MATHEMATICS

| Paper 9709/11 |
| Paper 11 |

Key Messages

Candidates should ensure that when a question asks for solutions or values that they consider other possible solutions which may arise from their working.

In this paper there were several instances where the inability to deal with fractions within equations caused marks to be unnecessarily lost.

General Comments

Presentation of solutions was an issue for some candidates. Working down a page in logical steps is highly recommended. When answers or conclusions are given it is very important to show each stage in the process of reaching them.

Comments on Specific Questions

Question 1

This question provided a good start to the paper for most candidates. Finding the appropriate two terms of the expansion of $(a + x)^5$ and multiplying these by 1 and $\dfrac{-2x}{a}$ respectively provided the most direct route to the given result. Those who chose to calculate every term and select the terms in x^2 were generally less successful.

Question 2

The candidates who realised integration with a constant of integration was required scored well on this question. y was sometimes used instead of f(x) but this was not penalised.

Answer: f(x) = x^3 + 7x – 1

Question 3

The use of sin^{-1} confused some candidates but those who realised $sin^{-1}A = \dfrac{1}{6}\pi \Rightarrow A = sin\dfrac{\pi}{6}$ were able to obtain the required quartic equation. It was expected that this would be solved as a quadratic in x^2. The successful candidates expressed this clearly; usually by use of a substitution such as $u = x^2$. Those who reached the only viable solution $x^2 = \dfrac{1}{4}$ often then omitted the negative square root.

Answer: $x = \pm\dfrac{1}{2}$

Question 4

In **part (i)** the appropriate use of $tan\theta = sin\theta / cos\theta$ and $cos^2\theta = 1 - sin^2\theta$ were often seen but the final elimination of the denominator was not as successful.

The use of the result from **part (i)** in **part (ii)** was generally appreciated enabling the one possible value of $sin\theta$ to be found. When this negative value was dealt with correctly both solutions within the given range were usually reached. Those who quoted the correct solutions in radians were given some credit.

Answers: **(ii)** 194.5 *and* 345.5

Question 5

This question was answered well by the majority of candidates.

In **part (i)** the differentiation of y and $\dfrac{dy}{dx}$ caused few problems and in **part (ii)** many correct solutions were seen. Some candidates lost marks by only failing to find the y coordinates of the stationary points.

It should be emphasised again that a complete and unambiguous solution is required if a turning point is to be identified using the change in sign of the gradient rather than the previously calculated expression for $\dfrac{d^2y}{dx^2}$.

Answer: **(i)** $\dfrac{dy}{dx} = \dfrac{-8}{x^2} + 2, \dfrac{d^2y}{dx^2} = \dfrac{16}{x^2}$ **(ii)** (2,8) *Minimum*, (−2,−8) *Maximum*

Question 6

In **part (i)** the two expressions for y were usually equated and rearranged correctly to produce the given answer.

Part (ii) inspired the use of a variety of methods with many choosing to substitute $x = -1$ into the given result to find a and then solving the quadratic for the second value of x. Those who appreciated the sum of the roots of the quadratic equalled 4 arrived at the value in one step.

The form of the result in **part (i)** was there to suggest the use of the discriminant in **part (iii)** and this was often seen. Those who chose to equate the gradients of the line and curve at point P were equally successful.

Answers: **(ii)** 5   **(iii)** −1, (2,5)

Question 7

This question proved to be very testing for many candidates.

The given result in **part (i)** was reached using Pythagoras' theorem or trigonometry.

In **part (ii)** the request for an answer in terms of r was not always adhered to with answers being presented in terms of r, π and even θ. The best solutions were clearly explained with reference to the given letters and/or the names of the parts of the circle being considered. The use of diagrams of these parts was equally successful. Those who subtracted the segment area of the large circle from the area of the small semi-circle and appreciated that *CBD* was a right angle usually went on to a correct final solution.

Answer: **(ii)** r^2

Question 8

Candidates were generally able to select the appropriate formulae required in both parts of this question.

In **part (i)** the required quadratic equation was often stated initially although some used the nth term formula to derive it. From these starting points completely correct solutions were often seen. Those who 'spotted' and used the positive value of x received some credit.

Part (ii) required good algebraic skills to eliminate x or r. As in **question 4(i)** those able to deal with variables in a denominator were usually able to reach the final answer.

Answers: **(i)** −2 and 6, 16 and 48   **(ii)** $\dfrac{16}{27}$

Question 9

A variety of methods were used successfully to complete the square in **part (i)**. Most adhered to the required form stated in the question.

In **part (ii)** the value of $-b$ was required. When this was appreciated the correct or followed through answer was usually reached. Some arrived at the same result using calculus or detailed sketches.

In **part (iii)** those who realised they should use their answer to **part (i)** quickly reached the required expression. Some unnecessarily restarted the process of completing the square. Those who attempted to use flow diagrams were invariably unsuccessful.

To obtain the domain it was important to note the given value, $m = 5$. The frequency with which this part was omitted suggests some confusion over the idea of a domain.

Answers: **(i)** $a = -1$, $b = -3$, $c = 4$ **(ii)** 3   **(iii)** $3+\sqrt{4-x}$, $x \leqslant 0$

Question 10

The need to find the vector path from P to M in **part (i)** was well understood and well attempted. After finding $\overrightarrow{PM}$ it was quite common to see no attempt at the unit vector. Some found the magnitude of $\overrightarrow{PM}$ but did not go on to use it to find the unit vector.

Of those who attempted **part (ii)** most used the intended vector method (scalar product) rather than the cosine rule. The form in which angle ATP was presented caused some problems as did the algebraic manipulation. The correct final answer was seen only occasionally.

Answers: **(i)** $\dfrac{1}{2}(2\boldsymbol{i} + 3\boldsymbol{j} - 6\boldsymbol{k})$ **(ii)** 3

Question 11

The required differentiation in **part (i)** was often seen correctly carried out. The gradient relationship between perpendicular lines was the favoured route to the given conclusion but occasionally candidates used the gradient of the curve at P to find the gradient of the perpendicular through P and verified that Q was on the line. In some cases more explanation of the method used would have allowed all the marks to be awarded.

In **part (ii)** the correct squaring of y led to a straightforward integration with relatively obvious limits. The suggested route for the area under the line was not always used and there was some confusion over which length was the height and which the radius. Those who chose to use the equation of PQ often found the integral and substitution very difficult to complete. Some neat, concise and well-explained answers were seen.

Answer: **(ii)** $\dfrac{284\pi}{3}$

MATHEMATICS

| Paper 9709/12 |
| Paper 12 |

Key Messages

Candidates should try not to be thrown off course by questions set in context. The mathematics involved in such questions is often quite straightforward providing candidates do not panic but concentrate on reading the question very carefully when they are extracting the relevant information.

Extra care needs to be taken to avoid careless numerical errors which can cause a considerable loss of marks.

General Comments

The paper did not seem to be as well received by the candidates as in previous years although many good and excellent scripts were seen. Questions **3**, **4(i)**, **6(i)**, **7(i)** and **8** proved to be a difficult challenge for many. The standard of presentation was generally good with candidates setting their work out in a clear readable fashion. The paper seemed to work quite well with a number of questions being reasonably straightforward giving all candidates the opportunity to show what they had learned and understood, but also some questions which provided more of a challenge, even for candidates of good ability. Most candidates appeared to have sufficient time to complete the paper although a few good candidates did seem to struggle. Those candidates who started questions on the bottom line of a page often made errors in transferring information to the next page. It would be helpful to advise these candidates not to start work when they can tell that they will not have enough room to complete a section of the question. Questions with given answers do need to be fully justified and this wasn't always done sufficiently.

Comments on Specific Questions

Question 1

This question was generally done very well with candidates finding the appropriate inverse and composite functions, equating them and then solving the resultant equation correctly. Some candidates found fg(x) instead of gf(x) or multiplied f by g but this was rare. A more common mistake was $35x = 10 \Rightarrow x = 3.5$.

Answer: $x = \dfrac{2}{7}$

Question 2

This was a standard question on the binomial expansion and was a good source of marks for the vast majority of candidates. They generally could correctly pick out the relevant terms in the expansions, equate them and then solve the resulting equation effectively. Errors such as ignoring the powers of 2 or including x's in the resulting equation were seen rarely but $10k = 3$ then $k = 10/3$ was surprisingly common.

Answer: $k = 0.3$

Question 3

This question set in context proved to be a real challenge for many candidates with a number making no attempt to answer it. In part **(i)** candidates seemed to struggle with the context and the diagram in spite of the end view, diagram 2. Many attempted to obtain the given answer without any clear justification. Candidates who found the area of the triangle either by $\frac{1}{2}$ base $\times$ height or $\frac{1}{2}$ $ab\sin C$ and then multiplied by 40 rather than worrying about the answer were much more successful. In part **(ii)** many candidates realised that the chain rule was needed and most attempted to differentiate the given answer from part **(i)**. After that a poor understanding of the notation often led to the terms being wrongly divided or multiplied

Answer: **(ii)** 0.289

Question 4

In part **(i)** many candidates found proving the given identity very difficult. Those who started with the right hand side were very rarely successful but very many correct solutions starting with the left hand side were seen. Candidates who combined the two fractions before squaring were able to obtain the given answer more quickly generally. Many candidates obtained $\cos^2 x - 2\cos x + 1 = 0$ for the numerator but factorised it as $(\cos x - 1)(\cos x - 1)$ rather than $(1 - \cos x)(1 - \cos x)$ and then struggled to obtain the given answer correctly. Those who combined the fractions first avoided the potential for this error. In part **(ii)** the vast majority of candidates used the answer from part **(i)** to obtain the correct value of $\cos x$ but some then did not give the answers to 3 significant figures and many more used degrees instead of radians.

Answer: **(ii)** x = 1.13, 5.16

Question 5

This question proved to be straightforward for the vast majority of candidates. In part **(ii)** some candidates did not use the given answer from part **(i)** but used a less accurate approximation of their own. The arc length was usually found successfully but some candidates used 6 as the radius or used $\pi - 0.6$ rather than $\frac{\pi}{2} - 0.6$ as the angle. Some did not realise that OB was not part of the perimeter and for others premature approximation lead to accuracy marks being lost. Changing from radians to degrees was usually not successful. Similar errors occurred in part **(iii)**.

Answers: **(ii)** 24.4, **(iii)** 38.0

Question 6

Many candidates struggled with part **(i)** due to not appreciating that it was the lengths of *AB* and *BC* that were equal and not the gradients. Those who understood the situation were able to solve the resulting quadratic equation with relative ease. Many candidates were able to score full marks in part **(ii)** even if they had made no progress with part **(i)**. Errors included not using the perpendicular gradient or the midpoint, setting x = 0 at the end rather than y = 0 and making $\sqrt{6^2 + (1-k)^2}$ equal to 6 + (1 − *k*).

Answers: **(i)** k = –7 *and* 9 **(ii)** (–2,0)

Question 7

The vast majority of candidates were able to score some marks on this question although scoring full marks was quite rare. A minority of candidates were unsure how to start the question or used the position vectors of the original points and so received no credit. Part **(i)** proved to be most problematic with the majority of candidates being unsure how to make a start on it. Many worked out two of the vectors $\overrightarrow{AB}, \overrightarrow{AC}$ or $\overrightarrow{BC}$ correctly but then often equated them rather than realising that one had to be a scalar multiple of the other. Those who realised the link were easily able to complete the question. Candidates were more successful in part **(ii)** although a sizeable minority calculated the scalar product of $\overrightarrow{AB}$ and $\overrightarrow{BC}$ thereby making the right angle to be at *B* rather than at *A*. Part **(iii)** was generally successfully completed with a correct quadratic equation being formed and solved although many sign errors occurred particularly with the **k** component of vector $\overrightarrow{AB}$ which should have been –2 – –3 = 1 but was often found to be –5 instead. It was not uncommon to see $q^2 + 5q + 1 = 0$ factorised to give $(q - 5)(q - 1) = 0$. Candidates should take extra care to ensure that accuracy marks are not lost through this type of error.

Answers: **(i)** *p* = 6.5 *and q* = –1.5 **(ii)** *q* = –3*p* – 3 **(iii)** *q* = –1 *or* –5

Question 8

Again in this question the vast majority of candidates were able to score some marks but scoring full marks was quite rare. In part **(i)** the large majority of candidates realised the need to complete the square in order to find the range although this was not always done correctly and some failed to see the correct link with the calculated values, with a number using the 3 rather than the –17. It should be noted that the range should not be expressed as $x \geqslant$ (i.e. with *x* the subject of the inequality) and this was penalised. Those simply attempting to substitute different values in a trial and improvement method were generally not successful. In part **(ii)** the mention of roots led many candidates to incorrectly consider that $b^2 - 4ac = 0$ and thus obtain no credit. Those who realised that the factors would be (*x* – *k*)(*x* + 2*k*) and then equated the coefficients of the expansion with the given equation were most successful and with less work needed. Alternative approaches such as substituting *k* and –2*k* into the given equation and solving simultaneously were more complex and rarely fully successful. In part **(iii)** many candidates seemed unable to deal with f(*x* + *a*) correctly and f(*x* + *a*) = f(*x*) + f(*a*) was sometimes seen. This meant that no progress could be made with the question although the vast majority realised that $b^2 - 4ac < 0$ was needed. Some obtained the wrong value for *c* (not including the –*a*) as they did not obtain a quadratic equation = 0 before using $b^2 - 4ac < 0$. A few very good candidates realised that a quadratic equation in (*x* + *a*) was obtained and therefore $b^2 - 4ac < 0$ could be applied directly.

Answers: **(i)** f(*x*) $\geqslant$ –17 **(ii)** *k* = 5, *b* = –50

Question 9

The majority of candidates realised the need to integrate twice and many were able to score full marks on this fairly standard question. Some candidates forgot about the constant of integration or incorrectly used the point (2,10) after only integrating once. This also led to lost marks in the second and third parts of the question. Some candidates failed to correctly label f(*x*) and f′(*x*) and so the wrong expression for the first derivative was then used in part **(ii)**. A few more able candidates did not answer the question fully and only stated the *x* co-ordinate. In part **(iii)** comments such as: "The first is a min and the second is a max", were too vague to receive any credit. A minority of candidates did not seem to understand the f″(*x*) notation. Candidates would perhaps benefit from more practice on the type of question.

Answers: **(i)** $f(x) = \dfrac{6}{x} + \dfrac{3x}{2} + 4$ **(ii)** (–2,–2) **(iii)** *x* = 2 *min, x* = –2 *max*

Question 10

In part **(i)** many candidates realised the need to differentiate in order to find the gradient of the curve and then subsequently found the perpendicular gradient of the normal. A common mistake was forgetting to multiply by ($-4x$) in the differentiation or multiplying by -4, x, $2x$ or $-2x$ instead. Some candidates also failed to justify the given statement sufficiently. In part **(ii)** many candidates integrated about the x-axis in spite of **y-axis** being in bold type. Those who realised what was required were usually successful although some struggled to obtain x on its own and then square it, whereas those finding x^2 to begin with were much more successful. Some candidates had problems with the limits and a minority failed to give the final answer exactly.

