equations by putting them in the same linear form as each other, usually with the μ and σ on one side of the = and the constants on the other side.

Candidates are again reminded to ensure they provide an answer to at least 3 significant figures.

(b) Most strong candidates realised that the z-values they needed to work with were 2 and –2. Others used the values for μ and σ from **part (a)** even though the question clearly stated it was a different type of tree. This approach did not always lead to the correct *z*-values. Those who worked with 2 and –2 usually calculated the correct area, but a significant number did not show the Φs they worked with (0.9772 and 0.0228) to arrive at the correct answer of 0.9544.

Candidates are reminded to check the question to ensure their solution gives the required answer. This question required candidates to give an integer solution for the number of leaves, obtained from multiplying their probability correct to at least 4 significant figures. Many candidates stated the answer as being approximate or correct to 3 significant figures.

Question 6

(a) Candidates found this question to be particularly accessible with most obtaining the correct answer. Only a few candidates omitted to divide 11! by 2! and 3!.

(b) This part was generally well answered with most candidates realising that if the Es and Rs were together they would be arranging 8 items. A small number incorrectly treated the Es and Rs as individual items or multiplied the correct answer of 8! by 2! and 3!.

(c) Candidates found this question part to be particularly challenging, with a variety of approaches seen. The most successful approach was to see that there were 7 possible positions for the first R and then $\frac{9!}{3!}$ ways of arranging the other letters. Another common approach was to see that there were 9C3 different sets of 3 letters between the Rs and then 7! ways of arranging the block and the 6 other letters. With this approach they needed to multiply by 3! to take account of the number of arrangements of the 3 letters between the Rs and divide by 3! because there are three Es. These two 3!s conveniently cancelled out. However, many candidates chose to consider the number of Es that could be positioned between the two Rs. This was a more challenging approach and only the most careful obtained the correct final answer.

(d) This question was well and clearly answered by many candidates with the four scenarios and their associated number of ways clearly stated and totalled. As in previous parts of this question, some candidates thought they needed to distinguish between the Es and the Rs once they had identified the four scenarios.

Just a few simplified the required answer to 8C2, having appreciated that if there were 2Es and an R the other two letters could be chosen without further restriction from the remaining 8 different letters. A very neat solution.

Question 7

(a) (i) This question was generally well answered. However, several candidates misread the question and calculated the probability of a Shan resident having poor broadband service.

(ii) Most candidates recognised the need for a conditional probability and only a minority calculated P(Shan/Good service) rather than the required P(Good service/Shan). A common error was to misread the 177 as 117 with many candidates using both values within the one calculation.

(b) (i) This question was well answered with most candidates gaining full marks. Almost all recognised the need to use a binomial distribution with $n = 10$ and $p = 0.35$ and they knew to show the unsimplified expression for each term before adding the values, each of which had to be to at least 4 significant figures, when evaluated, to avoid premature rounding errors.

(ii) Most candidates seem to be well prepared for this type of question and knew to use a normal approximation to the binomial distribution. Many gained full marks, showing their working clearly as they calculated the mean and standard deviation for the approximation and correctly used a continuity correction in their standardisation. Only a few chose the wrong tail when calculating the final answer.

MATHEMATICS

Paper 9709/61
Probability and Statistics 2

Key messages

- All relevant working must be clearly shown.
- Candidates should be able to recognise a distribution from a given scenario and correctly choose a valid approximating distribution when required.
- Candidates should be aware of when and how to apply a continuity correction.
- Candidates must ensure that they are aware of the differences between standard deviation and variance.
- Solutions should be clear and neatly presented.

General comments

This paper generated a large range of responses from candidates. Candidates must ensure that they are familiar with all aspects of the syllabus requirements for Probability and Statistics 2 to ensure they are able to cope with the demands of the question paper. Questions that were, in general, relatively well attempted were **Questions 1**, **4** and **8(a)**, whilst **Questions 2(b)**, **3**, **5(b)**, **8(c)** and **(d)** proved to be particularly demanding.

