Cambridge International A Level

**Cambridge International Examinations**
Cambridge International Advanced Level

**MATHEMATICS** **9709/32**

Paper 3 **May/June 2017**

MARK SCHEME

Maximum Mark: 75

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking $g$ equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

## Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

| Question | Answer | Marks |
|---|---|---|
| 1 | Use law of the logarithm of a power or a quotient | **M1** |
| | Remove logarithms and obtain a correct equation in $x$. e.g. $x^2+1=\mathrm{e}x^2$ | **A1** |
| | Obtain answer 0.763 and no other | **A1** |
| | **Total**: | **3** |
| 2 | *EITHER*:<br>State or imply non-modular inequality $(x-3)^2<(3x-4)^2$, or corresponding equation | **(B1** |
| | Make reasonable attempt at solving a three term quadratic | **M1** |
| | Obtain critical value $x=\frac{7}{4}$ | **A1** |
| | State final answer $x>\frac{7}{4}$ only | **A1)** |
| | *OR*1:<br>State the relevant critical inequality $3-x<3x-4$, or corresponding equation | **(B1** |
| | Solve for $x$ | **M1** |
| | Obtain critical value $x=\frac{7}{4}$ | **A1** |
| | State final answer $x>\frac{7}{4}$ only | **A1)** |
| | *OR*2:<br>Make recognizable sketches of $y=\lvert x-3\rvert$ and $y=3x-4$ on a single diagram | **(B1** |
| | Find $x$-coordinate of the intersection | **M1** |
| | Obtain $x=\frac{7}{4}$ | **A1** |
| | State final answer $x>\frac{7}{4}$ only | **A1)** |
| | **Total:** | **4** |

| Question | Answer | Marks |
|---|---|---|
| 3(i) | Use correct formulae to express the equation in terms of cos $\theta$ and sin $\theta$ | M1 |
| | Use Pythagoras and express the equation in terms of cos $\theta$ only | M1 |
| | Obtain correct 3-term equation, e.g. $2\cos^4\theta + \cos^2\theta - 2 = 0$ | A1 |
| | **Total:** | **3** |
| 3(ii) | Solve a 3-term quadratic in $\cos^2\theta$ for cos $\theta$ | M1 |
| | Obtain answer $\theta = 152.1°$ only | A1 |
| | **Total:** | **2** |
| 4(i) | State $\frac{\mathrm{d}y}{\mathrm{d}t} = 4 + \frac{2}{2t-1}$ | B1 |
| | Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$ | M1 |
| | Obtain answer $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8t-2}{2t(2t-1)}$, or equivalent e.g. $\frac{2}{t} + \frac{2}{4t^2 - 2t}$ | A1 |
| | **Total:** | **3** |
| 4(ii) | Use correct method to find the gradient of the normal at $t = 1$ | M1 |
| | Use a correct method to form an equation for the normal at $t = 1$ | M1 |
| | Obtain final answer $x + 3y - 14 = 0$, or horizontal equivalent | A1 |
| | **Total:** | **3** |

| Question | Answer | Marks |
|---|---|---|
| 5(i) | State $\dfrac{dy}{dt} = -\dfrac{2y}{(1+t)^2}$, or equivalent | **B1** |
| | Separate variables correctly and attempt integration of one side | **M1** |
| | Obtain term $\ln y$, or equivalent | **A1** |
| | Obtain term $\dfrac{2}{(1+t)}$, or equivalent | **A1** |
| | Use $y = 100$ and $t = 0$ to evaluate a constant, or as limits in an expression containing terms of the form $a \ln y$ and $\dfrac{b}{1+t}$ | **M1** |
| | Obtain correct solution in any form, e.g. $\ln y = \dfrac{2}{1+t} - 2 + \ln 100$ | **A1** |
| | **Total:** | **6** |
| 5(ii) | State that the mass of $B$ approaches $\dfrac{100}{e^2}$, or **exact** equivalent | **B1** |
| | State or imply that the mass of $A$ tends to zero | **B1** |
| | **Total:** | **2** |