Answer: **(ii)** $\dfrac{14\pi}{3}$

MATHEMATICS

| Paper 9709/13 |
| Paper 13 |

Key Messages

1.   Whilst the majority of candidates presented their work well and in an organised way, there was nevertheless a significant minority whose work was presented poorly. For example, many of these candidates attempted to compress their work onto two sheets of paper and this practice often leads to the omission of essential working and, potentially, the loss of marks. If this compression of the work is further exacerbated by crossings out with amendments squeezed into small spaces, the work can get extremely difficult, sometimes impossible, to read. In the same vein, Examiners are still seeing scripts in which the candidates have presented their work on some, or all, of the pages in two columns. Candidates should be encouraged to present their work reasonably spaced out so that, if necessary, they can amend their work more easily and also so that there is less chance that the solution of one question gets mixed up with the solution of another question.

2.   **Question 6** was a question set in context and it was very poorly done. It is an important skill to be able to translate everyday situations into a form that can be expressed mathematically and to be able to extract information from the context correctly. For this reason it is suggested that candidates are given practice in tackling such questions and that they are encouraged to read the wording of questions very carefully. For example it was emphasised in **Question 6** that the distance required was 'up and down' but this fact was overlooked by a substantial proportion of candidates.

General Comments

The paper was generally well received by candidates and many excellent scripts were seen. All candidates seemed to have sufficient time to finish the paper. Some questions (e.g., **Questions 6(ii)** and **7(a)**), require candidates to show that a particular result is true. When a result is given it is important that candidates understand that each step needs to be shown clearly so that Examiners are left in no doubt at all that the result has been obtained legitimately. Marks are frequently lost in trigonometry questions in particular when candidates are asked to prove a given identity and steps, which might seem obvious to candidates, are omitted.

Comments on Specific Questions

Question 1

Most candidates managed to score the first two marks in this question but the final mark was frequently not scored. This was most often due to failing to reverse the inequality having reached $-4c < -8$ - giving the answer as $c < 2$. Other common errors included setting the discriminant to ' $= 0$ ' or even ' > 0 ' as well as errors made when removing the bracket from $36 - 4(c + 7)$. Other valid methods, not involving the use of the discriminant, were also occasionally seen.

Answer: $c > 2$.

Question 2

This question was mainly very well done with many candidates scoring full marks. Some candidates wrote out all the terms of the expansion before recognising the required term.

Answer: 7.

Question 3

Part **(i)** was answered well and most candidates scored full marks for this part of the question. In contrast, only a small proportion of candidates achieved full marks for part **(ii)**. Almost all candidates obtained the correct derivative of the function but only a small number went on to make the connection between part **(i)** and part **(ii)** and express the derivative as $3(x-1)^2 + 4$ which is > 0 for all values of *x*. The majority of candidates who did not spot the connection usually recognised that it was necessary to show that f '(*x*) > 0 but tried to do this substituting one or two values of *x*, which is not a rigorous approach. Others tried to work with the second derivative but could make no progress.

Answers: **(i)** $3(x-1)^2 - 1$; **(ii)** f '(*x*) = $3x^2 - 6x + 7$; f is an increasing function.

Question 4

This was often well done with many correct and well explained arguments. There was evidence to support the fact that the given answer in part **(i)** was used constructively in prompting some candidates successfully to rethink their approach, not only to part **(i)**, but to part **(ii)** also. There were, however, some candidates who were not able to make progress with part **(ii)** despite having been successful with part **(i)**. Indeed, there were some who did not attempt the second part despite having done the first part. Although most candidates adopted the expected route to the answer, there were a number of less orthodox but nonetheless valid approaches seen. Weaker candidates mixed radians and degrees in their attempts. There were surprising numbers of candidates who insisted on using the approach that the required arc length or sector area was the fraction ($\theta/2\pi$) of the relevant circumference or area of a circle. While this is not incorrect, of course, it overlooks completely section 4 of the syllabus on circular measure (radians) 'Use the formulae *s* = *r*θ and *A* = ½*r*²θ in solving problems .....etc.'. Furthermore, candidates who used the former approach frequently made mistakes in simplifying the expressions that this approach produced.

Answer: **(ii)** 2π*r* + 4*r*.

Question 5

Part **(i)** was done very well with the majority finding the scalar product, accurately simplifying, setting this to zero and solving the resulting linear equation. There were some algebraic errors and some instances of the scalar product being equated to 1 or to –1.

Part **(ii),** by contrast, did cause problems. Only the stronger candidates, or perhaps those who had seen similar questions before, followed the straightforward route of comparing components, noting that the components of **OB** were double those of **OA**. Some candidates simply equated the components, whilst many others used *p* = 2.2 (the value found in part **(i)**). Other candidates attempted to equate the scalar product to 1 or to –1, but without including the moduli of the vectors. A few did include the moduli of the vectors and tried to pursue what was essentially a correct method but eventually had to abandon the attempt because of very complex algebra. Those candidates who did get a vector **OA** (using a correct or incorrect value of *p*) were usually able to demonstrate that they knew how to find the magnitude of a vector.

Answer: **(i)** *p* = 2.2; **(ii)** **OA** = $\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$, **OB** = $\begin{pmatrix} -4 \\ 4 \\ 2 \end{pmatrix}$, length of *OA* is 3.

Question 6

This question was set in context and a very large proportion of candidates made incorrect interpretations. Many considered only one direction rather than up and down as stated in the question. Many used the wrong starting value and many used the wrong number of terms. Most candidates used the correct formulae for which 2 method marks were available but unfortunately this was all too often the total score for this question.

Many candidates realised their initial answers needed further adjustment and attempted to do this by adding or subtracting 1, but this rarely met with success.

In part **(i)(a)** the most straightforward approach is to use $a = 1.92$ and $n = 20$ with $d = -0.08$, but this was seen rarely. If one direction only is considered then values used should be $a = 0.96$, $n = 20$ and $d = -0.04$ with the result doubled. Unfortunately $a = 1$ and $n = 21$ were most often used. In addition, $d = +0.04$ was another common error.

In part **(i)(b)** the same errors were made with respect to a and n. The value for r should be 0.96 but 0.04 was often seen.

In part **(ii)** most candidates appreciated that the formula for sum to infinity was appropriate here, but wrong values of a and/or r were again frequently used.

Answers: **(i)(a)** 23.2; **(i)(b)** 26.8.

Question 7

Most candidates attempted at least some part of this question and again there were many wholly correct solutions. The first part required candidates to use three main steps before simplifying and rearranging to reach the given answer. These were (in various orders):

- Replace $\tan\theta$ by $\sin\theta/\cos\theta$
- Multiply by $\cos\theta$
- Use the identity $\cos^2\theta + \sin^2\theta = 1$

Those candidates who could not reach the given form of the equation were still able to use it to solve the equation, but there were a few scripts on which the solution was not attempted. Where the solution was attempted it was generally successful, although occasionally a mark was lost because only one value of θ was given or the values of θ were given to the nearest degree rather than to one decimal place, as specified in the rubric of the paper

Part **(b)** produced many correct responses but there were quite a few candidates who gave the value of d to be $-b$ and a few who said that c was $\cos^{-1}(b/a)$ or $\cos[\cos^{-1}(b/a)]$.

Answers: **(a)** 131.8º, 228.2º; **(b)** $c = b/a$, $d = a - b$.

Question 8

Part **(i)** was found very difficult with most candidates failing to score any marks. The majority of candidates focused on rearranging gf rather than considering $3x + 1$ and the restriction on x.

In part **(ii)** the first 2 marks were usually scored with candidates finding fg correctly and going on to simplify the inequality. However, the restriction on x was then often not taken into account for the final mark.

In part **(iii)** most candidates again found the composite function correctly (gf) and started simplifying the inequality correctly. However, the same misunderstanding of inequalities that was apparent in **Question 1** appeared also here where multiplication or division by a negative quantity requires the inequality to be reversed. Even the candidates who had not made this mistake very often continued by initially taking the region between the two critical values instead of the regions outside the critical values. There was yet a further hurdle to negotiate; the restriction on x required the candidate to reject one of the regions found.

Only a relatively small proportion of candidates were able to reach the correct answer for this part.

Answers: **(i)** Largest value of a is $-2/3$; **(ii)** $x = -2$; **(iii)** $x \le -8/3$.

Question 9

This question was generally answered quite well and provided a good source of marks - even for the less strong candidates. In part **(i)** most candidates used the chain rule appropriately but some candidates made it more awkward for themselves when they stated the chain rule by making $\dfrac{dy}{dx}$ the subject, rather than $\dfrac{dy}{dt}$. A surprising number of candidates applied the chain correctly by multiplying the given expression for $\dfrac{dy}{dx}$ by 3 but left this as their answer without substituting $x = 4$ into their expression.

Part **(ii)** was generally answered well with most candidates handling the integration accurately. Most candidates remembered the constant of integration and were able to evaluate it successfully. In part **(iii)** a good proportion of the candidates were able to find correctly equations of the tangent and normal. Errors did occur with finding the coordinates of *B* and *C* - mostly due to candidates carelessly substituting $x = 0$, instead of $y = 0$, into one or both of the equations. The more obvious route to the area of the triangle is to use $\frac{1}{2}BC \times$ 6, but some used the fact that angle *A* is a right angle and calculated the lengths of *AB* and *AC* to find the area.

Answers: **(i)** 6; **(ii)** $y = x + 4x^{\frac{1}{2}} - 6$; **(iii)** 45.

Question 10

The question proved accessible to most candidates, at least in part, and a good number of candidates scored highly. In part **(i)** the differentiation was done well by all but the weakest candidates but there were many who failed to get all 4 marks in this part. The 3rd mark proved elusive to quite a few who did nothing to show or verify that there was a stationary value when $x = 0$. Of those who gained this mark many opted to solve f '(x) = 0 rather than the simple substitution of $x = 0$ into f '(x). The final mark was more frequently scored by the substitution but there were a few instances where the candidate argued successfully that the second derivative was greater than zero for all *x* so any stationary value would be a minimum.

Part **(ii)** was mostly well answered. Some weaker candidates added the coordinates and a few spoiled good work by rounding incorrectly when transferring the answer from their calculator to the paper. All equivalent forms of the answer were well represented.

Part **(iii)** again was often done well. Some candidates, however, despite the wording of the question, did not show the actual integration step so were unable to access most or all of the marks in this part of the question. Candidates needed to find the area under the curve by integration and subtract this from the area of the trapezium. The area of the trapezium is quite easy to find by direct methods, but substantial numbers of candidates opted to follow the longer and error-strewn route of finding the equation of line *AB* (often inaccurately) and then finding the area under the line by integration. Some weaker candidates had trouble with the integration of the curve and showed some confusion when dealing with the negative power.

Answers: **(i)** f '(x) $= 2 - 2(x + 1)^{-3}$, f ''(x) $= 6(x + 1)^{-4}$,
  f '(0) = 0 hence stationary at $x = 0$, f ''(0) = 6 hence minimum at $x = 0$;
  (ii) *AB* = 1.68 (or equivalent answers);
  (iii) 27/16 (or 1.69).

MATHEMATICS

Key Messages

Candidates should be reminded of the necessity of reading each question carefully and ensuring that they have given the required answer in the correct form and to the correct level of accuracy. There is still an evident misunderstanding of the request for an exact solution.

General Comments

A relatively small cohort took this paper. Lack of time did not seem to be a problem. There were some candidates who produced well thought out and well-presented solutions thus showing a good understanding of the syllabus aims and objectives.

Comments on Specific Questions

Question 1

Most candidates realised that they needed to use logarithms, together with the power rule. Problems arose with the use of basic algebra. Omission of appropriate brackets gave rise to incorrect simplification and evaluation. Often answers were not given to the level of accuracy asked for in the question.

Answer: 20.1

Question 2

Most candidates recognised the need to differentiate the given function as a quotient and were able to make a reasonable if not completely correct attempt. Some candidates do not know the correct formula and sign errors and terms incorrectly positioned in the numerator occurred. Some candidates chose to equate their gradient function to zero rather than -4, highlighting the importance of reading the question carefully.

Answer: $(3,-5)$ and $(7,-11)$

Question 3

(i) Most candidates were able to deal with standard piece of trigonometric work, gaining all 3 marks available. Some candidates lost marks due to answers not being to the required level of accuracy or an incorrect angle of 28.07°.

(ii) Many candidates did not cope as well with this part of the question, a few not realising the significance of part (i), but mostly because of the inability to deal correctly with the first solution that most obtained being -43.3°. There were few completely correct solutions, showing that this is an area that centres may need to concentrate upon when solving basic trigonometric equations.

Answer: (i) $R = 17$, $\alpha = 61.93^{\circ}$, (ii) 97.4° and 318.7°

Question 4

(i) Many candidates omitted this part, but there were also many correct sketches of $y = \ln x$ and

$y = 4 - \dfrac{1}{2}x$. Candidates are to be encouraged to make a comment about there only being one

point of intersection, hence only one real root to the given equation, rather than just offering a sketch of the 2 functions. Unfortunately some candidates chose to draw their graphs on 2 separate diagrams which did not really help show only one real root.

(ii) Most candidates were able to make a valid attempt to show the range within which, the real root lies, by using change of sign methods. However, they should again be encouraged to make conclusions about what their work has shown e.g. 'there is a change of sign, so the root lies in the given interval' or words to that effect.

(ii) Most candidates do the iteration parts of questions well and this was no exception apart from the candidates who either did not do enough iterations to warrant the solution they gave, or gave their iterations to the wrong level of accuracy.

Answer: **(iii)** 4.84

Question 5

(a) Not done well, many candidates were unable to integrate both terms correctly. Many made use of the fact that $\tan^2 x = \sec^2 x - 1$ and hence were able to make an attempt to integrate each term, but candidates often had problems integrating $\sin 2x$.

(b) Many candidates made a good attempt at integration of the exponential function, realising that their answer should be of the form $k\mathrm{e}^{1-2x}$. There were sign errors and errors in the value of *k*, with most candidates applying the limits correctly. Some candidates did not realise the implication of the word 'exact' and used their calculator to evaluate the integral.

Answer: **(a)** $\tan x - x - \dfrac{1}{2}\cos 2x + c$, **(b)** $-\dfrac{3}{2}\mathrm{e}^{-1} + \dfrac{3}{2}\mathrm{e}$

Question 6

(i) Most candidates were able to attempt this, using algebraic long division, with many gaining all 4 marks. There were occasional sign errors which resulted in loss of accuracy marks.

(ii) Very few candidates actually related this part of the question to the previous part, choosing to start the question again and usually making errors in the long division. Candidates should be encouraged to look at the mark allocation for a question part as it is usually a good indicator of the amount of work that needs to be done. Very few correct solutions were seen.

(iii) This question part was rarely attempted as many candidates did not have a value for *p* or for *q* to use. Again this part of the question was not related by those candidates who did have values for *p* and for *q*, to the first part of the question. This resulted in very few correct statements about one of the quadratic factors being such that it had no real roots when equated to zero and the other one having equal roots when equated to zero.

Answer: **(i)** Quotient $x^2 + 2x + 1$, remainder $5x + 2$, **(ii)** $p = 7$, $q = 4$, **(iii)** $x = -1$

Question 7

(i)   Many candidates had difficulty with the differentiation of $x = 6\sin^2 t$ with respect to t, failing to recognise the need for the use of the chain rule. Most had more success with the differentiation of $y = 2\sin 2t + 3\cos 2t$, although there were sign errors and errors in the coefficients. This meant that few candidates were able to obtain the given result.

(ii)   Few candidates attempted this part of the question even though they had the given result in part **(i)** to work from. For those that did attempt this part, most were unable to deal with the equation $\cot 2t = \dfrac{3}{2}$ correctly to give valid solutions.

(iii)   Very few candidates attempted this part of the question and those that did failed to realise that they needed to be considering $\dfrac{dy}{dt} = 0$.