It is important that all necessary working is shown with solutions and is presented clearly and logically.

The comments below indicate common errors and misconceptions, however, it should be noted there were also some full and correct solutions presented.

Comments on specific questions

Question 1

This was a generally well attempted question. Most candidates identified that a Poisson distribution was required and were able to successfully find the value for λ, 9.6. Common errors included omission of '1–…' in the Poisson expression and incorrect values for λ. It was important that all necessary working was shown; there were some occasions when candidates gave unsupported answers and were therefore unable to gain some of the marks available.

Question 2

(a) Most candidates found this question to be accessible. Common errors included standard deviation and variance mixes, the use of an unrequired continuity correction and also finding the wrong probability area. By using a diagram this could help candidates to minimise errors in finding the correct probability area. It was noted that not all candidates used $\sqrt{40}$ in their standardising equation.

(b) This question was found to be particularly challenging for many candidates, with many candidates not being precise enough in their responses. The first requirement of this question was to say that it was not necessary to use the Central Limit Theorem, and then it was required to say that this was because we knew that the population was normally distributed. Many candidates were unclear in their statements, saying that 'the data' was normally distributed or 'it' was normally distributed, or 'the distribution' was normal; these statements do not clearly specify that it is the parent population that is normal.

Question 3

This was a question which tested candidates' understanding of probability density functions. Responses which used diagrams were often more logical and successful in their approach. There were many candidates who did not know how to approach the question, and many did not attempt the question at all.

Question 4

This question was found to be reasonably straightforward and was generally well attempted. Most candidates successfully found unbiased estimates of the mean and variance, though there were some candidates who found and incorrectly used the biased estimate of the population variance. The formula for the confidence interval was often used correctly; common errors included an incorrect *z* value and omission of $\sqrt{100}$. Candidates were required to give their final answer as an interval, and this was done in most cases.

Question 5

(a) Candidates did not always recognise that the given distribution was binomial, and that the appropriate approximating distribution was Poisson. It is important that candidates recognise a binomial distribution from information given and are able to find a valid approximating distribution. Those who identified that $\text{Po}\left(\frac{2}{15}\right)$ was the correct distribution to use were usually successful in finding the required probability. Common errors included use of a binomial distribution to find the required probability (i.e. not using an approximating distribution at all) or attempting a normal distribution. It was important that all relevant working, including writing the full Poisson expression, was clearly shown; use of a binomial distribution was incorrect, as this was not as requested in the question. Candidates who did use a binomial to find the probability were able to achieve some, but not full, credit for their correct work.

(b) Many candidates did not know how to approach the question, and many did not attempt it. There were some candidates who approached the question correctly, either by using a Poisson distribution or a binomial distribution, either of which was valid for this question. Once the initial equation was found these candidates were usually successful in using logarithms to find *n*. As the question required the value of the largest *n*, the final answer needed to be correctly rounded and given as a whole number. Some candidates attempted a trial and improvement method, which was rarely successful.

Question 6

Candidates are reminded to think carefully about the requirements in a question; especially when candidates are required to identify the relevant information to use and calculate. E(X) and a value for k needed to be found before Var(X) could be attempted. Many candidates did not recognise the symmetry of the function and rather than state E(X) they calculated it once a value of k had been found. This was a lengthy but acceptable method. Many candidates did not realise the need to find k and left their answers in terms of k; some method marks were available for correct work in term of k. There were some candidates who gave full and well-presented answers, but equally others did not approach the question logically and answers were poorly presented.

Question 7

(a) There were many well attempted responses to this question. Common errors included calculating the variance incorrectly, standard deviation and variance mixes, use of unrequired continuity corrections and calculating the wrong probability area. Candidates are encouraged to use of a diagram which could help to reduce errors particularly when determined the correct probability area to calculate.

