UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2009 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/72   Paper 72, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

UNIVERSITY *of* CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M   Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B   Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
                    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF   Any Equivalent Form (of answer is equally acceptable)

AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD   Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO   Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO   Correct Working Only – often written by a 'fortuitous' answer

ISW   Ignore Subsequent Working

MR   Misread

PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS   See Other Solution (the candidate makes a better attempt at the same question)

SR   Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1   A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1   This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	(i)	doubling only gives even numbers so odd numbers not included	B1 [1]	Needs to be aware that odd numbers aren't included
	(ii)	98% CI = $1.62 \pm 2.326 \times \dfrac{\sqrt{0.0052}}{\sqrt{5}}$ $= 1.62 \pm 0.0750$ $= (1.54, 1.70)$	M1 B1 A1 [3]	Correct shape with $\sqrt{5}$ seen in denom and their evaluated mean and sd (condone unbiased estimate of sample variance). 2.326 seen correct answer, cwo. Accept 1.55 and 1.70 .
2	(i)	$\lambda = 6.35$ $P(>1) = 1 - e^{-6.35}(1 + 6.35)$ $= 0.987$	M1 A1 [2]	Attempt at Poisson, $1 - (P(0) + P(1))$ with their λ. Accept $1 - P(0)$. Correct answer.
	(ii)	$\lambda = 889$ $P(>850) = 1 - \Phi\left(\dfrac{850.5 - 889}{\sqrt{889}}\right)$ $= \Phi(1.29)$ or $\Phi(1.31)$ with no cc $= 0.902$	B1 M1* M1*dep A1 [4]	Correct mean Standardising, with or without continuity correction using their N(889, 889). Condone sd/var mixes. Correct area (> 0.5) with or without cc Correct answer
3	(i)	Poisson mean 4.26 P(overbooked) = P(0, 1, 2 not arrive) $= e^{-4.26}\left(1 + 4.26 + \dfrac{4.26^2}{2}\right)$ $= 0.202$	B1 M1 A1 ft A1 [4]	Correct mean Poisson attempt at P(0,1,2), their 4.26, allow end errors. Correct unsimplified answer, ft their mean. Correct answer, as final answer.
	(ii)	mean = 135/75 = 1.8 Σ (Poissons) new mean = 4.26 + 1.8 = 6.06 $P(5) = e^{-6.06}\left(\dfrac{6.06^5}{5!}\right)$ $= 0.159$	M1 M1 A1 [3]	Adding their two means. Attempt at Poisson P(5) with their mean. Correct answer

(i)	SR Normal	B1	N(4.26, 4.17(5))	or	N(208.74 (209), 4.17(5))
		B1	Prob = 0.195		
	SR Binomial	B(213 , 1/50)	B1	correct unsimplified expression for P(0,1,2)	
			B1	Prob = 0.200 or 0.199 .	
(ii)	SR Binomials	M2	all cases of binomial products		
		A1	prob = 0.160		

4	(i)	$P(0) = 0.5^{10} = 0.000976 = P(10)$ $P(1) = {}^{10}C_1 \times 0.5^1 \times 0.5^9 = 0.00976 = P(9)$ Σ 4 probs = 0.02147 < 10% $P(2) = {}^{10}C_2 \times 0.5^2 \times 0.5^8 = 0.0439 = P(8)$ Σ 6 probs = 0.1093 > 10% Values of h are 2, 3, 4, 5, 6, 7, 8	M1 M1 M1 A1 **[4]**	Evaluating two Binomial end probs of 0 and 1, (or 10 and 9). Summing either/both ends and comparing to 10% (doesn't have to be two-tailed here). Summing a third/last end probability and comparing to 10% (needn't be two-tailed here) Correct answer only. A1 dep on all 3 Ms. No errors seen.
	(ii)	$P(\text{Type II error}) =$ $P(2, 3, 4, 5, 6, 7, 8) \; (= 1 - P(0, 1, 9, 10))$ $= 1 - \left(\begin{array}{c} (0.3)^{10} + {}^{10}C_1(0.7)(0.3)^9 \\ + {}^{10}C_9(0.7)^9(0.3) + (0.7)^{10} \end{array} \right)$ $= 1 - 0.149$ $= 0.851$	B1ft M1 A1 **[3]**	Identifying correct probs ft their ans **(i)** Binomial expression for their P(Type II error) with powers summing to 10, ${}^{10}C$ somethings and 0.3 and 0.7 seen. Correct final answer.
5	(i)	$H_0: \mu = 750$ $H_1: \mu < 750$ Test statistic $z = \dfrac{746 - 750}{(11/\sqrt{20})} = +/-1.626$ CV $z = -1.751$ or -1.752 Not enough evidence to say mean is less.	B1 B1 M1 A1ft **[4]**	H_0 and H_1 correct Correct test statistic used in comparison. oe. Correct CV and valid comparison. oe. [0.0520 or 745.7] [2 tail 2.054/5] Correct conclusion ft their test statistic. No contradictions. Condone cc's.
	(ii)	$z = +/-1.881$ or $+/-1.882$ $-1.881 > \dfrac{745 - 750}{(11/\sqrt{n})}$ $n = 18$	B1 M1* M1*dep A1 **[4]**	Accept 1.881 or 1.882 Equation or inequality relating their z to standardised value. Condone cc's. Solving attempt for n Correct answer. cwo. 3 sf accuracy.