| Question | Answer | Marks |
|---|---|---|
| 6(i) | *EITHER:*<br>Substitute $x = 2 - \mathrm{i}$ (or $x = 2 + \mathrm{i}$) in the equation and attempt expansions of $x^2$ and $x^3$ | **(M1** |
| | Equate real and/or imaginary parts to zero | **M1** |
| | Obtain $a = -2$ | **A1** |
| | Obtain $b = 10$ | **A1)** |
| | *OR*1:<br>Substitute $x = 2 - \mathrm{i}$ in the equation and attempt expansions of $x^2$ and $x^3$ | **(M1** |
| | Substitute $x = 2 + \mathrm{i}$ in the equation and add/subtract the two equations | **M1** |
| | Obtain $a = -2$ | **A1** |
| | Obtain $b = 10$ | **A1)** |
| | *OR*2:<br>Factorise to obtain $(x-2+\mathrm{i})(x-2-\mathrm{i})(x-p)$ $\left(=\left(x^2-4x+5\right)(x-p)\right)$ | **(M1** |
| | Compare coefficients | **M1** |
| | Obtain $a = -2$ | **A1** |
| | Obtain $b = 10$ | **A1)** |
| | *OR*3:<br>Obtain the quadratic factor $(x^2 - 4x + 5)$ | **(M1** |
| | Use algebraic division to obtain a real linear factor of the form $x - p$ and set the remainder equal to zero | **M1** |
| | Obtain $a = -2$ | **A1** |
| | Obtain $b = 10$ | **A1)** |
| | *OR*4:<br>Use $\alpha\beta = 5$ and $\alpha + \beta = 4$ in $\alpha\beta + \beta\gamma + \gamma\alpha = -3$ | **(M1** |
| | Solve for $\gamma$ and use in $\alpha\beta\gamma = -b$ and/or $\alpha + \beta + \gamma = -a$ | **M1** |
| | Obtain $a = -2$ | **A1** |
| | Obtain $b = 10$ | **A1)** |

| Question | Answer | Marks |
|---|---|---|
| | *OR5*:<br>Factorise as $(x – (2\text{-i}))(x^2 + ex + g)$ and compare coefficients to form an equation in $a$ and $b$ | **(M1** |
| | Equate real and/or imaginary parts to zero | **M1** |
| | Obtain $a = –2$ | **A1** |
| | Obtain $b = 10$ | **A1)** |
| | **Total:** | **4** |
| 6(ii) | Show a circle with centre $2 - \text{i}$ in a relatively correct position | **B1** |
| | Show a circle with radius 1 and centre not at the origin | **B1** |
| | Show the perpendicular bisector of the line segment joining 0 to – i | **B1** |
| | Shade the correct region | **B1** |
| | **Total:** | **4** |
| 7(i) | Use quotient or chain rule | **M1** |
| | Obtain given answer correctly | **A1** |
| | **Total:** | **2** |
| 7(ii) | *EITHER*:<br>Multiply numerator and denominator of LHS by $1+\sin\theta$ | **(M1** |
| | Use Pythagoras and express LHS in terms of sec $\theta$ and tan $\theta$ | **M1** |
| | Complete the proof | **A1)** |
| | *OR1:*<br>Express RHS in terms of cos $\theta$ and sin $\theta$ | **(M1** |
| | Use Pythagoras and express RHS in terms of sin $\theta$ | **M1** |
| | Complete the proof | **A1)** |
| | *OR2:*<br>Express LHS in terms of sec$\theta$ and tan$\theta$ | **(M1** |
| | Multiply numerator and denominator by sec$\theta$ + tan$\theta$ and use Pythagoras | **M1** |
| | Complete the proof | **A1)** |
| | **Total:** | **3** |

| Question | Answer | Marks |
|---|---|---|
| 7(iii) | Use the identity and obtain integral $2\tan\theta + 2\sec\theta - \theta$ | **B2** |
| | Use correct limits correctly in an integral containing terms $a\tan\theta$ and $b\sec\theta$ | **M1** |
| | Obtain answer $2\sqrt{2} - \frac{1}{4}\pi$ | **A1** |
| | **Total:** | **4** |
| 8(i) | State or imply the form $\frac{A}{3x+2} + \frac{Bx+C}{x^2+5}$ | **B1** |
| | Use a relevant method to determine a constant | **M1** |
| | Obtain one of the values $A = 2$, $B = 1$, $C = -3$ | **A1** |
| | Obtain a second value | **A1** |
| | Obtain the third value | **A1** |
| | **Total:** | **5** |
| 8(ii) | Use correct method to find the first two terms of the expansion of $(3x+2)^{-1}$, $(1+\frac{3}{2}x)^{-1}$, $(5+x^2)^{-1}$ or $(1+\frac{1}{5}x^2)^{-1}$<br>[Symbolic coefficients, e.g. $\begin{pmatrix}-1\\2\end{pmatrix}$ are not sufficient] | **M1** |
| | Obtain correct unsimplified expansions up to the term in $x^2$ of each partial fraction.<br>The FT is on $A$, $B$, $C$. from part (i) | **A1FT + A1FT** |
| | Multiply out up to the term in $x^2$ by $Bx + C$, where $BC \neq 0$ | **M1** |
| | Obtain **final answer** $\frac{2}{5} - \frac{13}{10}x + \frac{237}{100}x^2$, or equivalent | **A1** |
| | **Total:** | **5** |
| 9(i) | *EITHER*:<br>Find $\overrightarrow{AP}$ for a general point $P$ on $l$ with parameter $\lambda$, e.g.$(8 + 3\lambda, -3 - \lambda, 4 + 2\lambda)$ | **(B1** |
| | Equate scalar product of $\overrightarrow{AP}$ and direction vector of $l$ to zero and solve for $\lambda$ | **M1** |
| | Obtain $\lambda = -\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$ | **A1** |
| | Carry out a complete method for finding the position vector of the reflection of $A$ in $l$ | **M1** |
| | Obtain answer $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ | **A1)** |