(b) This question was reasonably well attempted by well-prepared. As in **part (a)** similar errors were made with the most common being use of a variance of 34 (from $3^2 + 2^2 \times 2.5^2$) rather than the correct value of 21.5 (from $3^2 + 2 \times 2.5^2$). It is important that candidates can correctly distinguish between a sum of normal random variables and a multiple of a normal random variable.

Question 8

(a) Many candidates were able to fully explain why the sample would not be appropriate; it was important that the idea of not representing the whole school was conveyed.

(b) This was reasonably well attempted; common errors included omission of 'P' (i.e. writing $H_0 = 0.15$) and incorrect use of μ instead of P. A one-tailed test was usually declared.

(c) The question clearly stated that a binomial distribution was to be used, and many candidates correctly used B(50,0.15). Some candidates found $P(X \leqslant 3)$ but others only found individual probabilities rather than calculating cumulative probabilities. Not all candidates gave fully supported answers. To fully justify the answer the probability of $P(X \leqslant 4)$ needed to be calculated and not all candidates did this. Candidates generally found this question part to be particularly challenging.

(d) Some candidates also found this question part to be particularly challenging. Candidates who had been successful or partially successful on **part (c)** often were able to make a good attempt at this part. Weaker responses often did not realise that a comparison, either with the critical region or with 0.05, was necessary before a conclusion could be drawn.

(e) Some candidates did not answer this question, but for those who did a reasonable attempt was made. Whatever conclusion was drawn in **part (d)**, either reject or accept H_0 this was followed through to the candidates' answer in **part (e)**; full marks could be obtained if the candidate gave a correct answer based on what they had said in **part (d)**.

MATHEMATICS

Paper 9709/62
Probability and Statistics 2

Key messages

- Candidates must show all relevant working.
- In general, only one solution should be offered. Candidates should ensure that they clearly indicate which is their final answer.
- Conclusions to hypothesis tests must always be in context and not definite/
- Candidates should always check the sensibility of their answers, to ensure it fits with the context of the question and if the value obtained generally looks reasonable. Checking answers against the question can help to minimise errors.
- Candidates should be able to correctly choose and fully justify a valid approximating distribution when required.
- Candidates should be aware of when and how to apply a continuity correction.

General comments

On this paper, many candidates demonstrated and applied their knowledge in the situations presented effectively. It was noted that there was a large range of responses from candidates. In general, candidates found **Questions 4, 6(a) (b) and 7** most accessible, whilst **Questions 1 (b), 2 (a) (b)(i) and 6(c)** proved to be most challenging.

It is important that all necessary working is shown with solutions presented clearly and logically. Candidates are advised to clearly selected their final answers, it was noted that several candidates offered more than one solution particularly in **Question 1(b)**.

It is important that candidates show all necessary working. For example, in **Question 2(b)(ii)** the correct approximating distribution was Poisson, so the Poisson expression needs to be clearly stated before any numerical answers.

D The comments below indicate common errors and misconceptions, however, it should be noted that there were also some very strong and complete answers presented.

Comments on specific questions

Question 1

(a) Candidates should be reminded to take care with their presentation, as many candidates were not careful enough in answering this question. Many candidates declared their hypotheses as H_0 and H_0 (with no mention of H_1), rather than H_0 and H_1. For candidates using H_a it is important that the 'a' is clearly written so as not to look like H_0. There was also confusion between μ and p, and occasionally a one-tailed test was declared.

(b) Several errors were noted in response to this question. Common errors included using an incorrect normal distribution (the given distribution, B(100,0.25), should have been approximated to N(25,18.75)), incorrect standardisation attempts often with incorrect use of $\frac{18.75}{100}$ and a wrong or no continuity correction. Some candidates did not show a clear comparison, used an incorrect z value or gave a final conclusion which was not in context or not with the required level of uncertainty in the language used. On occasions incorrect conclusions were given, such as concluding that the ball would land in slot A. Candidates should be reminded to clearly select and identify the solution that they wanted to offer as their final answer, as some candidates made more than one attempt at standardising in this question.