Answer: **(ii)** $t = 0.294$ and $t = 1.865$, **(iii)** $-\dfrac{13}{9}$

MATHEMATICS

| Paper 9709/22 |
| Paper 22 |

Key Messages

Candidates should be reminded of the necessity of reading each question carefully and ensuring that they have given the required answer in the correct form and to the correct level of accuracy. There is still an evident misunderstanding of the request for an exact solution.

General Comments

It was evident from the large number of low-scoring scripts that many candidates were either not adequately prepared for this examination, or had not covered the syllabus in sufficient detail. Lack of time did not seem to be a problem; lack of knowledge seemed to be the issue. However, there were candidates who had clearly covered the syllabus aims and objectives well, producing scripts with clearly set out solutions resulting in marks that gained the higher grades.

Comments on Specific Questions

Question 1

(i) The most common approach was to deal with the modulus by squaring each side of the given equation. Unfortunately, a large number of candidates only squared the left-hand side of the equation. These candidates were usually able to gain a method mark for the solution of their resulting quadratic equation. Fewer candidates tried to set up two linear equations but these candidates generally had more success in gaining a correct solution.

(ii) For those candidates that saw a connection with part **(i)**, their lack of knowledge concerning logarithms and general poor algebraic skills meant that very few correctly reached a statement of the form $5^y = k$ correctly in order to be able to proceed.

Answer: **(i)** $-1, \dfrac{7}{3}$, **(ii)** 0.526

Question 2

(i) A good proportion of candidates were able to gain the first three marks for the iteration process. However, there were also many who lost marks due to either not showing the correct level of accuracy as requested in the paper, or incorrectly rounding their solution to 2.290. On occasion, both errors were seen.

(ii) Very few candidates were able to even start this part of the question. Of those that did, all too often a calculator was used to give a rounded answer, rather than the specified exact answer.

Answer: **(i)** 2.289, **(ii)** $\sqrt[3]{12}$

Question 3

Most candidates earned the marks associated with the gradient and the intercept, namely the straight line work, without showing any deeper understanding of the situation and its context. Few candidates gained full marks but those that did had often taken a pure approach of setting up simultaneous equations and solving them correctly. This question again highlighted the lack of understanding of logarithms by many candidates and also the fact that many solutions were not given to the required level of accuracy.

Answer: $m = -1.4$ and $K = 71$ or 72

Question 4

(i)   Most candidates were able to gain the marks for this part of the question. However, it should be noted that candidates should attempt to demonstrate the application of the factor theorem more clearly; very often terms were not equated to zero until the end of the work, or the equating to zero was implicit.

(i) (a) The most common and successful approach to determine the quadratic factor was by algebraic long division. However, many candidates failed to show the full factorisation of the given expression using all three linear factors.

(b)   Many candidates failed to see the connection between this part of the question and the preceding part. For those that did, many were unsure of how to deal with secant and many invalid solutions were often given. Many candidates started again and produced very long and often completely meaningless solutions. Centres should impress upon candidates the links between question parts; work done in a previous part will often be of use in a future question part.

Answer: **(ii) (a)** $(x + 2)(3x - 2)(2x + 1)$, **(b)** $120°$

Question 5

(i)   This question part should have been a straightforward application of the product rule but many candidates failed to recognise the structure of the function as a product and were often unable to differentiate the exponential function correctly. For those that did manage to differentiate as a product, many then failed to proceed by equating to zero in order to find the stationary point. It is essential that candidates check that they have done what is asked for in a question.

(ii)   This question part should have been a straightforward application of the quotient rule but many candidates failed to apply the quotient rule correctly. Those that rewrote the given function did not do much better, being unable to apply the appropriate algebraic skills to simplify their work correctly. Many candidates omitted brackets where there should have been brackets and careless expansions often lead to loss of accuracy.

Answer: **(i)** $x = \dfrac{1}{3}$, **(ii)** $x = 0$ and $x = -2$

Question 6

(i)   An encouraging number of candidates earned up to the first three marks for correct derivatives and assembly of $\dfrac{dy}{dx}$. However, many candidates were unsure of how to deal with the compound angle $\left(\theta - \dfrac{\pi}{6}\right)$ and went on to give false expansions. An encouraging number of candidates were able to work through to the given answer robustly, showing excellent knowledge and manipulation of trigonometric expressions.

(ii) Very few candidates were able to offer any useful work to find the equation of the tangent as required, being unable to find the value of the parameter at the required point. Very few correct solutions were seen.

Answer: **(ii)** $y = \dfrac{\sqrt{3}}{3}x + 1$

Question 7

(i) Very few correct attempts at integration of both parts of the function were seen. Many candidates used the correct double angle formula for $\cos^2 x$ and then incorrectly made use of this when dealing with $\sec^2 x$, rather than recognising that $\sec^2 x$ has a standard integral.

(ii) While many candidates realised that they needed to consider $\pi \int y^2 dx$, very few were able to deal with y^2 correctly, giving a two-term expansion rather than a correct three-term expansion. This again shows a lack of basic algebraic skills. As a result, very few correct solutions were seen with most candidates not recognising the link between the two parts of the question.

Answer: **(ii)** $\dfrac{5}{6}\pi^2 + \dfrac{9}{8}\sqrt{3}\pi$

MATHEMATICS

Key Messages

Candidates should be reminded of the necessity of reading each question carefully and ensuring that they have given the required answer in the correct form and to the correct level of accuracy. There is still an evident misunderstanding of the request for an exact solution.

General Comments

Very few candidates sat this particular paper, making it very difficult to make comments on overall performance. For each of the questions, a comment will be made on what was expected from candidates in order to achieve.

Comments on Specific Questions

Question 1

Candidates are expected to recognise the integral as being a form of $k\ln(2x+5)$ and to be able to apply the given limits correctly, together with the logarithmic law for $\ln a - \ln b$. An exact answer is expected, so a candidate should be able to do this question without having to resort to the use of a calculator.

Answer: ln125

Question 2

(i) Candidates are expected to use one of two methods to solve the equation involving a modulus. Solutions should be obtained by either squaring each side of the given equation and solving the resulting quadratic equation to give the two required values, or by forming two linear equations and solving each one separately. Graphical solutions or those by inspection are also acceptable.

(ii) It is important that candidates realise that a part **(ii)** for a question usually involves the use of part **(i)** of that question and this is no exception. Candidates are expected to equate their value of *x* in part **(i)** to 2^y and then solve the equation $2^y = 5$, making use of logarithms. Any invalid extra solutions would be penalised by withholding the final accuracy mark.

Answer: **(i)** $x = 5$ and $x = -\dfrac{11}{3}$ **(ii)** 2.32

Question 3

Differentiation of both parametric equations, with respect to *t*, is expected as a first step. Candidates should be encouraged to use the correct notation when differentiating with respect to *t* and also recognise the need to differentiate a product and an equation using the chain rule. Substitution of $t = 0$ into the parametric equation for both *x* and *y* together with the expression gained from a division of $\dfrac{dy}{dt}$ by $\dfrac{dx}{dt}$ would yield enough information to obtain the equation of the required tangent. The actual required form of the tangent is important, highlighting the fact that candidates should ensure that they are answering the question fully.

Answer: $3x - 4y + 45 = 0$

Question 4

   (i)   A completely correct algebraic long division is expected, clearly showing the quotient and also verifying that the remainder is 39.

   (ii) Again, candidates are expected to make use of their answers to part **(i)** to complete the solution for this part, which is reinforced by the use of the word 'Hence'. The problem reduces to an equation involving the linear factor $x - 2$ and a quadratic factor, which has no real roots when equated to zero. A confirmation of only one real root is needed by making use of the discriminant or an attempted solution of the quadratic factor equated to zero, together with an appropriate comment.

Answer: **(i)** quotient = $3x^2 + 11x + 20$

Question 5

   (i)   Correct integration of each term would gain 1 accuracy mark, with candidates needing to show a correct substitution of limits into their integral, which must be in the form $k_1 e^{3x} + k_2 e^x$, for a method mark. Candidates are expected to show their algebraic skills by correctly manipulating the result and introducing logarithms to obtain the given answer.

   (ii) Most candidates answer questions involving iterations really well. In this case, the form to use is given in part **(i)**. It is important that candidates do enough iterations at the required level of accuracy to justify the answer, which must also be given to the required level of accuracy. Many candidates often lose marks needlessly by not taking enough care where accuracy is involved.

Answer: **(ii)** 1.477

Question 6

   (i)   Many candidates usually do this type of trigonometric problem really well. Care must be taken when calculating the angle and also ensuring that the correct level of accuracy is used for the angle, as well as the correct units.

   (ii)(a) Most candidates will attempt to use their result from part **(i)** to help in part **(ii)**.

   (b)   Centres should encourage candidates to look at mark allocations to question parts to help gauge the amount of work that might be expected. In this case, the mark allocation is 2 marks, which should suggest to candidates that there is a limit to how much work is required and that they should also use the result they obtained in part **(i)**; this is reinforced by the use of the word 'Hence' and the word 'state'. Candidates would not be expected to use calculus to solve this part of the question.

Answer: **(i)** $R = 3$, $\alpha = 41.81^\circ$, **(ii)(a)** 30.7° and 245.6°, **(ii)(b)** greatest value 13, least value 7

Question 7

   (i)   Candidates are expected to realise that they need to differentiate the given equation making use of the product rule. It is usually more beneficial to leave a function like this as a quotient rather than rewrite it as a product, which then usually has a need for more demanding algebraic skills when coming to use it. Candidates are also expected to make use of the appropriate double angle formulae in order to obtain the given result.

   (ii) Even if candidates fail to achieve the expected result in part **(i)**, they can make use of the given result to carry on with part **(ii)**. All too often candidates do not make use of given answers and thus preclude themselves from gaining marks that they might have otherwise obtained. Attention should be brought to the required accuracy that is necessary as the units being used are radians – a fact that is reinforced by the given range for the expected solutions.

Answer: **(ii)** ± 0.905

MATHEMATICS

General comments

The standard of work on this paper varied considerably and resulted in a wide spread of marks from zero to full marks. Overall the paper was less difficult than in the previous November and the percentage of candidates with very low scores was correspondingly smaller. The questions or parts of questions that were generally found easy were **Question 4 (iii)** (iteration), **Question 6** (algebra) and **Question 10 (i)** (differentiation). Those that were done least well were **Question 3** (trigonometry), **Question 7** (vector geometry), **Question 8** (differential equation) and **Question 10 (ii)** (integration).

In general the presentation of work was good, though there were some untidy scripts and occasionally work was presented in a double columnar format that was difficult for Examiners to follow and assess. Some candidates were over reliant on the use of a calculator to solve quadratic equations and even simultaneous equations. Candidates need to be aware that should errors occur in such work, written evidence of the methods used is required before any method marks become available. Simply writing down incorrect answers from a calculator therefore leads to no marks being awarded for the piece of work in question.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on Specific Questions

Question 1

Considering that this is a standard piece of work it was not always done particularly well. Most candidates attempted to obtain a non-modular equation by squaring both sides. Errors such as failing to square the factor of 3 or taking the modulus of, say, $2x + 1$ to be $\sqrt{(2x+1)}$ or $(2x+1)^2$ were not infrequent. Having squared both sides, candidates reduced the problem to a 3-term quadratic inequality or equation. This involved finding at least 9 coefficients and errors were common. Very few considered the initial expression as a difference of two squares so that the inequality becomes $(2x - 5 - (6x+3))(2x - 5 + (6x+3)) > 0$. It then follows that $(-4x - 8)(8x - 2) > 0$, from which the critical values and solution are easily found. The few attempts using graphs were usually successful as were those working with a pair of linear equations. Those that set out with linear inequalities, almost always omitting the ranges for which they were valid, usually found the critical values but were often unable to derive the correct final answer.

Answer: $-2 < x < \dfrac{1}{4}$

Question 2

This was well answered by many candidates. However there was also some poor work with indices and logarithms in both stages of the problem. Some found a value for u and then stopped. Candidates generally gave the final answer to the required accuracy but premature approximation of the positive root to $u = 1.62$ led to $x = 0.439$. Candidates generally need to be more secure in their knowledge and manipulation of indices and logarithms.

Answer: 0.438

Question 3

This question discriminated well. There were some good attempts, but many went astray because of basic errors such as taking the first equation to be $\tan\theta - \tan\phi = 3$ or the second to be $\tan(\theta + \phi) = 1$. Sign errors in the use of the $\tan(A \pm B)$ formula were also frequent. Those that reached a correct quadratic in $\tan\theta$ or $\tan\phi$ sometimes incorrectly rejected negative roots such as –1 believing that they produced angles outside the given interval. However many found a pair of correct angles. For full marks the correct pairs of angles in each solution had to be seen or implied.

Answer: $\theta = 53.1°, \phi = 161.6°$; $\theta = 135°, \phi = 63.4°$

Question 4

   (i)   There were many correct attempts. However some attempted to solve the equation by various unsound methods. Lack of understanding of the terms 'consecutive' and 'integer' was evident. Following correct work, the answer was quite often given as 'between f(2) and f(3)' or 'between 2 and – 12'.

   (ii)   This was generally well answered by those who attempted it.

   (iii)   The iteration was carried out properly by most. Common errors included truncation to 2.218 and giving the answer 2.22 to 3 s.f.

Answers: **(i)** 2, 3; **(iii)** 2.219

Question 5

   (i)   Most candidates made a correct start using the product rule. Some did not seem to know, or failed to think to use, the identity $\sec^2 x \equiv 1 + \tan^2 x$, however there were a good number of correct answers.

   (ii)   Many candidates ignored the exponential term and said that the square factor was positive rather than positive or zero.

   (iii)   Some gave the angle in degrees rather than radians, hence spoliing their good work.

Answer: $e^{-2x}(-1 + \tan x)^2$; $\frac{1}{4}\pi$

Question 6

  (i)   Candidates usually applied the factor and remainder theorems correctly and obtained the two constants. A few equated $p(-\frac{1}{2})$ to 0, or changed 1 to 0 in the course of the working.

  (ii)   Whatever the values of a and b, most started the division by $(x + 1)$ correctly but some attempted division by $(2x + 1)$. A few candidates with correct working did not factorise the quotient $8x^2 - 2x - 1$.

Answers: **(i)** $a = 6$, $b = -3$ **(ii)** $(x + 1)(4x + 1)(2x - 1)$

Question 7

This was only moderately well answered. Some of the errors might have been avoided if candidates had drawn a diagram showing the points, line and plane involved. In part **(i)** most understood the structure needed for the line equation but quite often omitted "**r** = ". A minority simply stated the vector $\overrightarrow{AB}$. Part **(ii)** was quite often correctly answered. However many candidates found the equation of the wrong plane, e.g. the plane perpendicular to $\overrightarrow{OC}$ and $\overrightarrow{AB}$. In part **(iii)** most used their answers to **(i)** and **(ii)** to find N and CN and usually made good progress, though some did not give sufficient working to justify the given answer for CN. An alternative method, which began by using a scalar product to find AN (or BN), was also seen.

Answers: **(i)** $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$; **(ii)** $2x - 2y + z = 4$; **(iii)** $\overrightarrow{ON} = \frac{7}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$

Question 8

Though most separated correctly and reached $\ln(x + 2)$, there were many unsound attempts at integrating $\sin^2 2\theta$, which surely would have been eliminated had candidates taken the precaution of checking by differentiation. Only a minority tried to express this term in terms of $\cos 4\theta$. There were errors of sign and/or factor in the use of the $\cos 2A$ formula and the error of taking the integral of $\frac{1}{2}$ to be $\frac{1}{2}x$ rather than $\frac{1}{2}\theta$ was quite often seen and proved damaging. The final stage for solving a log equation was usually well done.

