Cambridge International Examinations
Cambridge International Advanced Subsidiary Level

MATHEMATICS **9709/22**

Paper 2 **October/November 2016**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	State non-modulus equation $(0.4x-0.8)^2=4$ or equivalent or corresponding pair of linear equations	**B1**		SR One solution only – B1
	Solve 3-term quadratic equation or pair of linear equations	**M1**		Must see some evidence of attempt to solve the quadratic for M1 for at least one value of x For a pair of linear equations, there must be a sign difference
	Obtain -3 and 7	**A1**	[3]	If extra solutions are given then A0
2 (i)	Use $4^y=2^{2y}$	**B1**		
	Attempt solution of quadratic equation in 2^y	**M1**		
	Obtain finally $2^y=7$ only	**A1**	[3]	
(ii)	Apply logarithms to solve equation of form $2^y=k$ where $k>0$	**M1**		Must be using their positive answer for (i)
	Obtain 2.81	**A1**	[2]	
3 (i)	Obtain integral of form $k_1\mathrm{e}^{\frac{1}{2}x}+k_2x$	**M1**		Allow $k_1=4$
	Obtain correct $8\mathrm{e}^{\frac{1}{2}x}+3x$ oe	**A1**		
	Use limits correctly to confirm $8\mathrm{e}-2$	**A1**	[3]	
(ii)	Draw increasing curve in first quadrant	**M1**		If incorrect y intercept used then M1 A0
	Draw more or less accurate sketch with correct curvature, gradient at $x=0$ must be >0	**A1**	[2]	Allow if no intercept stated
(iii)	State more and refer to top(s) of trapezium(s) above curve	**B1**	[1]	Can be shown using a diagram. Reference to a trapezium must be made

4 **(i)**	Substitute $x=-1$ and simplify	**M1**		Allow attempt at long division , must get down to a remainder Allow M1 if at least 2 numerical values of a are used May equate to $(x+1)\left(Ax^2+Bx+C\right)+R$ - allow M1 if they get as far as finding R
	Obtain $-4+a-a+4=0$ and conclude appropriately	**A1**	[2]	Must have a conclusion - allow 'hence shown', or made a statement of intent at the start of the question
(ii)	Substitute $x=2$ and equate to -42 and attempt to solve	**M1**		May equate to $(x-2)\left(Ax^2+Bx+C\right)$, must have a complete method to get as far as $a=$ ... to obtain M1
	Obtain $a=-13$	**A1**	[2]	
(iii)	Divide $\text{p}(x)$ with their a at least as far as $4x^2+kx$	**M1**		
	Obtain $4x^2-17x+4$	**A1**		
	Obtain $(x+1)(4x-1)(x-4)$ or equivalent if x^2 already involved	**A1**		If $(x+1)(4x-1)(x-4)$ seen with no evidence of long division then allow the marks
	Obtain $(x^2+1)(2x-1)(2x+1)(x-2)(x+2)$	**A1**	[4]	
5 **(i)**	Use quotient rule (or product rule) to find first derivative	**M1**		Quotient: Must have a difference in the numerator and $(x^2+1)^2$ in the denominator
	Obtain $\dfrac{\frac{4}{x}(x^2+1)-8x\ln x}{(x^2+1)^2}$ or equivalent	**A1**		Product: Must see an application of the chain rule.
	State $\frac{4}{x}(x^2+1)-8x\ln x=0$ or equivalent	**A1**		Condone missing brackets if correct use is implied by correct work later
	Carry out correct process to produce equation without ln, without any incorrect working	**M1**		
	Confirm $m=\text{e}^{0.5(1+m^{-2})}$ or $x=\text{e}^{0.5(1+x^{-2})}$	**A1**	[5]	

(ii)	Use iterative formula correctly at least once	**M1**		Should not be attempting to use $x_0 = 0$, but if used and 'recovered' then SC M1 A1- usually see $m_1 = 1.6487$
	Obtain final answer 1.895	**A1**		
	Show sufficient iterations to 6 sf to justify answer or show sign change in interval $(1.8945, 1.8955)$	**A1**	[3]	
6 (i)	Use $\cos 2\theta = 2\cos^2\theta - 1$ appropriately twice	**B1**		Alternative method $\frac{1-2\sin^2\theta}{2\cos^2\theta} = \frac{1}{2}\sec^2\theta - \tan^2\theta$ or $\frac{1}{2\cos^2\theta} - \tan^2\theta$ B1
	Simplify to confirm $1 - \frac{1}{2}\sec^2\theta$	**B1**	[2]	then as for 2nd B1
(ii)	Use $\sec^2\alpha = 1 + \tan^2\alpha$	**B1**		
	Obtain equation $\tan^2\alpha + 10\tan\alpha + 25 = 0$ or equivalent	**B1**		
	Attempt solution of 3-term quadratic equation for $\tan\alpha$ and use correct process for finding value of α from negative value of $\tan\alpha$	**M1**		If quadratic is incorrect, need to see evidence of attempt to solve as required to obtain M1
	Obtain 1.77	**A1**	[4]	Allow better or in terms of π $\left(\frac{1013\pi}{1800}\right)$
(iii)	State or imply integrand $1 - \frac{1}{2}\sec^2\frac{1}{2}x$	**B1**		
	Obtain integral of form $k_1x - k_2\tan\frac{1}{2}x$	**M1**		
	Obtain correct $x - \tan\frac{1}{2}x$	**A1**		
	Apply limits correctly to obtain $\pi - 2$	**A1**	[4]	

7 **(i)**	Use correct addition formula for either $\cos(\theta + \frac{1}{6}\pi)$ or, after diffn, $\sin(\theta + \frac{1}{6}\pi)$	**B1**		Condone 'missing brackets'
	Differentiate to obtain $\frac{dy}{d\theta}$ of form $k_1 \sin\theta + k_2 \cos\theta$ or $k\sin(\theta + \frac{1}{6}\pi)$	**M1**		
	Divide attempt at $\frac{dy}{d\theta}$ by attempt at $\frac{dx}{d\theta}$	**M1**		
	Obtain $\dfrac{-\frac{3\sqrt{3}}{2}\sin\theta - \frac{3}{2}\cos\theta}{4\cos\theta}$ or equivalent	**A1**		
	Simplify to obtain $-\frac{3}{8}(1+\sqrt{3}\tan\theta)$	**A1**	[5]	
(ii)	Identify $\theta = 0$	**B1**		soi
	Substitute 0 into formula for $\frac{dy}{dx}$ and take negative reciprocal	**M1**		be implied by $y = 1 + \frac{3\sqrt{3}}{2}$ or 3.6 Must be from correct **(i)**
	Obtain gradient of normal $\frac{8}{3}$	**A1**		
	Form equation of normal through point $(0, 1 + \frac{3\sqrt{3}}{2})$	**M1**		
	Obtain $y = \frac{8}{3}x + 1 + \frac{3\sqrt{3}}{2}$ or equivalent	**A1**	[5]	