Question 2

(a) Many candidates did not calculate Var(X) thinking it was 0.01, rather than $400 \times 0.01 \times 0.99$. Another common error was to calculate Var($4X + 2$) as $16 \times$ Var(X) + 2.

(b) In **part (i)** many candidates realised a Poisson distribution was a suitable approximating distribution, but some did not give the parameter, and many candidates did not fully justify this approximation using the context of the question. It is not sufficient to say, for example, n is large or n is greater than 50 as these are the generic conditions. Candidates should ensure they relate their solution to the question by saying $n = 400$ so $n > 50$ and $np = 4$ so $np < 5$, or equivalent.

Part (ii) was generally well answered with the majority of candidates using Po(4) and writing the correct Poisson expression. There were some candidates who gave unsupported answers and did not state the Poisson terms.

Question 3

(a) Many candidates used the fact that the area below the line, a triangle, was equal to 1, and used this to show that $p = 2$. It was important that the resulting quadratic equation was clearly rearranged to equal zero for solving as the answer had been given; 'show that' questions require candidates to show full and complete working. Some candidates assumed that $p = 2$ and tried to verify that with this value of p the area was 1. In order to answer this question successfully via a verification method, this required a full explanation along with a clear statement saying that it was to be assumed that p was 2 and then verifying that it must be true. Very few convincing and complete verification methods were seen. A particularly long method involving finding the equation of the line in terms of p and integrating this between 1 and p and equating to 1 was also seen on occasion, but candidates were rarely successful with this approach.

(b) Some candidates successfully found the equation of the line ($y = 2x - 2$) but errors were made finding the gradient; some candidates did not make any attempt at finding the equation of the line and merely assumed $y = x$ or $y = p = 2$. Candidates are advised to use the diagram provided as a guide, for example to note that the gradient was positive and y-intercept had to be negative. Most candidates realised that to find E(X) they needed to integrate xf(x) between 1 and 2, so marks were gained for this using their expression for f(x). Some candidates incorrectly integrated from 0 to 2.

Question 4

(a) This part of the question was particularly well answered. Nearly all candidates correctly added the three mean value and the three variance values. Incorrectly dividing these values by 3 was also occasionally seen.

(b) Standardising was usually done correctly, and the correct probability area found. On occasions use of an unrequired continuity correction was seen, or the wrong tail probability found.

(c) This part was also reasonably well attempted, though a common error was to omit $\sqrt{15}$ when standardising. Some candidates who did omit $\sqrt{15}$ thought their probability should be multiplied by 15, giving a final probability greater than 1. Candidates should make sure they always check that answers are sensible and appropriate.

Question 5

(a) It was important that the context of the question was clearly used in the answer here. Generic definitions of a Type 1 error were not acceptable. It was necessary to say what the conclusion of the test would be, in context, and to say what the reality was, again in context, for a Type I error to have been made.

(b) Errors were generally made in this part when declaring the hypotheses. The standardising was mostly well attempted with many candidates remembering to use $\sqrt{20}$. The comparison with 2.240

needed to be clearly shown; a common error was to use the one-tailed value of 1.96. The conclusion needed to be in context and not definite, for example 'there is evidence that the mean time has changed' is acceptable but 'the mean time has changed' is not.

Question 6

(a) This was a very well attempted question with candidates demonstrating they knew the difference between the biased and unbiased estimate of the population variance. There was some confusion noted between the two alternative formulae for the unbiased estimate of the population variance, however this was infrequent.

(b) This part was also well attempted; candidates knew the formula to use and mainly used the correct *z* value, though on occasions a *z*-value of 2.326 was used instead of 2.576. The final answer was required as an interval, and in general candidates knew to do this.

(c) Candidates found this question part to be more challenging. Many candidates did not realise that the probability of a 99% confidence interval containing the true value of μ was 0.99, so for all the 4 confidence intervals to contain the true value of μ the probability would be $(0.99)^4$. Some candidates attempted to find a 99% confidence interval, whilst others stated, for example, 0.99×4 or 0.01×4 or $(0.01)^4$. Many candidates omitted this part completely.