Answer: 0.962

Question 9

Candidates with a good grasp of complex numbers did well here. In part **(i)** there were a fair number of good answers. However there was also a considerable amount of confused work. It was surprising to find the correct quadrilateral given a wrong nameIn part **(ii),** many candidates were successful, though some substituted incorrectly or multiplied by $\frac{3-i}{3-i}$ or by $\frac{3+i}{3-i}$. In part **(iii)** many found $\arg\left(\frac{u^*}{u}\right)$ correctly and the best candidates went on to complete sound proofs of the given result. Some attempted inexact work using computation of angles.

Answers: **(i)** Parallelogram **(ii)** $\frac{4}{5} + \frac{3}{5}i$

Question 10

(i) Algebraic errors sometimes produced insoluble equations when the derivative was equated to zero. However there were many correct solutions. Following correct work some candidates gave the incorrect answer $\sqrt{2}$, while others gave the correct exact answer and then followed it with a decimal approximation. Candidates need to understand the meaning of the adjective "exact".

(ii) A substantial number of candidates worked with an indefinite integral of the form $k(1 + x^3)$. Had candidates checked their attempts at the integral by differentiation, wrong values such as $k = 1$ might have been corrected.

Answers: **(i)** $\sqrt[3]{2}$; **(ii)** 3.40

MATHEMATICS

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Answers: **(i)** $\sqrt[3]{2}$; **(ii)** 3.40

MATHEMATICS

General comments

The paper was accessible to almost all candidates, with many of the stronger candidates scoring well across all topics. Weaker candidates were able to tackle parts of most questions and it was unusual to see scripts that did not offer a full set of solutions. Many candidates scored full marks on **Question 2** (binomial expansion), **Question 3** (differentiation and equation of a straight line), and **Question 6** (trigonometric formulae). The response to **Question 1** (sketch of an exponential function) was particularly disappointing; these two marks were often the only marks lost by the most able candidates. Other questions that candidates found challenging were **Question 5** (integration by substitution), **Question 8(iii)** (position vector of a point), and **Question 9(b)** (Argand diagram).

Many candidates used graph paper for their responses to **Question 1** (sketch of a function) and **Question 9(b)** (sketch of an Argand diagram). When a question asks for a sketch, the use of graph paper is not required - the Examiners are looking for a diagram which shows the shape and the key features, not an accurate plot.

In **Question 4(i)**, **Question 4(ii)**, **Question 7(i)** and **Question 9(a)** candidates were asked to demonstrate a given answer. It is important for candidates to realise that in these circumstances they need to show full reasoning to support their conclusions in order to score full marks. Similarly, when a question asks candidates to "show all necessary working" or to "find the exact value..." this is a clear message to candidates that an answer copied from a calculator will not be sufficient to score the marks available.

The standard of presentation of the work was often very good, and the best candidates often made their reasoning clear by the use of diagrams. Work which follows sequentially is much easier to follow, and to mark, than work which is written partially or completely in double columns on the page, or where afterthoughts are written in gaps on the page.

The detailed comments that follow draw attention to common errors and might lead to a cumulative impression of indifferent work on a difficult paper. In fact there were many scripts showing a complete understanding of all the topics being tested.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on Specific Questions

Question 1

This question drew a wide variety of responses. Many candidates attempted an accurate plot of the function but did not sketch enough of it to show the key features. The Examiners were looking for the basic shape of an exponential function, and for the sketch to pass through the origin. Many sketches existed only for positive values of x.

Question 2

Many candidates demonstrated a good understanding of how to use the binomial expansion for a rational index. Most solutions started with a correct unsimplified expansion as far as the term in x^3, but there were several slips in the arithmetic in reaching the final answers. The most common error in the expansion was to use powers of x rather than of $9x$. A small number of candidates attempted to find the cube of the right hand side and compare coefficients rather than start by expressing the cube root in index form.

Answer: $a = -9$, $b = 45$

Question 3

Most candidates started this question with a correct application of the quotient rule or product rule. It is important to note that some candidates applied the quotient rule incorrectly (usually with a sign error) without ever stating the rule. No credit can be gained for the use of an incorrect formula - the correct form is given in the formula booklet. Having completed the differentiation many candidates embarked on unnecessary simplification of the expression, often making errors in the process. Use of the incorrect identity $(1 + \tan x)^2 = \sec^2 x$ was quite common. Further errors were caused by the use of incorrect values for $\tan \frac{\pi}{4}$ and $\cos \frac{\pi}{4}$. A small number of candidates started by rewriting the original function as $-1 + \dfrac{3}{1 + \tan x}$, which makes the differentiation more straight forward.

The process for finding the equation of the straight line was well understood, but some candidates found the equation of the normal, not the tangent, and some found the equation of the tangent at $x = 0$.

Answer: $y = -1.5x + 1.68$

Question 4

Many candidates scored full marks for this question. Most errors arose from a lack of detail in the response.

(i)   Most candidates demonstrated a good understanding of how to differentiate a function expressed in parametric form. There were some errors in differentiating t^4. Most candidates moved from the parameter t to the specific value p with no difficulty, however there were several answers with a mixture of t and p.

(ii)   Most candidates who considered roots of a function of the form f(p) = 0 obtained two values and reached a correct conclusion by referring to the change of sign. It was not sufficient to obtain the two values and give no indication of how this supported the required conclusion. A small number of candidates who considered roots of a function of the form f(p) = p did reach a correct conclusion. However, several concluded that there was no root because they had two values of the same sign. If this approach is used then candidates need to compare their answers with the values of p as part of the method.

(iii)   The majority of answers were fully correct. A few candidates did not work to the required accuracy - in some cases there appeared to be confusion between "significant figures" and "decimal places".

Answer: **(iii)** 1.89

Question 5

The majority of candidates demonstrated a good understanding of the process for integration by substitution. Most scored the first mark for a correct expression for $\dfrac{\mathrm{d}u}{\mathrm{d}x}$. Several candidates did not deal with sin $2x$ correctly. Those who did not manage to express the integral solely in terms of u often went on to try to integrate with a mixture of x and u. Many candidates did complete the substitution correctly. By rewriting $\dfrac{8 - 2u}{\sqrt{u}}$ as $8u^{-\frac{1}{2}} - 2u^{\frac{1}{2}}$ the integration becomes straight forward. However a large number of candidates worked through integration by parts instead. Having completed the integration, most candidates went on to

use limits for *u* to find the value of the integral. Some candidates preferred to rewrite their answer in terms of *x* and use the original limits.

This question asked candidates to use a particular method to find the exact value of the integral. A small number of candidates showed no working or incorrect working followed by a "correct" answer. These candidates scored no marks.

Answer: $\dfrac{20}{3}$

Question 6

Candidates with a good knowledge of the trigonometric formulae found this question very straight forward. The value of $\tan A$ was usually found correctly. Most errors were caused by confusion in working with $\sqrt{2}$, and a small number of candidates appeared to think that $\tan A = \dfrac{\cos A}{\sin A}$. Candidates who did not recognise that they could use $\sec^2 B = 1 + \tan^2 B$ often made no progress after forming an equation in $\cos B$ and $\sin B$. The formula for $\tan(A - B)$ was often misquoted, and a small number of candidates produced answers involving $\tan 3$ and $\tan 1.5$.

The instruction not to use a calculator was ignored by several candidates. Where an answer depended on values obtained using a calculator it scored no marks.

Answer: $\dfrac{3}{11}$

Question 7

(i)   Many candidates approached this question by finding the value of the cubic for $x = -1$ and most of them obtained the required answer of zero. However, the majority of them did not then make any comment about the significance of the answer in relation to the task. Similarly, several of the candidates who completed the long division and obtained no remainder did not make any comment about how their work demonstrates that $(x + 1)$ is a factor.

(ii)   The work on partial fractions was generally well done. Many candidates did not use the most efficient route to finding the values of the constants - several worked through the solution of three equations in three unknowns rather than making the substitutions $x = -1$, $x = 2$, and $x = -\dfrac{3}{4}$.

Some candidates who wrote down three equations in three unknowns showed no working, wrote down an incorrect solution and scored no marks - method must be demonstrated in order to score a method mark, an incorrect solution by calculator or by guesswork will gain no credit. The final integral was often completed correctly, but there were some errors in the coefficients and the final "$+C$" was often missing.

Some candidates used the hint from part **(i)**, factorised the cubic as far as $(x + 1)(4x^2 - 5x - 6)$, and used this for their partial fractions. Although a few realised later on that they needed to split the fraction further in order to integrate, most just stopped.

Many candidates who started with roots -1, 2 and $-\dfrac{3}{4}$ did not show where these values came from and they went on to use the incorrect factorised form $(x + 1)(x - 2)\left(x + \dfrac{3}{4}\right)$ so they gained very little credit.

A small number of candidates made no progress at all with the integral, not recognising the potential use of partial fractions.

Answer: $-2\ln(x + 1) + \ln(x - 2) + 2\ln(4x + 3) + C$

Question 8

(i) The method for finding the point of intersection of the line with the plane was often completed successfully, but there were several arithmetic errors in the process ($2 - 9 = 11$ was seen several times), and some candidates had $\mathbf{r} = \mathbf{i} - 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{j} - 8\mathbf{k})$ as the equation of their line. The final answer was often the position vector of B, not the coordinates of B as asked for in the question.

(ii) Most candidates knew that they needed to use the scalar product to find the angle between two vectors, and many used an appropriate pair of vectors. The process was usually completed correctly. The final answer given was often the angle between the line and the normal rather than the angle between the line and the plane. Although the process of calculating the scalar product is simple mental arithmetic, candidates should be advised to show their working because if they make a slip with no working shown then there is no evidence that they used the correct process.

(iii) The most successful candidates were those who drew a diagram and found the position vectors of the two points using $(3\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}) \pm 2(3\mathbf{i} - 9\mathbf{j} + 12\mathbf{k})$. Most candidates preferred to use Pythagoras' theorem for the distance between two points, and set up and solve a quadratic equation. There were several slips in the algebra and in the arithmetic.

Answers: **(i)** $(3, -7, 4)$ **(ii)** $54.8°$ **(iii)** $(-3, 11, -20)$ and $(9, -25, 28)$

Question 9

(a) The candidates adopted a wide variety of methods to find $\left|w^2\right|$ and $\arg\left(w^2\right)$ and many of them scored full marks. Apart from arithmetic errors, the common reasons for loss of marks were a lack of clarity in demonstrating that $\left|w^2\right| = 2$ and the value of $\arg\left(w^2\right)$ not matching the correct quadrant. Because the question tells the candidates that $\left|w^2\right| = 2$, if they start by finding that $|w| = \sqrt{2}$ they have to say that they are using $\left|w^2\right| = |w|^2$ to confirm the use of the correct method. The question asks candidates to "show all necessary working", so use of a calculator for division of complex numbers was not acceptable.

(b) Most candidates gave a correct sketch of the locus $|z| = 5$. The straight line proved to be more difficult - the most common errors were to draw it as a second circle, or to draw a horizontal straight line. Many candidates with a correct diagram then went on to find the coordinates of the points of intersection by measuring the y values, apparently not recognising the equilateral triangle in their diagram. Some candidates need to be reminded that the form $re^{i\theta}$ requires θ to be in radians, not degrees.

Answer: **(a)** -0.284 radians, **(b)** $5e^{\pm i\frac{\pi}{3}}$

Question 10

(i)   The question asks candidates to translate the description of the scenario into a differential equation. Many candidates did this successfully. The common errors were to omit the constant of proportionality, to use *t* as the constant, or to include an additional factor of *t*. Some candidates retained the symbol for proportionality and did not get as far as an equation.

(ii)   The separation of variables required in this example was straight forward, and many candidates were successful in reaching a correct expression for *N* in terms of *t*. Some candidates reached a correct expression for *t* in terms of *N* and went no further, others made errors in changing the subject of their equation. It was quite common to see solutions with $e^{\ln 500}$ not simplified to 500. The most common error was in finding the value for *k*, usually because candidates overlooked the information that 60 was the rate of change of the number of plants, not a value for *N* or for *t*. Several candidates made errors in copying their own work, with 150 becoming 50 in the course of the working.

(iii)   Most candidates substituted $t = 15$ or $N = 2000$ into their equation connecting *N* and *t*, but several misinterpreted the question and thought that they were aiming for $N = 1350 (= 2000 - 650)$.

Answers: **(i)** $\dfrac{dN}{dT} = k(N - 150)$ **(ii)** $N = 500e^{0.08t} + 150$

MATHEMATICS

General Comments

About half of the candidates produced reasonably good answers on this paper, although a number of candidates were clearly not well prepared for the examination.

Some candidates lost marks due to not giving answers to 3 sf as requested and also due to prematurely approximating within their calculations leading to the final answer. Candidates should be reminded that if an answer is required to 3 sf then their working should be performed to at least 4 sf.

One of the rubrics on this paper is to take $g = 10$ and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve the correct answer unless this value is used.

It should be noted that if a calculator is used within a question, for example in solving a quadratic equation, then some detail of working should also be shown. This is because if a calculator has been used and an incorrect answer is merely stated then this cannot earn any method marks.

Comments on Specific Questions

Question 1

(i) Candidates usually applied the definition of work done to the force of 200 g N as WD = Force x distance moved in the direction of the force. An error seen was to use 200 as the force giving a dimensionally incorrect answer.

Answer: Work done by the weightlifter = 1400 J

(ii) In this part most candidates applied the definition of Average Power = Work Done / Time and the majority scored well here.

Answer: Average Power developed by the weightlifter = 1170 W

Question 2

(i) Most candidates who attempted this question correctly resolved forces down the plane to find that the acceleration of the particle was 5 ms^{-2} and then used the constant acceleration equations to find the time taken to reach the given speed.

Answer: Time taken for the particle to reach a speed of 2.5 ms^{-1} is 0.5 s

(ii) A number of different methods could be applied to solve this part of the question. It is possible to find the distance travelled along the rough plane without considering the two stages of motion. This can be done by equating the loss of Potential Energy to the work done against the frictional force. Other methods require that the speed, v, of the particle as it reaches the rough plane is determined as $v^2 = 30$ and then applying either work/energy methods or Newton's second law of motion and the constant acceleration equations to determine the distance travelled.

Answer: Distance travelled along the ground before coming to rest = 7.5 m

Question 3

(i) This question required the use of Newton's second law applied to the lorry in the direction up the hill. Many candidates forgot to include the effect of the component of the weight which is acting down the hill. Also some attempted to use the constant acceleration equations which are not valid as the acceleration given is only applied at that instant. The equation of motion gives a value for the driving force, F, which then has to be related to the power, P, using the formula $P = Fv$

Answer: Power developed by the lorry's engine is 514 kW

(ii) In this part candidates needed to realise that the acceleration is zero and that the given power, P, has to be used in Newton's second law in the form of driving force = P/v, where v is the required steady speed. Again a common error was to ignore the component of the weight down the hill.