| Question | Answer | Marks |
|---|---|---|
| | *OR:* Find $\overrightarrow{AP}$ for a general point $P$ on $l$ with parameter $\lambda$, e.g.$(8 + 3\lambda, -3 - \lambda, 4 + 2\lambda)$ | (B1 |
| | Differentiate $\lvert AP \rvert^2$ and solve for $\lambda$ at minimum | M1 |
| | Obtain $\lambda = -\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$ | A1 |
| | Carry out a complete method for finding the position vector of the reflection of $A$ in $l$ | M1 |
| | Obtain answer $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ | A1) |
| | **Total:** | **5** |
| 9(ii) | *EITHER:* Use scalar product to obtain an equation in $a$, $b$ and $c$, e.g. $3a - b + 2c = 0$ | (B1 |
| | Form a second relevant equation, e.g. $9a - b + 8c = 0$ and solve for one ratio, e.g. $a : b$ | M1 |
| | Obtain final answer $a : b : c = 1 : 1 : -1$ and state plane equation $x + y - z = 0$ | A1) |
| | *OR*1: Attempt to calculate vector product of two relevant vectors, e.g. $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \times (9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$ | (M1 |
| | Obtain two correct components | A1 |
| | Obtain correct answer, e.g. $-6\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$, and state plane equation $-x - y + z = 0$ | A1) |
| | *OR*2: Using a relevant point and relevant vectors, attempt to form a 2-parameter equation for the plane, e.g. $\mathbf{r} = 6\mathbf{i} + 6\mathbf{k} + s(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + t(9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$ | (M1 |
| | State 3 correct equations in $x$, $y$, $z$, $s$ and $t$ | A1 |
| | Eliminate $s$ and $t$ and state plane equation $x + y - z = 0$, or equivalent | A1) |
| | *OR*3: Using a relevant point and relevant vectors, attempt to form a determinant equation for the plane, e.g. $\begin{vmatrix} x-3 & y-1 & z-4 \\ 3 & -1 & 2 \\ 9 & -1 & 8 \end{vmatrix} = 0$ | (M1 |
| | Expand a correct determinant and obtain two correct cofactors | A1 |
| | Obtain answer $-6x - 6y + 6z = 0$, or equivalent | A1) |
| | **Total:** | **3** |

| Question | Answer | Marks |
|---|---|---|
| 9(iii) | *EITHER*: Using the correct processes, divide the scalar product of $\overrightarrow{OA}$ and a normal to the plane by the modulus of the normal or make a recognisable attempt to apply the perpendicular formula | **(M1** |
| | Obtain a correct expression in any form, e.g. $\frac{1+2-4}{\sqrt{(1^2+1^2+(-1)^2)}}$, or equivalent | **A1 FT** |
| | Obtain answer $1/\sqrt{3}$, or exact equivalent | **A1)** |
| | *OR1*: Obtain equation of the parallel plane through $A$, e.g. $x+y-z=-1$ [The f.t. is on the plane found in part (**ii**).] | **(B1 FT** |
| | Use correct method to find its distance from the origin | **M1** |
| | Obtain answer $1/\sqrt{3}$, or exact equivalent | **A1)** |
| | *OR2:* Form equation for the intersection of the perpendicular through $A$ and the plane [FT on their **n**] | **(B1 FT** |
| | Solve for $\lambda$ | **M1** |
| | $\lvert\lambda\mathbf{n}\rvert=\frac{1}{\sqrt{3}}$ | **A1)** |
| | **Total:** | **3** |
| 10(i) | Use correct product rule | **M1** |
| | Obtain correct derivative in any form $\left(y'=2x\cos 2x-2x^2\sin 2x\right)$ | **A1** |
| | Equate to zero and derive the given equation | **A1** |
| | **Total**: | **3** |
| 10(ii) | Use the iterative formula correctly at least once e.g. $0.5\to 0.55357\to 0.53261\to 0.54070\to 0.53755$ | **M1** |
| | Obtain final answer 0.54 | **A1** |
| | Show sufficient iterations to 4 d.p. to justify 0.54 to 2 d.p., or show there is a sign change in the interval (0.535, 0.545) | **A1** |
| | **Total:** | **3** |

| Question | Answer | Marks |
|---|---|---|
| 10(iii) | Integrate by parts and reach $ax^2 \sin 2x + b\int x \sin 2x \, dx$ | *M1 |
| | Obtain $\frac{1}{2}x^2 \sin 2x - \int 2x.\frac{1}{2}\sin 2x \, dx$ | A1 |
| | Complete integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x$, or equivalent | A1 |
| | Substitute limits $x = 0$, $x = \frac{1}{4}\pi$, having integrated twice | DM1 |
| | Obtain answer $\frac{1}{32}(\pi^2 - 8)$, or exact equivalent | A1 |
| | **Total:** | **5** |