Question 7

(a) This part was generally well attempted. Many candidates correctly used Po(4.2) and used the correct Poisson expression.

(b) This part was also well attempted. The correct Poisson expression was usually seen and used; errors included omission of '1–…', an incorrect value for λ, or inclusion of an extra incorrect term. In most cases candidates showed all necessary working; unsupported answers, where answers did not show a clear Poisson expression, we not able to gain all of the available marks.

(c) Candidates appeared to have a good understanding to answer this question. Use of N(50.4, 50.4) was generally seen and standardising was usually done correctly, though an incorrect, or omission, of a continuity correction was sometimes seen.

MATHEMATICS

Paper 9709/63
Probability and Statistics 2

Key messages

Candidates should ensure that they show clear and full working when answering questions, to ensure they are able to gain the available marks. It was noted that in some places throughout candidates responses so essential working was omitted.

General comments

The conclusions to the hypotheses tests were required to be written in context, not definite and without contradictions. These requirements applied in **Question 2(b)** and **Question 3(a)(ii).**

In order to gain the available marks for a question part it was necessary for the answer to be fully supported. This could include listing Poisson terms as in **Question 3(a)(ii)** and **Question 5(b)** as well as showing the substitution of limits in integration questions as in **Question 6(a)** and **Question 6(b)**.

Comments on specific questions

Question 1

This question involved both the sum of variables and the multiple of a variable. The total number of goals (T), could be denoted by $X_1 + X_2 + \ldots + X_{10}$ giving $E(T) = 13.6$. Then the total amount of money given (G) was related to this by $G = 5T$ giving $E(G) = 68$.

As the variable X was given as a Poisson variable, the variance of T was also 13.6. Then the variance of G was $5^2 \times 13.6 = 340$. It was important to note that the standard deviation was required for this question. A frequent incorrect answer was to use variance of G as 68, resulting in a standard deviation of 8.25.

Question 2

(a) A Type II error is defined as being made when we accept H_0 when in fact it is false. To answer the question this had to be expressed in the context of the train journey times. Candidates needed to show that they were expressing the conclusion of the test and relating to the mean journey time and whether there was a decrease in the time or not. Words other than 'concluding' could be used, such as 'claiming' or 'suggesting'. It was essential to refer to the decrease in the time, not merely to say that the time had not changed.

(b) Many candidates made good progress on this question. Performing the test required the hypotheses, standardisation, comparison and the conclusion. The hypotheses needed to be one-tailed and to use μ or 'population mean'. The standardisation had to include $\sqrt{50}$. The comparison using z values needed –1.96 or if using probabilities 0.025 or 0.975 if the large probabilities were used. The conclusion needed to be in context, not definite and with no contradictions. Use of 'there is evidence that …' is recommended.

(c) The reason was essential here in order to score any marks. It was expected for candidates to state for example that H_0 was rejected and hence a Type I error might have been made. After an error by the candidate in **part (b)** follow through marks were allowed here for if they interpreted their conclusion correctly.

Question 3

(a) (i) The hypotheses needed to be one-tailed and to use λ. The mean for 3 years was 2.4, but the mean 0.8 for 1 year was also accepted so long as it was clearly stated.

(ii) Performing the test required the calculation of the probability tail, the comparison and the conclusion. It was necessary to list the Poisson probabilities for $P(\geqslant 5)$ by using $1 - P(<5)$ and to calculate the sum, 0.0959. This value needed to be compared to 0.05 for the significance level. The conclusion needed to be in context, not definite and with no contradictions. Many candidates answered most of this well. A common error was to include the P(5) term in $P(<5)$ giving 0.0357 which resulted in the opposite conclusion, however some follow though marks were allowed for this. Some candidates attempted to use only the single term P(5) which was an invalid approach.

(b) Candidates were required to refer to the changing mean number of accidents per year and how this affected the possible use of the Poisson distribution or otherwise and how this affected the validity of the test. A few candidates mentioned all three of these points. Most candidates omitted some or all of them.