Answer: Steady speed of the lorry up the hill is 31.7 ms^{-1}

Question 4

In this problem candidates need to realise that the tension, T, in the string is determined by considering particle Q and as this particle is in equilibrium the tension balances the weight giving $T = 5\,g$. Secondly, in order to determine the frictional force, F, acting on particle P, the normal reaction, R, at P is required and then used in the equation $F = \mu R$. Most candidates followed this approach. It is then necessary to consider the two extreme cases for the particle P namely **(a)** P being on the point of slipping up the plane and **(b)** P being on the point of slipping down the plane. This gives the largest and smallest possible values for the mass m of particle P and these values provide the solution to the problem. Many candidates only applied the equilibrium condition in one of these two extreme cases.

Answers: The set of values of m for which the two blocks remain at rest is $6.78 \le m \le 12.2$

Question 5

(i) This problem involved resolving forces in two perpendicular directions. The majority of candidates attempted to resolve the given forces acting on the bead horizontally and vertically and equated the components to $R \cos 15$ and $R \sin 15$ respectively. Some chose to resolve along and perpendicular to the force F. Either method was acceptable. The two resulting equations could then be solved for F and R in either case. One error was to include all of the forces on the diagram including R when resolving and not knowing what this equated to. As the vertical component of R acts downwards, some resolved but had sign errors appearing in their solution. Overall most candidates produced good attempts at this question.

Answers: $F = 1.90$ and $R = 12.4$

(ii) In order to solve this problem and determine the mass m of the bead, candidates had to apply Newton's second law in the form $F = ma$ where F is the force acting on the particle along the wire AB and a is its acceleration. Since F is known to be $R \cos 15$, it is necessary to find the acceleration before the mass m can be found. As the applied force is constant, the constant acceleration equation $v^2 = u^2 + 2\,as$ can be used to find acceleration from the given values of $u = 0$, $v = 11.7$ and $s = 3$. Most candidates found the value of a correctly but several candidates merely used R rather than its component along AB when stating the force F.

Answer: Mass of the bead = 0.526 kg

Question 6

(i) Since the given form of velocity leads to a variable acceleration, it is not possible here to use the constant acceleration equations. Many candidates did not realise this and failed to use the correct method of integration of the given velocity to determine the displacement of the particle. Those who followed this method generally substituted correctly and verified the given value. Differentiation of the velocity was needed to find acceleration and again several candidates wrongly used constant acceleration equations to find this.

Answers: Given: At $t = 5$ the particle is 6.25 m from O Acceleration of the particle when $t = 5$ is 0.3 ms^{-2}

(ii) Candidates who were correctly using calculus to solve this problem often used the fact that the maximum velocity occurred when a = 0. Others considered the velocity-time graph and with either method it was found that the maximum velocity occurred at t = 10 when the velocity was 3 ms^{-1}. In order to complete the problem, the given form of velocity was equated to 1.5 and the resulting quadratic equation solved.

Answer: Values of t when the particle is travelling at half of its maximum velocity: t = 2.93 s and t = 17.07 s

Question 7

(i) This problem can be solved either with the use of constant acceleration formulae or by using a velocity-time graph and candidates were equally divided as to which method they used. Either method was acceptable. In this part it was first necessary to determine the time taken to reach the constant speed and to find the value of this speed. Once this had been achieved the total distance travelled before deceleration begins can be found. As deceleration starts it can be shown that the cyclist still has 24 m to travel and this distance can be used to determine the time taken for the deceleration stage. Finally the times for the three stages, namely constant acceleration, constant speed and constant deceleration can be added to determine the total time of travel. This part was well done by most candidates and even those who failed to find the correct total time generally scored more than half marks on this part for their working.

Answer: Total time taken by the cyclist to travel from A to B is 12 + 25 +8 = 45 seconds

(ii) Candidates found this part quite difficult and not many fully correct answers were seen. The usual method was to find the distance that the cyclist had travelled in the 24 seconds before the car starts and then to add to this the distance travelled at constant speed by the cyclist while the car is moving. Using the constant acceleration equations the distance travelled by the car can be found and then equated to the total distance travelled by the cyclist. If this is done correctly it leads to a quadratic equation. The solution leads to two values, one of which is impossible and so the other solution is the correct answer. An error frequently seen was that candidates did not correctly use the fact that there is a 24 second delay before the car starts.

Answer: t^2 – 51t + 594 = 0 has solutions t = 18 (rejected) and t = 33. Time taken to overtake is 33 seconds

MATHEMATICS

| Paper 9709/42 |
| Paper 42 |

General Comments

The paper was generally well done by many candidates although as usual a wide range of marks was seen. The presentation of the work was good in most cases.

Some candidates lost marks due to not giving answers to 3 sf as requested and also due to prematurely approximating within their calculations leading to the final answer, particularly in **Questions 1, 4, 5, 6** and **7**. Candidates should be reminded that if an answer is required to 3 sf then their working should be performed to at least 4 sf.

In **Questions 5** and **6** enough information was given such that the sine and cosine of a required angle could be evaluated exactly. In both questions it was not necessary to calculate the angle itself. However, many candidates often proceeded to find the relevant angles to 1 decimal place and immediately lost accuracy and in some cases marks.

One of the rubrics on this paper is to take $g = 10$ and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve the correct given answer unless this value is used.

Comments on Specific Questions

Question 1

(i) The majority of candidates chose to resolve forces horizontally as this did not involve the force G. In most cases this was done correctly and the given answer was achieved. In this case, as whenever an answer is given in the question, it is important to show all of your working. Most candidates scored well on this part.

Answer: Given: $F = 41.0$

(ii) In this part the value of F must be used as 41.0 and the majority of candidates resolved forces vertically in order to find G. Most found the correct value by this method.

An alternative approach to the whole question is to resolve forces in the two perpendicular directions defined by the two F forces. This leads to a pair of simultaneous equations for F and G which can be solved to produce the magnitudes of the forces F and G. This method is perfectly acceptable.

Answer: $G = 56.0$ or $G = 15\left(2 + \sqrt{3}\right)$

Question 2

(i) The majority of candidates correctly chose to study the final 35 m of travel in order to solve this part of the question. When using constant acceleration equations with $u = V - 10$ and $v = V$ the correct answer follows after simplification. However, a number of candidates wrongly chose $u = 0$ and $v = V$ or $v = V - 10$ and hence failed to obtain the correct answer.

Answer: $V = 40$

(ii) Once *V* is found from part **(i)** almost all candidates correctly used their value of *V* to consider the motion from start to finish over the distance *H* using constant acceleration equations with *a* = 10. Even those who failed to determine the correct value of *V* in part **(i)** were able to score the method mark.

Answer: *H* = 80

Question 3

(i) The majority of candidates scored well on this question. Most realised that differentiation was needed to find the acceleration and this was generally found correctly. Setting the acceleration equal to zero leads to a quadratic equation. Although the coefficients involved decimals, it could be simplified and factorised giving the two solutions. Some errors were seen due to miscopying the number of zeros in the decimals. Only a few candidates wrongly used constant acceleration equations for this part.

Answer: Values of *t* at which acceleration is zero are *t* = 40 and *t* = 60

(ii) Again most candidates correctly integrated the given expression for velocity in order to find the displacement. The resulting expression has to be evaluated between limits of *t* = 0 and *t* = 100 to find the required displacement. Although many candidates found the correct displacement, a good number lost marks due to incorrect calculations again possibly due to misreading the decimals. Once again only a small number of candidates attempted this part using the constant acceleration equations by finding *v*(100) = 8.8 and using *s* = (0 + 8.8)/2 × 100 which then fortuitously gave the correct answer but did not score any marks.

Answer: The displacement of *P* from *O* when *t* = 100 is 440 m

Question 4

Candidates generally found this question to be the most difficult on the paper. There are several methods of approach to the problem. Whichever method is used it is essential to determine the frictional force whilst the particle, *P*, travels on the rough surface. Once the normal reaction is found as $\sqrt{2}$, the frictional force can be found using $F = \mu R = 0.4\sqrt{2}$. The neatest solution is found by using the work/energy principle where the initial total potential energy at *A* is equated to the final kinetic energy at *C* plus the work done against the frictional force and the value of the velocity at *C* is obtained. Alternative methods involve calculation of the velocity at *B* and then either applying work/energy principles or using Newton's second law and the constant acceleration equations on *BC* to find the speed at *C*. Any of these methods can obtain full marks. An error often seen when candidates attempted to find the speed at *B* was the use of constant acceleration equations which only apply to motion in a straight line. It is necessary to use energy principles to find the speed at *B*. Another error often seen was the use of incorrect signs in the energy equation.

Answer: The speed with which *P* reaches *C* is 9.16 ms^{-1}

Question 5

(i) The most straightforward approach to this part of the question is to apply Newton's second law to each particle and then solve the resulting pair of simultaneous equations to find the required values of *T* and *a*. This was the choice of method of almost all candidates. It is possible to solve this problem using work and energy but it becomes much more involved. An error which was often seen occurred when the equation of motion of particle *P* was considered. Many candidates did not take into account the fact that as the equation is applied down the plane it required the component of the weight down the plane and not simply 0.5 *g*. Those who wrote down the correct equations and solved them often gave the value of the acceleration, *a*, to 2 sf and hence lost a mark.

Answer: The tension in the string $T = \dfrac{16}{15} = 1.07\,\text{N}$ Magnitude of acceleration $a = \dfrac{2}{3} = 0.667\,\text{ms}^{-2}$

(ii) Very few candidates achieved the correct final answer here. Using the value of *a* found in **(i)** in the constant acceleration equations enables the speed of the particles as the string breaks to be found. After the break the acceleration changes and becomes the component of *g* down the plane and this was often taken in error either as *g* or as the same acceleration as found in **(i)**. The further distance, *x*, travelled down the plane by *P* can now be found and hence the length of the string is given by 2.5 – *x*

A common error seen was candidates not realising that the kinematics had to be applied separately before and after the break with some wrongly trying to combine both parts of the motion.

Answer: The length of the string is 1.95 m

Question 6

(i) This question was well done by many candidates. Horizontal resolution of forces is needed to find the frictional force *F*. Some wrongly equated *F* to the vertical component of the force 0.195 N and some thought that *F* was 0.195 N. Vertical resolution of forces gives the normal reaction *R*. Some wrongly thought that R was merely the weight instead of a combination of the weight and the vertical component of the 0.195 N force. Finally use of the equation $F = \mu R$ gives the required value of μ.

Answer: The coefficient of friction between the ring and the rod is $\mu = \dfrac{4}{7} = 0.571$

(ii) The error made by most candidates in this part was thinking that the normal reaction is the same as in part **(i)** whereas it has to be re-evaluated. Once the new value of *R* has been found, a three term Newton's law equation enables the acceleration to be found.

Answers: Acceleration of the ring $= \dfrac{25}{7} = 3.57 \ \text{ms}^{-2}$

Question 7

(i) This part was correctly found by many who used the definition that 'Work Done = Power x Time'. An error which was seen was to misquote the formula as Power/Time. Some attempted to find the distance travelled in order to use the formula 'Work Done = Force x Distance' moved but in doing so assumed that it was a constant acceleration problem and so even though the correct answer was achieved, it was from an invalid method with fortuitous cancelling.

Answer: Work Done by the car's engine = 350000 J = 350 kJ

(ii) For this part of the question, candidates had to write down Newton's second law for the motion of the car at the two instances given. The driving force in each case has to be written as 14000/*v* and the equation is then used to determine the velocity of the car at the two given times. Once the velocities are found the definition of kinetic energy is used to complete the solution. An error often seen was to not include the resistance term which leads to velocities of 17.5 ms^{-1} and 35 ms^{-1}.

Answer: V_A = 2800/207 = 13.53 ms^{-1} and V_B = 2800/127 = 22.05 ms^{-1} Gain in Kinetic Energy = 242490 J

(iii) In order to complete this part of the question, energy principles must be used and a common error seen was an attempt to find the distance *AB* by using constant acceleration formulae. The work/energy principle states that the work done by the car's engine = increase in kinetic energy plus the work done against resistance and by applying this to the given situation enables the required distance *AB* to be found.

Answer: The distance *AB* = 457 m

MATHEMATICS

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates are also reminded to maintain sufficient accuracy in their working to achieve this level of accuracy in their final answers (e.g. **Question 2, Question 5(ii)** and **Question 7(ii) and (iii)**).

- Candidates are reminded of the importance of a clear and complete force diagram in problem solving and a consideration of all forces when resolving or when writing down equations of motion or work / energy equations.

- Candidates should take note of key words in each question e.g. 'equilibrium' in **Question 1**, 'resultant' in **Question 3**, 'speed' in **Question 6** and 'work and energy' in **Question 7**.

General comments

This examination provided the opportunity for candidates to demonstrate what they knew and most candidates attempted all questions. The standard of work seen was very varied including, as usual, some excellent, well presented scripts. A number of candidates, however, were clearly not prepared for problem solving at this level. **Question 4** was found to be the easiest question whilst **Question 1,** even for good candidates, was often a zero mark question.

Comments on specific questions

Question 1

This proved to be a challenging first question. Candidates were expected to 'state' the tension by considering the forces acting on particle P. Many overlooked the state of equilibrium and set up incorrect equations of motion for P and for B, e.g. $4g – T = 0.4a$ and $T – 3g = 0.3a$, thus obtaining an acceleration and also ignoring the significance of the inclined plane. Some candidates gave two different values for T (e.g. T_P=30 N and T_B=40 N) suggesting that the tension throughout the string was not constant. Other incorrect values for the tension were 10 N (40-30), 35 N (½ (30+40)), and 70 N (30+40).

Candidates frequently concluded that the normal component of the contact force was 32 N, without considering the component of the tension perpendicular to the plane. Many solutions did not include a force diagram which could have helped to ensure that all relevant forces were considered.

Answer: 30 N   8 N

Question 2

Whilst the majority of candidates attempted to apply $F = \mu R$ to the situation, a high proportion oversimplified the vertical resolution of forces, with no T component, concluding that $R = 2g$ rather than $R = 2g – 2g\sin\alpha$. Others, who did include R, $2g$ and $2g\sin\alpha$ believed that $R = 2g\sin\alpha – 2g$ or $R = 2g + 2g\sin\alpha$. A few candidates mistakenly used $0.2g$ as 20 N whilst many found an approximation for the angle (16.3°) without realising that $\sqrt{(1 – 0.96^{2})}$ or $\sin(\cos^{-1}0.96)$ would give an exact value for $\sin\alpha$. A large number of solutions were completed correct to two significant figures (0.49 N) instead of the required three significant figures.

Answer: 0.485 N

Question 3

Although some candidates completed clear, concise and accurate solutions, the question proved to be difficult for many, either with equations set up but then not solved or with inaccurate equations solved. The majority of candidates attempted to resolve the forces in two perpendicular directions to form a pair of simultaneous equations in P and in θ as expected. However, the equations used were frequently $100 + P\cos\theta = 150 + 120\cos75^\circ$ and $P\sin\theta + 120\sin75^\circ = 0$, suggesting that the resultant was part of a system of four forces in equilibrium. Those candidates who formed equations using $\tan75^\circ = 'Y/X'$ and $120^2 = 'X^2 + Y^2'$ were often unable to eliminate either P or θ from their pair of equations despite sometimes lengthy attempts. An alternative solution which worked well for a few candidates was to form a triangle of forces with P N, 50 N and the resultant 120 N and then to apply the sine and cosine rules.

Answer: 117   80.7

Question 4

This question was a straightforward question for the majority of candidates with many gaining full marks.

(i)   Most candidates formed the equations of motion for particles A and B and then solved them. Occasionally the acceleration was found but the tension omitted and occasionally the acceleration was found to be negative from equations set up in the opposite direction to motion. Other errors included occasional sign errors in one or both equations and slips in solving the equations. Those who found the acceleration from a single equation in a only, did not always form a second equation for finding the tension.