6	(i)	$\int_1^2 \dfrac{x}{3}(k-x)\,dx = 1$	M1	Equating to 1 and attempting to integrate, ignore limits here.
		$\left[\dfrac{kx^2}{6} - \dfrac{x^3}{9}\right]_1^2 = 1$	A1	Correct integration and correct limits.
		$\left[\dfrac{4k}{6} - \dfrac{8}{9}\right] - \left[\dfrac{k}{6} - \dfrac{1}{9}\right] = 1$		
		$k = 32/9$ AG	A1 **[3]**	32/9 correctly obtained.
	(ii)	$E(X) = \int_1^2 \dfrac{x^2}{3}(k-x)\,dx$	M1	attempting to evaluate $\int_1^2 \dfrac{x^2}{3}(k-x)\,dx$
		$= \left[\dfrac{kx^3}{9} - \dfrac{x^4}{12}\right]_1^2$		
		$= 1.52$ (491/324)	A1 **[2]**	correct answer.
	(iii)	$\int_1^{1.5} \dfrac{x}{3}(k-x)\,dx = \left[\dfrac{kx^2}{6} - \dfrac{x^3}{9}\right]_1^{1.5}$	M1	attempt to integrate using 1 and 1.5 or 1.5 and 2
		$= 0.477$ (or 0.523)	A1	
		this is < 0.5 so 1.5 is $<$ median	A1ft **[3]**	correct reasoning and conclusion, ft their integration
		Alternative method	M1	$\int f(x)\,dx$ limits 1 to m (or m to 2) and equate to 0.5 attempted.
			A1	Correct cubic in m in simplified form (no k's).
			A1	$m = 1.52(2)$, so median > 1.5, or, 2 trials showing that median > 1.5.
7	(a)	$\text{Var}(Y) = b^2\text{Var}(X) = 4\text{Var}(X)$ $b = 2\ (> 0)$	B1	Correct answer
		$E(Y) = a + 2E(X) = E(X) + 0.8$ $a + 14.24 = 7.92$ $a = -6.32$	M1 A1 **[3]**	Using $E(Y) = a + 2E(X)$, their "2". Correct equation in a, b subst in, using 0.8 correctly. Correct answer
	(b) (i)	$R + S \sim N(3\mu, 13)$ $z = -1.282$ or -1.281	B1 B1	Correct mean and variance. Correct z, must be $-$ve ($-$ve implied).
		$1.282 = \dfrac{3\mu - 1}{\sqrt{13}}$	M1	Standardising and solving, their mean, z, 13 (but combined), allow "13" or "$\sqrt{13}$".
		$\mu = 1.87$	A1 **[4]**	Correct answer.
	(ii)	$S - R \sim N(\mu, 13)$	B1 ft	Correct mean and variance, ft their mean.
		$P[(S - R) > 0] = P\left(z > \dfrac{0 - \mu}{\sqrt{13}}\right)$	M1	Standardising and correct area (i.e. > 0.5 for correct), but consistent with their values.
		$= \Phi(0.5197)$ $= 0.699$	A1 **[3]**	Accept 0.698