Question 4

(a) Most candidates found the unbiased estimates correctly. A few candidates found the sample or biased variance, 0.5455… and so did not gain those marks.

(b) To find a 98% confidence interval for the population mean required an expression of the correct form and the correct value of z, 2.326. The answer needed to be an interval. Many candidates found this interval correctly.

(c) The distribution of the population of the masses of flour in the sacks was not given, so it was necessary to use the central limit theorem in **part (b)**.

(d) To find this probability it was necessary to consider the tails of the distribution and then to select the single relevant tail. Some candidates obtained $1 - 0.98 = 0.02$ but did not take the next step. Some candidates made detailed calculations involving standardisation which did not lead to the answer.

Question 5

(a) The given distribution was a binomial distribution B(25 000, 0.0 001). Some candidates gave this as their answer; however, the question required an approximating distribution. For the values involved this was a Poisson distribution Po(2.5). Here the required parameter was the mean $\lambda = 2.5$. The justification involved comparing the sample size 25000 to 50 and comparing the value of $np = 2.5$ to 5. Alternatively, the 25000 and the probability 0.0001 could be used. Some candidates gave only partial answers here and did not fully answer the question.

(b) Most candidates found this question straightforward and listed the Poisson terms and obtained the correct answer. A few candidates used an incorrect mean or omitted the P(3) term. Some candidates attempted to follow through from an incorrect distribution in **part (a)**, for which a follow through mark could be obtained if done correctly, however these candidates often made subsequent errors.

(c) Many candidates obtained the expressions for both Poisson terms correctly and put the first term equal to twice the second term. Many of these candidates worked through correctly to obtain the correct value for k. Other candidates made errors such as cancelling the factorial signs to reduce the denominators to k and (k + 1) resulting in $k = \frac{1}{4}$. Some candidates attempted a numerical approach and calculated $P(4) = 0.134$ and $2 \times P(5) = 0.134$ showing that $k = 4$.

(d) The suitable approximating distribution was Poisson $\text{Po}\left(\frac{n}{10000}\right)$. To deal with $P(Y \geqslant 1)$ it was necessary to consider $1 - P(Y = 0)$ and hence obtain $e^{-\lambda} = 0.037$. The process was then to take natural logs of both sides and to use $\lambda = \frac{n}{10000}$. The answer for *n* needed to be an integer. Some candidates omitted the '1 – ' step, whilst other candidates omitted the conversion from λ to *n*.

Question 6

(a) Candidates were required to consider one day for the lateness of the train and then to square that probability answer to gain the final answer for the two days. To find the first probability candidates needed to integrate f(*x*) between the limits 10 and 20. The integration could be done by integrating the given form to get $\frac{(x-20)^3}{3}$ or by expanding the given form and then integrating term by term. It was necessary to show the substitution of the limits as well as the answer. Many candidates did all of this correctly. Some candidates incorrectly used limits of 11 to 20. Other candidates made an error in expanding $(x - 20)^2$ initially. Some candidates made an error in trying to convert from one day to two days, including doubling instead of squaring or trying a binomial conversion.

(b) To find E(*X*) it was necessary to integrate *x*f(*x*) between the limits 0 and 20. The integration could be done by expanding *x*f(*x*) and then integrating term by term or by parts. It was necessary to show the substitution of the limits as well as the answer. Many candidates did this correctly.

(c) To work with the median, it was necessary to integrate f(x) and to equate this to 0.5. The limits could be either from 0 to *m* or from *m* to 20. The integration could be done by integrating the given form to get $\frac{(x-20)^3}{3}$ or by expanding the given form and then integrating term by term. A certain amount of convincing manipulation was needed to produce the given result. The final answer for *m* was obtained by solving the given equation.

(d) Many candidates stated a reasonable way in which the model may have been unrealistic. Examples included not allowing for trains to be more than 20 minutes late or not allowing for trains to be early.