(ii)   Candidates who used constant acceleration formulae often completed the solution successfully although some continued with their value for acceleration rather than using $a = g$ after A reached the floor. Those who attempted an energy solution frequently omitted to consider the work done by the tension.

Answer: **(i)** 4 ms^{-2} 2.1 N **(ii)** 2.24 m

Question 5

(i)   Whilst many candidates were able to apply Newton's Second Law to the situation to achieve the required result, some omitted 'ma' to obtain $R = 420/v + 90g(0.05)$ whilst others omitted the component of weight down the hill to obtain $R = 420/v - 90(0.016)$. Although the given answer allowed candidates to check their working, inaccuracies and sign errors were seen. Those who calculated the angle of incline of the hill to be 2.9° and then used $\sin 2.9^\circ$ instead of 0.05 could not achieve the required 43.56. Candidates were expected to give sufficient working to achieve the given result, but some solutions were too minimal to gain all marks. A significant number of candidates attempted a work / energy solution, including $500R$ and $500(420/v)$, despite the varying R and v.

(ii)   Although the speed at the mid-point of the hill could be found using $v^2 = u^2 + 2as$, some candidates mistakenly averaged the speeds at the top and bottom of the hill to get $v = (3+5)/2 = 4$ ms^{-1} at the mid-point. Whilst the given equation in R and v was used, some candidates found only one of the two decreases in the value of R. It was also common to see approximations in working which led to insufficient accuracy in the final answers.

Answer: **(ii)** 4.12 ms^{-1} 38.1   17.9

Question 6

(i)   Part **(i)** was often answered well with candidates achieving a velocity of –32 ms^{-1} although the speed of 32 ms^{-1} was not always stated. Whilst candidates usually differentiated to find the velocity *v*, there was some variation in the value found for *t*. Some found and used the time when *v*=0 or when *a*=0 rather than solving *x*=0 to find the time taken to return to *O*.

(ii)   A common error in this part of the question was to assume that the furthest distance from *O* was reached in half of the time (*t*=200) rather than when *v*=0. Those who correctly found *x* when *t*=800/3 sometimes added this to *x*(400/3) to find the total journey time rather than doubling the distance to the furthest point. Some simply omitted to consider the return journey or rounded before doubling, leading to an error in accuracy.

(iii)   Candidates generally knew that 'total distance/total time' was required and those who used a suitable method to find the total distance in part **(ii)** were usually able to complete part **(iii)**. Some candidates used only half the distance whilst others identified an incorrect time.

Answer: **(i)** 400 s   32 ms^{-1} **(ii) (a)** 3790 m **(ii) (b)** 9.48 ms^{-1}

Question 7

This question differentiated well with most candidates able to make some progress particularly with parts **(i)** and **(ii)**

(i)   Since the question asked for a work and energy solution, the minority of candidates who attempted solutions based only on Newton's Second Law were unable to gain full marks. The work/energy solutions were often correct but the usual range of errors was also seen. These included sign errors e.g. KE gain + PE loss, missing terms (usually 2000 x 400 or 1250 g x 400 sin 4°), or an extra term if 'work done by the weight component' appeared as well as the potential energy loss. Some candidates used the resistance 2000 N rather than the work done by the resistance in their work/energy equation. Others found the work done by the driving force, omitting to divide by 400 to find the driving force itself.

(ii)   This part was well answered, usually with a suitable application of Newton's Second Law. Candidates occasionally overlooked the doubling of the driving force or omitted to include the 2000 N resistance in their equation. A few candidates treated the 'work done by the driving force' as if it was a force, thus creating a dimensionally incorrect equation. The answers 0.3 and 0.304 were both regularly seen. Candidates are expected to give answers correct to three significant figures and to maintain sufficient accuracy in their working to achieve this.

(iii)   Most candidates calculated the power correctly using *P*=*Fv* as a simpler method than using *P*/*v*–*R*=*ma*. The main problem was inaccuracy following any previous error or over approximation. 54300 W following from *a*=0.304 in part **(ii)** was a common answer whilst 54000 W corrected to two significant figures was also seen. *v*=8 ms^{-1} was used occasionally instead of the velocity at C, and the driving force was not always doubled as required.

Answer: **(i)** 1190 N **(ii)** 0.302 ms^{-2} **(iii)** 54100 W

MATHEMATICS

| Paper 9709/51 |
| Paper 51 |

General Comments

This paper proved to be slightly harder than the one set last November.

The work was generally well presented and easy to follow with a few exceptions.

Some candidates pre-approximated their values while working in the middle phase of their solutions. This eventually produced errors in their final answers. If a 3 significant answer is required, it is recommended that at least 4 significant figures are used in all intermediate working.

g = 10 in now being used as requested by most candidates.

A good clear diagram can often help with the solution to a question.

The easier questions were **1, 2(i), 3, 4(i)** and **5(i)**.

The harder questions were **4(ii), 5(ii), (iii), 6** and **7**.

Comments on Specific Questions

Question 1

This question was generally well done with many candidates scoring full marks.

Answer: 2.4(0) ms^{-1}

Question 2

   (i)   This part was usually well done. Some candidates simply quoted the answer. This was not allowed as the answer was given in the question.

   (ii)   Very few candidates were able to give a clear explanation for this part of the question.

   (iii)   Candidates clearly knew that they had to take moments and often did so correctly.

Answers: **(i)** 20sinθ **(ii)** no friction of ring smooth   **(iii)** θ = 63.3°

Question 3

   (i)   Too many candidates said that 0.3 $v\dfrac{dv}{dx} = 2x$ and not -2x. A few candidates did not state the value of k as requested.

   (ii)   This part of the question was well done by separating the variables, integrating and using the correct limits.

Answers: **(i)** $v\dfrac{dv}{dx} = \dfrac{-20x}{3}$, k = –20/3   **(ii)** x = 3.1(0)

Question 4

(i)   This part was generally well done. *T* and *v* were found by resolving vertically and using Newton's Second Law horizontally respectively.

(ii)   Very few candidates were able to find the length of the string and so could not find the radius of the circle. The length of the string should be 0.6/sin30 + 0.6/sin45. The radius would then be (0.6/sin30 + 0.6/sin45)sinθ. The final step was to use Newton's Second Law horizontally.

Answers: **(i)** Speed of B = 6.75 ms^{-1} **(ii)** Tension = 5.53 N

Question 5

(i)   This part of the question was generally well done

(ii)   Very few candidates were able to find the required extension.

At the instant when the particle leaves the surface, the normal reaction is zero. so by resolving vertically, *T*cosθ = 0.2*g*, where *T* is the tension and θ is the angle the string makes with the vertical.

If e is the extension then $\dfrac{21e}{0.75}$ x $\dfrac{0.8}{(e+0.75)}$ = 0.2*g*, resulting in *e* = 0.0735.

(iii)   An energy equation in 4 terms is now required.

This equation would be initial EE + initial KE = final KE + final EE. This when solved would give the required speed.

Answers: **(i)** 0.6 N   **(ii)** 0.0735 m   **(iii)** 2.93 ms^{-1}

Question 6

In parts **(i)** and **(ii)** it was necessary to find the area of the disc which should be $\pi(1.2^2 - 0.4^2 - 0.3^2) = 1.19\pi$. Too many candidates got $\pi(1.2^2 + 0.4^2 + 0.3^2) = 1.69\pi$.

Part **(i)** required candidates to take moments about the x-axis and part **(ii)** moments about the y-axis.

(iii)   By using tan^{-1} ((distance in part **(i)**)/(distance in part **(ii)**)), this gave the required angle.

Answers: **(i)** 0.0941 m   **(ii)** 0.0378 m   **(iii)** 68.1°

Question 7

This question proved to be the hardest question on the paper.

(i)   This part of the question could be done by equating horizontal distances.

(x=) *V*cos45*t* = *V*cos60(*t* + 1) should have been used. Some candidates used *V*cos45(*t* + 1) = *V*cos60*t*, which suggested that Q was in flight longer than P.

(ii)   Very few candidates were able to solve this part of the question. This part can be done by equating the vertical distances.

(y =) *V*sin45*t* – *gt*^{2}/2 = *V*sin60(*t* + 1) – *g*(*t* + 1)^2/2 with *t* = 2.414 from part **(i)**.

(iii)   Only a handful of candidates arrived at the correct answer.

To solve this part it was necessary to work out the greatest height of P, the height of the particles when the collision occurred and then to subtract them.

Answers: **(i)** t = 2.414   **(ii)** 23.3   **(iii)** 9.72 m

MATHEMATICS

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MATHEMATICS

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$g = 10$ is now being used as requested by most candidates.

A good clear diagram can often help with the solution to a question.

The easier questions proved to be **2**, **4(i)**, **(ii)(a)**, **5(i)** and **7(i)**.

The harder questions proved to be **4(ii)(b)**, **5(ii)**, **(iii)**, **6** and **7(ii)**, **(iii)**.

Comments on Specific Questions

Question 1

This question can be solved in a number of different ways. The most popular approach was to use horizontal and vertical motion to find the two velocities. Having done this the resultant of these is equal to 18. This leads directly to the required time.

Answer: 1.1(0) s

Question 2

This question was generally well done by most candidates.

(i)   This part was done by equating vertically for the particle.

(ii)   By using Newton's Second Law in the direction of the centre O, the required velocity was found.

Answers: **(i)** 2.5 N   **(ii)** 1.5 ms^{-1}

Question 3

To solve this question candidates were required to use horizontal and vertical motion. The horizontal and vertical velocities at *A* are $V\cos\theta$ and $V\sin\theta$.

Horizontal motion gives 40 cos30 = $V\cos\theta$ x 4 i.e. $V\cos\theta$ = 10 cos30

Vertical motion gives 40 sin30 =$V\sin\theta$ x 4 $-\dfrac{1}{2}$ x 10 x 4^2 i.e. $V\sin\theta$ = 25.

By squaring and adding these *V* can be found and by dividing θ can be found.

Answers: V = 26.5 and θ = 70.9

Question 4

Parts **(i)** and **(ii)(a)** often scored full marks. Part **(ii)(b)** proved more difficult.

(i)   By using $v = r\omega$, r can be calculated and then by trigonometry of a right angled triangle, θ can be found.

(ii)(a) By resolving vertically for the particle the required angle can be calculated.

(b)   This part required the use of Newton's Second Law towards the centre of the circle travelled by the particle. This gives $6\sin\theta = 0.4\omega^2 \times 0.5\sin\theta$, where θ is the angle that OP makes with the vertical. Too many candidates used 0.5 and not $0.5\sin\theta$ for the radius.

Answers: **(i)** 53.1°   **(ii)(a)** 48.2°   **(ii)(b)** 5.48 rads^{-1}

Question 5

(i)   By using Newton's Second Law vertically upwards with $a = v\dfrac{\mathrm{d}v}{\mathrm{d}x}$ the required equation can be set up.

(ii)   The equation from part **(i)** can now be solved by separating the variables and integrating. By using the correct limits the greatest height can then be worked out.

(iii) Newton's Second Law applied vertically downward results in a new differential equation. Solving this results in finding the velocity just before P hits the surface.

Answers: **(i)** $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -10 - 0.4v^2$ **(ii)** 2.85 m   **(iii)** 7.14 ms^{-1}

Question 6

Candidates found this question very difficult. It proved to be one of the hardest questions on the paper.

(i)   Very few candidates were able to show that $d = h + 0.4/h$.

If θ is the semi-vertical angle of the cone then $d\cos\theta = \dfrac{h}{\cos\theta} = \sqrt{(h^2 + 0.2^2)}$

$\cos\theta = \dfrac{h}{\sqrt{h^2 + 0.2^2}}$ and $\cos^2\theta = \dfrac{h}{d}$ i.e. $d = \dfrac{h}{\cos^2\theta}$

Hence $d = \dfrac{h}{\left(\dfrac{h^2}{(h^2 + 0.2^2)}\right)} = \dfrac{h^2 + 0.4}{h} = h + 0.4/h$

(ii)   This part of the question can be solved by taking moments. If moments are taken about the vertex of the cone then $4 \times 0.6 + W \times 0.9 = (4 + W) \times 0.85$. This leads to $W = 20$.

Answers: **(i)** $d = h + 0.4/h$ **(ii)** $W = 20$

Question 7

(i) 2 marks were usually scored for this part of the question.

(ii) A 3 term energy equation, including PE, KE and EE, was required here. This often proved to be too difficult for many candidates.

The equation should be $Mg(0.8 + e) = M \times 4.4^2/2 + 12.5e^2/(2 \times 0.8)$. It was then necessary to use e = 0.64M from part **(i)** to set up an equation in M. Solving this equation results in the correct value for M.

(iii) This part also proved to be too difficult for many candidates.

A 2 term energy equation was needed here. This came from PE at A = EE at B.
i.e. $0.525 \, gd = 12.5(d – 0.8)^2/(2 \times 0.8)$, where $d = AB$. This gives a quadratic equation and when solved gave the correct value of d.

Answers: **(i)** $e = 0.64M$ **(ii)** $M = 0.525$ **(iii)** $AB = 1.94$ m

MATHEMATICS

> Paper 9709/61
> Paper 61

Key Messages

To do well in this paper candidates must work with 4 significant figures or more in order to achieve the accuracy required. Candidates should also show all working, so that in the event of a mistake being made, credit can be given for method; a wrong answer with no working shown, scores no marks. Candidates should label graphs and axes including units, and choose sensible scales.

General comments

It was pleasing to see that at least some of the candidates who took this paper had a good knowledge of the syllabus. However there is still evidence that further practice of past papers would help candidates with the language and style of questions used. The paper was straightforward and allowed candidates to demonstrate their knowledge of basic skills.

Comments on specific questions

Question 1

This was a routine question on the binomial distribution and was well attempted by the candidates who knew what a binomial distribution was.

Answer: 0.781

Question 2

This question on the normal distribution involved candidates reading the normal distribution tables backwards, to find a *z*-value, and then solving an equation. A diagram would have shown candidates immediately that if P(X < 54.1) is 0.5, then the mean must be 54.1. A few good candidates recognised this but the majority of those who attempted this question tried to solve it by simultaneous equations, which is a perfectly acceptable way of doing it.

Answers: 54.1, 2.88

Question 3

This question was answered better than the others, with many candidates displaying a knowledge of frequency density. The histogram was well drawn by those who obtained the correct values of *a* and *b*, though many lost marks because of poor scales and no labelling.

Answers: 4.5, 6

Question 4

This was a standard part question on coded means and standard deviations as it is shown in the Syllabus. Some candidates managed full marks here, others managed to find the standard deviation but not the mean. The second part was a routine normal distribution question.

Answers: 75.1, 2.78, 0.269

Question 5

A good question on permutations and combinations which was well attempted by many candidates. Candidates need to know that the word 'arranged' implies arrangements or permutations and that the words 'selected' and 'chosen' imply combinations.

Answers: 180, 210, 20

Question 6

This question was relatively straight forward, and indeed many candidates gained full marks for parts **(i)** and **(ii)**. Many however, did not appear to understand that the request 'Calculate the expected number of unsuccessful attempts' was the same as the command 'find $E(X)$'.

Answers: P(0) = 0.4, P(2) = 0.144, P(3) = 0.216. $E(X)$ = 1.176

Question 7

It was pleasing to see that many candidates used common sense and found the correct probability for **(i)**. Almost all managed to multiply three probabilities together to find the probability of two 5s and a 4. Some then omitted to multiply by 3, for the three options 554, 545, 455. The second part of this question was a normal approximation to the binomial which did require candidates to recognise the binomial in the first place. They were then told to use a suitable approximation, which means only 1 thing if they have covered the syllabus.

Answers: 2/9, 4/423, 0.812

MATHEMATICS

Key Messages

To do well in this paper candidates must work with 4 significant figures or more in order to achieve the accuracy required. Candidates should also show all working, so that in the event of a mistake being made, credit can be given for method; a wrong answer with no working shown, scores no marks. Candidates should label graphs and axes including units, and choose sensible scales.

General comments

This paper was well attempted by the majority of candidates who mainly showed a sound knowledge of the syllabus. There were very few candidates entered who did not manage to pick up some marks, and it was pleasing to see almost all gave their answers correct to 3 significant figures.

Comments on specific questions

Question 1

Most candidates scored something on this question; as nearly all were able to write down $2416/n$. Candidates who formed an equation $2416/n = 216/n + 100$ demonstrated frequent difficulties in rearranging the equation to solve for n while others seemed unsure of whether to add or subtract 100. Whilst many candidates expanded $\sum(x - 100)$ to $\sum x - 100n$ correctly, others tried to work with $\sum x - \sum 100$, introducing n only at the end of the working.

Answer: 22

Question 2

A large proportion of candidates gained 1 mark for appreciating that choosing no men meant choosing 6 women from 9, which involved $^9C_6$ giving an answer of 84. Candidates who introduced $^{16}C_6$ often did not realise that this needed to be the denominator of a fraction. The question asked for a probability but many candidates left the answer as 84. A few candidates did achieve a probability but gave their answer to 2 significant figures instead of 3, as requested in the instructions. Fortunately they mainly gave it as a fraction so full marks were awarded there, and the 2 significant figures was not penalised. It would have been had the fraction not been there. All equivalent fractions are acceptable.

Answer: 3/286 (0.0105)

Question 3

(i) Not surprisingly, over 99% of candidates got this answer correct.

(ii) This part was also correctly done by many candidates. A common error was to multiply $(3/4)^4(1/4)$ by $^5C_1$ with many other candidates just multiplying ¼ by 5 and giving a probability of 5/4. It would be helpful for candidates if they knew that probabilities cannot ever be greater than 1.

(iii) This part was better attempted than part **(ii)** and was a good discriminating question.. Many candidates got $(1/4)^4$ or $4(1/4)^4$ scoring a method mark but only a few multiplied their fraction by 4! Candidates who attempted 1 x 3/4 x 2/4 x 1/4 were generally more successful although a common error with this method was to reduce the value of the denominator each time rather than the numerator.

Answers: **(i)** ¼, **(ii)** 81/1084, **(iii)** 3/32

Question 4

(i) This question stretched most candidates Some candidates recognised the need to use $^6C_2$ or $^6C_4$ and some recognised that they needed to double their answer to account for the taxis being swapped round.

(ii) This question also proved difficult for most candidates. The most common error was to add the 4! (or number of arrangements) for Taxi Q to their arrangements for Taxi P rather than multiplying. Many failed to multiply by 5 for the number of ways that the fourth seat in Taxi P could be filled. Another error was to multiply 5 and 4! but then multiply by 2 rather than 4 for the number of ways Jon and Sarah could sit. Very few candidates gave a correct answer.

Answers: **(i)** 30, **(ii)** 480.

Question 5

(i) The stem-and-leaf diagram was usually done well, with a majority of candidates scoring 3 marks out of 4 as a few lost marks because the 'leaves' were not always consistently spaced down the page. The 'key' was often incomplete, with full details, including Team A and Team B and units (kg) needed to gain the mark.

(ii) This question was answered quite well although there were a few candidates who thought that the IQR was 12 – 4 = 8 or that the 8th value therefore represented the IQR.

(iii) This question was done well. A few tried to subtract the mean weight of the 15 players from the mean weight of the 16 players and found a very small weight for the 16th player! More than a few candidates misread the 93.9 in the question as 93.6.

Answers: **(ii)** 18, **(iii)** 103

Question 6

(i)   This question was generally well done by almost everybody who attempted it. Candidates completed the table correctly although a few assumed the last column was 4.

(ii)   Almost everybody was able to complete a correct table for the probability distribution of X although a few candidates completed a frequency distribution rather than a probability distribution.

(iii)   Not so many candidates were able to calculate $E(X)$ and $Var(X)$ successfully. They knew what to do but a surprising number made slips in multiplying fractions. It was pleasing that almost all remembered to subtract the square of the mean, thus gaining 2 method marks (1 for attempting to find the mean and one for attempting to find the variance by the correct formula). However, only a small percentage of candidates managed to evaluate the mean correctly or evaluate $\sum x^2 p$ correctly.

(iv)   Few candidates were able to find the conditional probability either by counting values in their tables or by using the conditional probability formula. There seemed to be a misunderstanding of the terms 'even' and 'positive' among many candidates who included 0 in both sets.

Answers: **(iii)** 23/8, **(iv)** 5/9

Question 7

(a) (i)   Almost all candidates knew how to standardise correctly. A sketch diagram might have helped some to see that the required probability had to be greater than 0.5. Credit was given to those who then multiplied their probability by 365.

(ii)   This part question was done well. Candidates were aware that they had to look up the probability backwards in the table and many did this correctly. Failure to use the tables accurately meant that candidates who wrote $z = 1.161$ instead of 1.165 lost an accuracy mark for the z-value but the value was close enough to give them the rest of the marks for this question.

(iii)   About half the candidates recognised that this was a binomial question and gained full marks for it. Of the rest, some looked for the probability of exactly 2 days rather than fewer than 2 days, and some attempted to use a normal distribution.

(b)   This question was well done by those stronger candidates who were able to standardise and also cope with simple algebraic fractions.

Answers: **(i)** 315 or 316, **(ii)** 7350, **(iii)** 0.840, **(iv)** 0.933

MATHEMATICS

| Paper 9709/63 |
| Paper 63 |

Key Messages

Candidates should be encouraged to show all necessary workings. A significant number of candidates did not show sufficient working to make their approach clear and were unable to gain full credit.

Candidates should be aware that they need to work to a greater degree of accuracy than the 3 significant figures required in the answer in order to ensure that final answers are correct.

When drawing graphs, candidates should be encouraged to use scales that enable accurate readings to be achieved.

General Comments

Answers to **Questions 2, 4** and **6** were generally stronger than answers to others questions.

The majority of candidates used the answer booklets provided effectively, however a number failed to utilise the available space appropriately either by answering the entire paper on a single page, or dividing the page into two columns, both made their reasoning difficult to follow.

A number of candidates made more than a single attempt at a question and then did not indicate which solution was to be submitted.

Comments on Specific Questions

Question 1

Good solutions initially calculated the coded mean from the data given and then used the variance formula with both terms coded. Many candidates attempted to expand $\Sigma(t - 2.5)^2$ and made algebraic errors resulting in an incorrect value for Σt^2. A few candidates appear to have truncated their final answer.

Answer: 0.543

Question 2

Many clear solutions were seen. Good solutions recognised that information calculated in **(ii)** could be used in **(iii)**.

(i) Most candidates interpreted the question correctly, although fractions involving the population in millions were seen frequently. A few candidates used country *Y* at this stage, and then consistently reversed the information through the remainder of the question.

(ii) The majority of candidates considered the sum of the two relevant probabilities. A few candidates failed to evaluate their stated expression correctly.

(iii) Good solutions utilised the calculations in **(ii)** to produce the conditional probability. Weaker solutions often started again and failed to calculate P(X∩F) accurately. The few candidates who considered the relevant populations were often successful. Exact answers were achieved where fractions were used throughout, however a surprising number of candidates provided a decimal answer that was never more accurate than 2 significant figures, which is not acceptable at this level.

Answers: **(i)** $\dfrac{5}{7}$ **(ii)** $\dfrac{7}{20}$ **(iii)** $\dfrac{25}{49}$

Question 3

Most candidates made some progress with this question, although there was often little clarity about the conditions necessary for independence and exclusivity. Good solutions contained clear sample space diagrams, with entries meeting the conditions identified.

(i) Most candidates identified the possible outcomes and stated P(S) and P(T) related to their information. Where the outcomes were listed, they were often incomplete. An unexpected number of candidates failed to used their values when considering P(S∩T), many stating that there was no overlap despite evidence in their work to suggest otherwise. Weaker solutions often failed to state an appropriate condition for independence and simply performed the calculations required. Many candidates assumed independence to calculate P(S∩T) and produced a circular argument. The best solutions stated the condition that was required initially, produced all the relevant data and then made a final substitution and evaluation, finishing with their conclusion.

(ii) Most candidates recognised that they could interpret P(S∩T) to determine exclusivity. A number failed to state what the requirement was and so could not gain credit. Candidates who had not attempted **(i)** were able to identify terms that satisfied both conditions and provide an appropriate counter-example here.

Answer: **(i)** not independent **(ii)** not exclusive

Question 4

Candidates of all abilities attempted this question with some success.

(i) Most candidates were able to quote the required normal standardisation formula and use the tables accurately to convert the stated probability. It was unfortunate that a number of solutions were seen where the *z*-value had been rounded to 3 significant figures at the start of the solution. Some candidates failed to use the tables appropriately.

(ii) Most candidates were able to use their values to calculate the required probability for meeting the condition. The best solutions had sketches of the normal curve with the areas identified to ensure that the correct calculation was performed. Weaker solutions often had one area 'reversed'. A few solutions with a probability greater than 1 were seen. A noticeable number of candidates replaced the mean of 125 with the total number of seeds. Many solutions calculated the number of seeds expected to germinate with a probability state to 3 significant figures, and so were not able to access the accuracy mark. It was noticeable that a large minority of solutions failed to answer the question set, with no attempt to calculate the expected value.

Answers: **(i)** 9.76 **(ii)** 37.2 (or 37 or 38)

Question 5

Most candidates attempted this question, although many solutions were not fully correct.

(a) Good solutions clearly set out the rationale for the approach being taken, considering separately the ordering of the vowels and the remaining letters and then multiplying the final answer by the 8 positions that the vowels could be inserted. A good number of candidates simplified the process by considering the vowels as an additional 'letter' in the ordering. Most candidates correctly recognised the need to remove the 'repeated letter' duplicates, although many solutions had missing terms.

(b) The best solutions listed the possible groups of students that fulfilled the conditions and then accurately used combinations to evaluate the number of ways each group could be picked before summing the answers. Unfortunately, many lists were incomplete, with some candidates omitting groups without Americans. A few candidates included groups with a total of students greater than 4. The alternative approach – calculating the total number of ways the 4 students can be picked and subtracting selections that did not meet the criteria – was attempted less successfully by a few candidates, except where a detailed rationale was stated initially.

Answers: **(a)** 604800 **(b)** 1841

Question 6

Many good solutions to this question were seen. Candidates generally used scales that enabled the histogram to be drawn accurately, using the graph paper effectively. The best solutions often had a table that was used to generate data for both parts of the question.

(i) The best solutions stated the class widths prior to calculating the frequency density, the use of a continuity correction was clearly identified, with accurate histograms being constructed using these values as a linear scale on the horizontal axis. The majority of histograms were labelled appropriately, although omitting 'metres' on the horizontal axis was a common error. Weaker solutions rarely had a consistent continuity correction and bars were not drawn accurately. Although still seen, the use of frequency on the vertical scale seemed to be reduced.

(ii) Almost all candidates attempted the calculation of the mean and variance appropriately. The best solutions completed a table of values and used the results in the correct formula. Common errors were in the calculation of the mid-points, (not always due to the continued use of the incorrect end points of **(i)**) and inconsistency in evaluating the stated variance formula. A number of candidates failed to show their workings, and then failed to gain credit because incorrect answers were stated.

Answers: **(ii)** μ = 52.7, σ = 12.2

Question 7

Many candidates scored well on this question, with solutions set out clearly. There were a number of incomplete attempts noted, although this did not always appear to be due to time constraints.

(i) Almost all candidates recognised that a Binomial Approximation was required. Weaker solutions often omitted 2 water pistols from the terms required. A small number of candidates failed to evaluate their expressions accurately.

(ii) The best solutions clearly stated the inequality and then used the properties of logarithms to solve the resolution power equation. The majority of candidates did not indicate their approach, although many appear to be using trial and improvement. A few candidates were unable to interpret the inequality correctly, or did not state an integer value for *n*.

(iii) Many good solutions were seen, which contained a clear calculation of the mean and variance initially, standardised accurately, then sketched the normal distribution to determine how to calculate the required probability from their value. A significant minority used the incorrect

continuity correction and a few failed to make any continuity correction. Again, there were often arithmetical errors in the evaluation of calculations stated.

(iv) The majority of candidates recognised that $np > 5$ was a necessary requirement, interpreting their calculation in **(iii)**. Good candidates stated $nq > 5$ was a requirement but also evaluated to confirm, as well as highlighting that $n > 30$ was also fulfilled. A significant proportion of candidates stated inaccurately that $npq > 5$ was required and were not given credit. A number of solutions that had checked the requirements prior to the calculation of **(iii)** did not use this as justification here.

Answers: **(i)** 0.809 **(ii)** 28 **(iii)** 0.257

MATHEMATICS

Paper 9709/71
Paper 71

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. There was a complete range of scripts from good ones to poor ones. In general, candidates scored well on **Questions 2(i), 3 and 6** whilst **Question 7** proved particularly demanding.

Most candidates kept to the required level of accuracy; there were a few cases where candidates lost marks for giving final answers to less than three significant figure accuracy, this was particularly seen on **Question 5(i)** and **6(ii)** where errors could have been made because of confusion between 3s.f. and 3d.p. (see comments below).

Timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also many good and complete answers.

Comments on specific questions

Question 1

Many candidates attempted to find the mean number of failures over the 6 month period (1.75). It was then required to find the probability of more than 1 and less than 4 errors (i.e. 2 or 3 errors). A large number of candidates successfully used a correct expression, though some included 1 and/or 4 in their Poisson expression, and some attempted incorrect calculations such as $P(>1) \times P(<4)$ or $P(>1) + P(<4)$.

Answer: 0.421

Question 2

Part **(i)** was well attempted. Most candidates successfully used $N(29, 6^2/120)$ to find the required probability.

Part **(ii)**, however, was not well attempted. Many candidates mistook what the question was asking. The question asked if it was necessary to assume that the population was normally distributed (the answer was therefore 'No' because n was large, meaning the CLT could be applied and the sample mean was then (approximately) Normally distributed). Many candidates did not understand what was required in the question, and it appeared that they thought they were being asked if the sample mean was normally distributed, and therefore answered 'Yes'. It is important that candidates read the question carefully, and understand which distribution is being considered.

Answer: 0.034
No; n is large; The CLT applies (sample mean approx. normally distributed)

Question 3

Part **(i)** was particularly well attempted, with very few candidates, this time, confusing the two formulae for the unbiased estimate of the population variance, and very few candidates merely calculating the biased variance. The 98% confidence interval was also well attempted, with just a small number of candidates using an incorrect *z* value.

Answer: 57, 4.41
56.4 to 57.6

Question 4

Part **(i)** was well attempted, though it is important that when an answer is given candidates work towards the answer legitimately with all necessary working shown and no errors seen. Some candidates lost marks due to omission of essential brackets in their working out.

Part **(ii)** was not as well attempted. The solution led to a quadratic equation in m (the median) which many candidates were unable to solve correctly. Once solved, it was necessary to reject one solution, which again was not always successfully done.

Answer: 1.42

Question 5

Candidates were required to approximate the given binomial distribution to a Poisson (Po(1.6)). Some candidates worked with the original binomial, not using an approximation at all, whilst some incorrectly used a normal approximation. It is important that candidates recognise the conditions for applying appropriate approximating distributions. Many marks can be lost in these situations. Candidates who successfully used Po(1.6) generally found the required probability P(X>3), though errors such as calculating P(X≥3) or P(X=3) were seen. As the answer was 0.0788 to 3.s.f., a common error made by some candidates was to give the answer as 0.079, possibly confusing 3.d.p with the 3.s.f. required.

Part **(ii)** could have been answered using either the Poisson approximating distribution, or using the original binomial. In both methods the solution required the use of logs and this was generally recognised and well attempted. Use of inequalities was not always well applied with many candidates therefore incorrectly choosing the smallest possible value of *n*.

Answer: 0.0788
         7490

Question 6

This question was well attempted by a large number of candidates. A common error in part **(i)** was to calculate the variance incorrectly (finding 9^2 x 49 + 7^2 x 25 rather than the correct calculation of 9 x 49 + 7 x 25) and often candidates calculated the wrong tail probability.

Similarly in part **(ii)** there were many correct solutions; similar errors, as noted above, were seen on occasions.

As in **Question 5**, the answer for part **(ii)** caused a loss of marks for some candidates who did not give full 3.s.f. accuracy of 0.0815 but rounded to 0.082 (another possible confusion with 3.d.p.)

In general this question was a good source of marks for many candidates.

Answer: 0.927
         0.0815

Question 7

This question was particularly poorly attempted. The first 4 marks required an understanding, in context, of practicalities in sampling techniques, hypothesis testing and a Type I error. Not all candidates were able to answer in context and show a full understanding of the given situation.

Finding the probability of a Type I error also caused difficulties for many candidates. The question stated that a binomial distribution should be used, but there were cases where candidates assumed that, as it was a significance test, standardising was needed. As in **Question 5**, it is important that candidates read the question, and use the appropriate distribution as otherwise a substantial loss of marks could be incurred. Many others used the correct B(30,0.2) but calculated only P($X \le 6$) or spoiled an otherwise correct approach by describing the critical region as P($X \le 2$). When finding the critical region, it is important that all steps are completed to fully justify the answer.

Part **(vi)** was also quite poorly attempted, even by those who identified a critical region. Many failed to connect part **(v)** to part **(vi)** and simply compared 0.1 to 0.2. A valid comparison (of 3 with the critical region found in **(iv)**, or of P($X \le 3$) with 0.05) was required in order to justify the conclusion of the test. No marks were available unless this justification was evident.

Answer: Probability could be different later in the day, or on a different day
Looking for a decrease H_0 p=0.2 H_1 p <0.2
Concluding that the probability has decreased when it has not
CR $X \le 2$ 0.0442
No evidence that p has decreased

MATHEMATICS

| Paper 9709/72 |
| Paper 72 |

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. There was a complete range of scripts from good ones to poor ones. In general, candidates scored well on **Questions 2(i), 3 and 6** whilst **Question 7** proved particularly demanding.

Most candidates kept to the required level of accuracy; there were a few cases where candidates lost marks for giving final answers to less than three significant figure accuracy, this was particularly seen on **Question 5(i)** and **6(ii)** where errors could have been made because of confusion between 3s.f. and 3d.p. (see comments below).

Timing did not appear to be a problem for candidates.

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Many candidates attempted to find the mean number of failures over the 6 month period (1.75). It was then required to find the probability of more than 1 and less than 4 errors (i.e. 2 or 3 errors). A large number of candidates successfully used a correct expression, though some included 1 and/or 4 in their Poisson expression, and some attempted incorrect calculations such as $P(>1) \times P(<4)$ or $P(>1) + P(<4)$.

Answer: 0.421

Question 2

Part **(i)** was well attempted. Most candidates successfully used $N(29, 6^2/120)$ to find the required probability.

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Answer: 0.034
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Question 3

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Answer: 0.0788
        7490

Question 6

This question was well attempted by a large number of candidates. A common error in part **(i)** was to calculate the variance incorrectly (finding $9^2 \times 49 + 7^2 \times 25$ rather than the correct calculation of $9 \times 49 + 7 \times 25$) and often candidates calculated the wrong tail probability.

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As in **Question 5**, the answer for part **(ii)** caused a loss of marks for some candidates who did not give full 3.s.f. accuracy of 0.0815 but rounded to 0.082 (another possible confusion with 3.d.p.)

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Finding the probability of a Type I error also caused difficulties for many candidates. The question stated that a binomial distribution should be used, but there were cases where candidates assumed that, as it was a significance test, standardising was needed. As in **Question 5**, it is important that candidates read the question, and use the appropriate distribution as otherwise a substantial loss of marks could be incurred. Many others used the correct B(30,0.2) but calculated only P($X \leq 6$) or spoiled an otherwise correct approach by describing the critical region as P($X \leq 2$). When finding the critical region, it is important that all steps are completed to fully justify the answer.

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Looking for a decrease H_0 p=0.2 H_1 p <0.2
Concluding that the probability has decreased when it has not
CR $X \leq 2$ 0.0442
No evidence that p has decreased

MATHEMATICS

Paper 9709/73
Paper 73

Key Messages

When finding the probability of a Type II error it is necessary to establish the acceptance region for H_0 in the context of the question. Thus for **Question 6(ii)** it was necessary to find the region x < 2.638 before continuing.

Final answers to questions are usually required to 3 significant figures. To achieve this it is sometimes necessary to work with more figures during the calculations in a question. These can be retained on the calculator.

In **Question 5(a)** more than 3 significant figures were required in the working in order to obtain the final answer.

In **Question 7(ii)** it was necessary to use the mean of X as 2.087 in order to obtain the final answer accurately.

General Comments

Many candidates showed a sound understanding of the syllabus and answered the questions in an efficient manner. In many cases work was well presented and methods were clear to follow. To assist this many candidates used a sensible amount of space in which to present their solutions.

Many candidates scored well on **Questions 1(ii)**, **4(i)**, **4(iii)**, **6(i)** and **7(i)** whilst some candidates found **Questions 2**, **3**, **5(a)** and **6(ii)** more demanding.

Most candidates attempted all of the questions in the available time.

Comments on Specific Questions

Question 1

(i)   To state the distribution of the sample mean fully, three properties were required. These were that the distribution was normal, the mean was 352 and the variance was 2.9.

Many candidates stated some of these, but not all three. Some candidates gave incorrect values such as N (3520, 290). Also the incorrect value 29/√10 was often seen.

Answers: normal   352   2.9

(ii)   Many candidates answered this part correctly, including some candidates who had not gained full marks in part **(i)**. Some candidates applied a continuity correction, which was not appropriate. Other candidates gave their answer as 0.8798 instead of finding the upper tail of the normal distribution. A diagram could help with this. Candidates who followed through from an incorrect value for the variance from part **(i)** could gain two method marks.

Answer: 0.120

Question 2

The question indicated that a normal approximation to the Poisson distribution should be used. It was necessary to realise that this was N(λ, λ). Many candidates did so correctly, or implied this in their standardisation equation. This equation required the use of $z = 2.326$ from the given probability value (0.01). Also the standardisation introduced the term $\sqrt{\lambda}$ and hence the quadratic equation in $\sqrt{\lambda}$. For this normal approximation the continuity correction (55.5) was also required.

Some candidates omitted this (using 55 itself) or used the incorrect value 54.5.

Some candidates solved the equation correctly and selected the single relevant value (40.7). Some candidates gave both quadratic equation answers, instead of rejecting the value 75.7 which was greater than 55 and came from the negative root for $\sqrt{\lambda}$.

Other candidates tried invalid methods of solution of their equation such as squaring individual terms, not whole expressions.

Other candidates briefly tried a Poisson calculation instead of the normal approximation. This gained no credit.

Answer: 40.7

Question 3

(i)   The limits (0.284 and 0.516) of the confidence interval were given. These values indicated that the proportion of people in the sample owning a bicycle was 0.4 as this was the mid value.

Sometimes this was obtained directly. Sometimes a longer approach was used, involving the full expressions for the confidence limit terms. Such expressions could then be used in part **(ii).**

Many candidates incorrectly gave 0.232 (the width) as the proportion.

Answer: 0.4

(ii)   As the limits of the confidence interval were given, the confidence interval Level α% could now be found. Various similar approaches could be followed. These all required $\sqrt{(pq/n)}$ with p=0.4 and q=0.6 and n=65. Candidates using an incorrect value of p could earn method marks.

The approaches were 0.4 + $z \times \sqrt{(\ )}$ = 0.516 or 0.4 – $z \times \sqrt{(\ )}$ = 0.284 using the given end values, or 2 $z \times \sqrt{(\ )}$ = 0.232 or $z \times \sqrt{(\ )}$ = 0.116 using the width or half width.

Some candidates used one of these methods correctly. Other candidates mixed up the methods or omitted the square root or omitted n.

Having found z and the corresponding φ value (0.9718), a complete method to find α was required. Again different approaches could be followed.

The approaches were $(2\varphi – 1) \times 100$ or $2(\varphi – 0.5) \times 100$ or $(1 – 2\times(1 – \varphi)) \times 100$.

Many candidates did not follow one of these full methods.

Answer: 94

Question 4

(i)   Most candidates answered this part very well, integrating correctly, using the limits, equating the expression to 1 (essential) and calculating exactly to obtain the exact value of *k*.

Answer: 3/32

(ii) Some candidates sketched a suitable curve with the correct shape and with the limits shown.

Many candidates made errors in one or more of the properties of the curve. For example, by not drawing an inverted parabola, or by omitting the x-limits (2 and –2), or by extending the parabola beyond these limits (f(x) was defined as 0 outside the range from –2 to 2).

It was not necessary to use graph paper to draw the sketch.

Using the symmetry of the curve, the value of the expectation E(X) could be written straight down. Some candidates did this correctly. Other candidates used integration of xf(x), taking more time here, though they could still score the mark. A few candidates incorrectly tried (3/32) x 4 = 3/8.

Answers: inverted parabola between –2 and 2 only   E(X) = 0

(iii) Many candidates realised that they needed to integrate f(x) and did so correctly. Some of these candidates used the correct limits (–2 and 1) and calculated correctly. Other candidates used incorrect limits, such as –2 and 0. Some candidates found the variance and then attempted to use a normal distribution. This was not a valid method.

Answer: 27/32   or   0.844

Question 5

(a) The initial distribution for throwing a 6 with the die was B(450, 1/100).

As n = 450 (> 50) and np = 4.5 (< 5), the approximating distribution was P_o(4.5).

To find the percentage error it was necessary to find the probability of no 6s in both distributions.

Thus $e^{-4.5}$ and $(99/100)^{450}$ were needed. Many candidates found one of these probabilities (often the Poisson probability), but not both, thereby gaining some of the marks. Even those candidates who found both probabilities needed to retain several significant figures of accuracy in their answers to be able to perform the final calculation accurately. Some candidates did so. Other candidates prematurely rounded and so did not obtain the final answer.

Many candidates attempted to use a normal distribution. This was not a valid method.

Answer: 2.29 %

(b) Some candidates produced very clear efficient solutions for this hypothesis test. They stated the null and alternative hypotheses in terms of p (H$_0$: p = 1/6 and H$_1$: p < 1/6), noticed and used the binomial distribution (B(25, 1/6) to calculate the sum of the probabilities of the three terms in the tail (P(0,1,2)), compared this sum (0.189) with 0.10 (for the given significance Level 10%) and stated the conclusion appropriately.

Other candidates made various errors in this process. For example when an incorrect binomial distribution (with p = 1/25 or p = 0.1 or p = 0.08) was used together with incorrect hypotheses.

Those candidates who omitted the P(2) term in their probability sum (so found P(0, 1)) could score method marks and follow through to their (often reverse) conclusion.

A particular error seen was to calculate only P(2) instead of P(0,1,2). This was not a valid method and could not score the remaining marks.

Another error seen was to try to compare a binomial probability with a normal distribution value such as 1.282.

Many candidates attempted to use a normal distribution. This was not a valid method.

An alternative valid method using the binomial distribution was to find the critical region (0,1) and to compare the test result (2) to this region to reach the conclusion. This approach still requires the probability sum (0.189) to be found. Also the calculations for this sum and the comparison of 2 with the critical region do need to be written down to show the method.

Many candidates did write their conclusion in suitable form (e.g. "no evidence to suggest that the die was biased" or "no evidence to suggest that the claim was justified").

Answer: no reason to believe that the die was biased

Question 6

(i)   Many candidates presented perfect solutions for this hypothesis test. The null and alternative hypotheses were stated in terms of μ ($H_0 : \mu = 2.60$ and $H_1 : \mu > 2.60$). The normal distribution of means of samples N(2.60, $0.2^2/75$) was used correctly and 1.732 was compared to 1.645 and the conclusion stated appropriately. Alternatively the comparison could be between the probabilities (0.0416 and 0.05). A few candidates made the wrong conclusion after correct work. Some candidates attempted to find x_{crit} by using 1.645 in the above normal distribution. Because of the closeness of the values, this critical value needed to be found to at least 2.638 so that a comparison between 2.64 and this value could be made. Some candidates worked to this accuracy. Other candidates used fewer figures and could not compare properly.

Answer: there was evidence that μ had increased

(ii)   To find the probability of a Type II error, it was necessary first to establish the acceptance region when $\mu = 2.60$ for the parcels. Using 1.645 gave $x < 2.638$ for this region. Many candidates did not find this. Instead they used the value 2.64 given in part (i).

Given that the value of μ had become 2.68 and that the standard deviation of W was unchanged, the critical value 2.638 could then be standardised in N(2.68, $0.2^2/75$). This gave $z = -1.819$ and hence the probability that $x < 2.638$ could be found.

Some candidates did attempt some of this work and did gain some marks.

To decide on the probability areas for acceptance / rejection, diagrams could be helpful – one with mean 2.60 and another with mean 2.68. Other candidates combined these diagrams with some success. Some candidates gave some of the above work and then ended with a comparison and a conclusion as though performing a hypothesis test.

Answer: 0.0345   or   0.0344

Question 7

(i)   The majority of candidates substituted correctly in the formulae and found the correct values for the unbiased estimates of μ and σ^2 to at least 3 significant figures. More significant figures were required for the work in the next part of the question. These could be retained on the calculator.

A few candidates made errors by mixing up the alternative forms of the formula for the variance.

Very few candidates gave only the biased sample variance (0.000131) as their answer.

Answers: 2.087   (2.09 accepted here)   0.000132

(ii) Many candidates realised that it was necessary to consider the distribution $Y - X$. They found the mean (0.033) and variance (0.00027632...) and standardised the limiting value 0.01 with these to obtain $z = -1.384$ which then gave the required probability. Many candidates dealt well with the decimal values involved in this question and followed the above process carefully.

Most candidates correctly added the variances. Only a few candidates subtracted them.

Some candidates did not use sufficient significant figures in this work, making their final answer inaccurate. For example the use of 2.09 as the mean of X gave $E(Y - X) = 0.03$ which was not accurate enough to obtain the final answer.

Some candidates used the distribution $Y - X - 0.01$ which was a sound approach, requiring the standardisation of 0 with $N(0.023, 0.00027632)$. Here also the mean needed to be 0.023 not just 0.02 in order to have sufficient accuracy.

Answer: 0.0832